COS 487: Theory of Computation **Fall 2008**

Assignment #3

Due: Tuesday, November 04 *Sanjeev Arora*

1. Show that the Post Correspondence Problem is decidable over the unary alphabet.

2. Let $J = \{w \mid \text{either } w = 0x \text{ for some } x \in A_{TM}, \text{ or } w = 1y \text{ for some } y \in \overline{A_{TM}}\}$. Show that neither J nor $\overline{J}$ is Turing recognizable.

3. Suppose the Church-Turing thesis is false and some advanced civilization has figured out how to solve the halting problem for TMs. They wish to send a brief message to the rest of universe allowing living beings everywhere to solve the halting problem. Imagine that the set of inputs for the halting problem are numbered $1, 2, 3, \ldots$ and everybody agrees about this numbering. Show that the length of the message needed to solve the halting problem on the first i inputs is only $\log i$.

4. (Rice's Theorem) Let P be a non-trivial property of the language of a Turing machine. Prove that the property of determining whether a TM's language has property P is undecidable.

More formally, let P be a language consisting of TM descriptions, where P fulfills two conditions. First, P is nontrivial – it contains some, but not all TM descriptions. Second, P is a property of the TM's *language* – if $L(M_1) = L(M_2)$ then $\langle M_1 \rangle \in P$ iff $\langle M_2 \rangle \in P$. Prove that P is an undecidable language.

(We will cover this in some detail in the precept)

5. Using Rice's theorem, prove the undecidability of the following languages.

1. $INFINITE_{TM} = \{\langle M \rangle \mid M \text{ is a TM and } L(M) \text{ is an infinite language}\}$.
2. $\{\langle M \rangle \mid M \text{ is a TM and } 1011 \in L(M)\}$.
3. $ALL_{TM} = \{\langle M \rangle \mid M \text{ is a TM and } L(M) = \Sigma^*\}$.