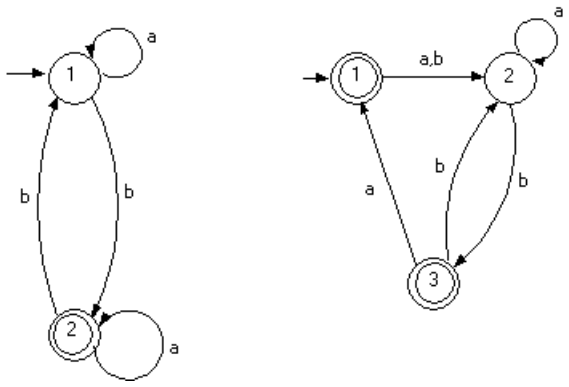
COS 487: Theory of Computation **Fall 2008**

Assignment #1

Due: Tuesday, September 30 *Sanjeev Arora*

1. **(a)** Give an NFA that recognizes the language $(01 \cup 001 \cup 010)^*$.
(b) Convert this NFA into an equivalent DFA.

2. Convert the following automata into regular expressions.



3. Prove that for every $k > 1$ a language $A_k \subseteq \{0,1\}^*$ exists that can be recognized by a DFA with k states but not by one with only $k-1$ states.

4. Let $\Sigma = \{0, 1, +, =\}$ and

$$ADD = \{x = y + z \mid x, y, z \text{ are binary integers, and } x \text{ is the sum of } y \text{ and } z\}.$$

Show that ADD is not regular.

5. Let $\Sigma = \{0, 1\}$ and let

$$D = \{w \mid w \text{ contains an equal number of occurances of the substrings 01 and 10}\}.$$

Thus $101 \in D$ because it contains a single 01 and a single 10 but $1010 \notin D$ because it contains two 10's and one 01. Show that D is a regular language.

6. Let $\Sigma = \{a, b\}$. Given a k, let C_k denote the language with an a in the kth position from last. More precisely, $C_k = \Sigma^* a \Sigma^{k-1}$.
(a) Describe an NFA with $k+1$ states that recognizes C_k.
(b) Show that any DFA that recognizes C_k has at least 2^k states.
(*Note:* This shows that in the worst case an exponential blow-up is needed in converting an NFA to a DFA).

7. (a) Let A be an infinite regular language. Prove that A can be split into two infinite disjoint, nonempty, regular subsets.

(b) Let B and D be two languages. Write $B \sqsubset D$ if $B \subseteq D$ and D contains infinitely many strings that are not in B. Show that if B and D are two regular languages with $B \sqsubset D$ then we can find a regular language C such that $B \sqsubset C \sqsubset D$.

8. If A is any language, let

$$A_{\frac{1}{2}\_} = \{x \mid \text{for some } y, |x| = |y| \text{ and } xy \in A\}.$$

Show that if A is regular, then so is $A_{\frac{1}{2}\_}$.

9. Give context-free grammars generating the following languages.
(a) The set of strings over the alphabet $\{a, b\}$ with more a's than b's.
(b) The complement of the language $\{a^n b^n \mid n \geq 0\}$.
(c) Give informal descriptions of PDA's for the languages above.

10. Let $G = (V, \Sigma, R, S)$ be the following grammar. $V = \{S, T, U\}$; $\Sigma = \{0, *\}$; and R is the set of rules:

$$\begin{aligned} S &\to TT \mid U \\ T &\to 0T \mid T0 \mid * \\ U &\to 0U00 \mid * \end{aligned}$$

(a) Describe $L(G)$ in English.
(b) Prove that $L(G)$ is not regular.