Tries

- tries
- TSTs
- applications

References:
Algorithms in Java, Chapter 15
http://www.cs.princeton.edu/algs4/62trie

Review: summary of the performance of symbol-table implementations

Frequency of operations.

implementation	typical case			ordered iteration?	operations on keys
	search	insert	delete		
BST	1.39 lg N	1.39 lg N	?	yes	`compareTo()`
red-black tree	1.00 lg N	1.00 lg N	1.00 lg N	yes	`compareTo()`
hashing	1 †	1 †	1 †	no	`equals()` `hashcode()`

† under uniform hashing assumption

Q. Can we do better?
A. Yes, if we can avoid examining the entire key, as with radix sorting.

Digital keys (review)

Digital key. Sequence of digits over fixed alphabet.
Radix. Number of digits in alphabet.

Applications.
- DNA: sequence of a, c, g, t.
- IPv6 address: sequence of 128 bits.
- ASCII: sequence of 7-bit characters.
- Protein: sequence of amino acids A, C, ..., Y.
- English words: sequence of lowercase letters.
- Credit card number: sequence of 16 decimal digits.
- International words: sequence of Unicode characters.
- Library call numbers: sequence of letters, numbers, periods.

This lecture. `String` of ASCII characters.

String set API

String set. Collection of distinct strings.

`public class StringSET`

	`StringSET()`	*create an empty set of strings*
`void`	`add(String key)`	*add a string to the set*
`boolean`	`contains(String key)`	*is key in the set?*

```
StringSET set = new StringSET();
while (!StdIn.isEmpty())
{
   String key = StdIn.readString();
   if (!set.contains(key))
   {
      set.add(key);
      StdOut.println(key);
   }
}
```
dedup client

Remark. Same idea extends to `StringST`.

String set implementations cost summary

	typical case			dedup	
implementation	search hit	insert	space	`moby.txt`	`actors.txt`
red-black	L + log N	log N	C	1.40	97.4
hashing	L	L	C	0.76	40.6

Parameters
- N = number of strings
- L = length of string
- C = number of characters in input
- R = radix

file	size	words	distinct
`moby.txt`	1.2 MB	210 K	32 K
`actors.txt`	82 MB	11.4 M	900 K

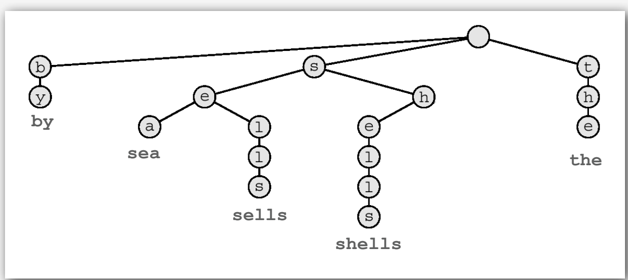
Challenge. Efficient performance for long keys (large L).

- tries
- TSTs
- applications

Tries

Tries. [from retrieval, but pronounced "try"]
- Store characters in nodes, not keys.
- Use the characters of the key to guide the search.
- Don't need to explicitly store key!

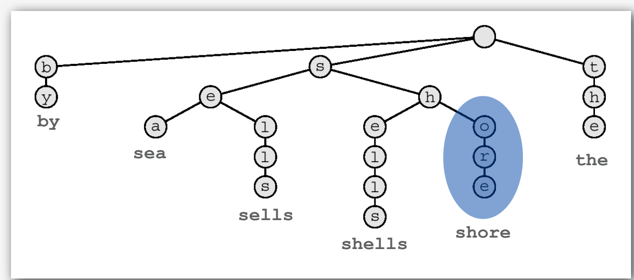
Ex. **`sells sea shells by the sea`**



Tries

Tries. [from retrieval, but pronounced "try"]
- Store characters in nodes, not keys.
- Use the characters of the key to guide the search.
- Don't need to explicitly store key!

Ex. **`sells sea shells by the sea shore`**



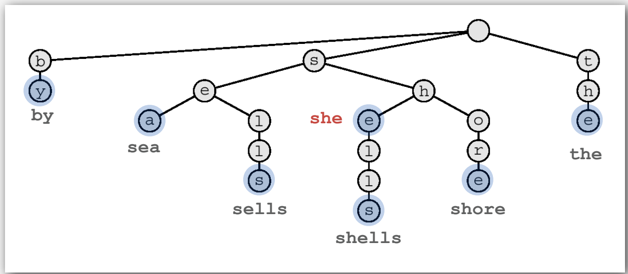
Tries

Q. How to handle case when one key is a prefix of another?

A1. Append sentinel character `'\0'` to every key so it never happens.

A2. Store extra bit to denote which nodes correspond to key ends.

Ex. `she sells sea shells by the sea shore`

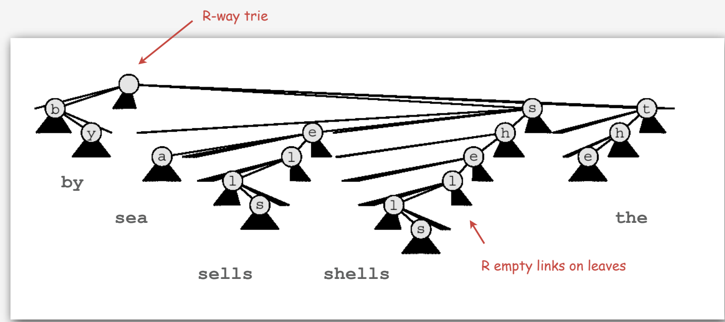


R-way trie

Q. How to branch to next level?

A. One link for each possible character.

Ex. `sells sea shells by the sea`



R-way trie

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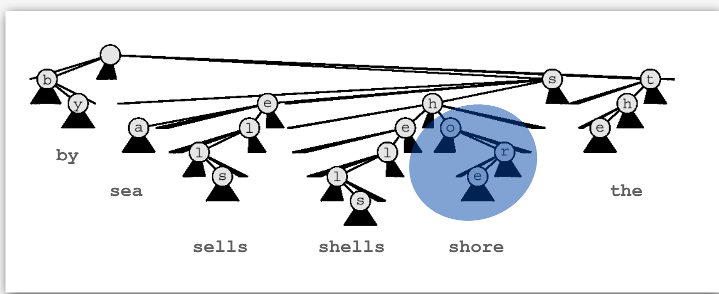
A. One link for each possible character.

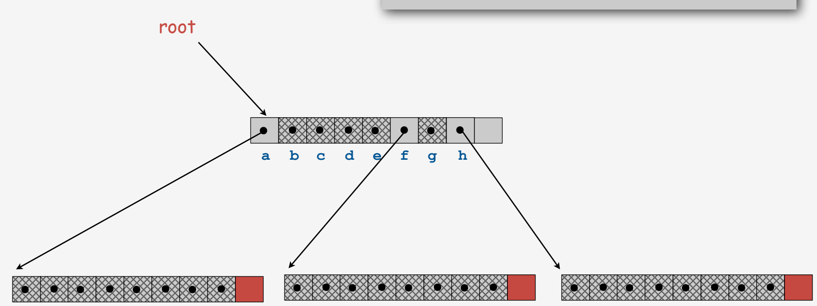
Ex. `sells sea shells by the sea shore`



R-way trie: Java implementation

Node. References to R nodes.

```
private class Node
{
   private Node[] next = new Node[R];
   private boolean end;
}
```



8-way trie that represents { `a, f, h` }

R-way trie: Java implementation

```
public class StringSET
{
   private static final int R = 128;
   private Node root = new Node();

   private class Node
   {
      private Node[] next = new Node[R];
      private boolean end;
   }

   public boolean contains(String s)
   {  return contains(root, s, 0);  }

   private boolean contains(Node x, String s, int d)
   {
      if (x == null) return false;
      if (d == s.length()) return x.end;
      char c = s.charAt(d);
      return contains(x.next[c], s, d+1);
   }
```

for ASCII strings

empty trie

current digit

R-way trie: Java implementation (cont)

```
public void add(String s)
{  root = add(root, s, 0);  }

private Node add(Node x, String s, int d)
{
   if (x == null) x = new Node();
   if (d == s.length()) x.end = true;
   else
   {
      char c = s.charAt(d);
      x.next[c] = add(x.next[c], s, d+1);
   }
   return x;
}
```

Q. How to implement delete?

Q. How to implement ST?

Sublinear search miss with R-way tries

Tries enable user to present string keys one char at a time.

Search miss.
- Could have mismatch on first character.
- Typical case: examine only a few characters.

Search hit.
- Need to examine all L characters for equality.
- Can present possible matches after a few characters.

auto-complete
(stay tuned)

Space.
- R null links at each leaf.
- Sublinear space possible if many short strings share common prefixes.

Bottom line. Fast search hit, sublinear-time search miss.

String set implementations cost summary

	typical case			dedup	
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red-black	L + log N	log N	C	1.40	97.4
hashing	L	L	C	0.76	40.6
R-way trie	L	L	RN + C	1.12	out of memory

R-way trie.
- Method of choice for small R.
- Too much memory for large R.

Challenge. Use less memory, e.g., 65,536-way trie for Unicode!

Digression: out of memory?

> *" 640 K ought to be enough for anybody. "*
> *— attributed to Bill Gates, 1981*
> *(commenting on the amount of RAM in personal computers)*

> *" 64 MB of RAM may limit performance of some Windows XP features; therefore, 128 MB or higher is recommended for best performance. "* *— Windows XP manual, 2002*

> *" 64 bit is coming to desktops, there is no doubt about that. But apart from Photoshop, I can't think of desktop applications where you would need more than 4GB of physical memory, which is what you have to have in order to benefit from this technology. Right now, it is costly. "* *— Bill Gates, 2003*

Digression: out of memory?

A short (approximate) history.

machine	year	address bits	addressable memory	typical actual memory	cost
PDP-8	1960s	12	6 KB	6 KB	\$16K
PDP-10	1970s	18	256 KB	256 KB	\$1M
IBM S/360	1970s	24	4 MB	512 KB	\$1M
VAX	1980s	32	4 GB	1 MB	\$1M
Pentium	1990s	32	4 GB	1 GB	\$1K
Xeon	2000s	64	enough	4 GB	\$100
??	future	128+	enough	enough	\$1

> *" 512-bit words ought to be enough for anybody. "*
> *— Kevin Wayne, 2003*

A modest proposal

Number of atoms in the universe (estimated). $\leq 2^{266}$.
Age of universe (estimated). 14 billion years ~ 2^{59} seconds $\leq 2^{89}$ nanoseconds.

Q. How many bits address every atom that ever existed?
A. Use a unique 512-bit address for every atom at every time quantum.

Observation. 512 bits ought to be enough.

266 bits	*89 bits*	*157 bits*
place	time	cushion for whatever

Use trie to map to current location.
- Represent location as 64 8-bit chars (512 bits).
- 256-way trie wastes 255/256 actual memory.
- Need better use of memory.

Ternary search tries

TST. [Bentley-Sedgewick, 1997]

- Store characters in nodes.
- Use the characters of the key to guide the search.
- Each node has three children: smaller (left), equal (middle), larger (right).

Ternary search tries

TST. [Bentley-Sedgewick, 1997]

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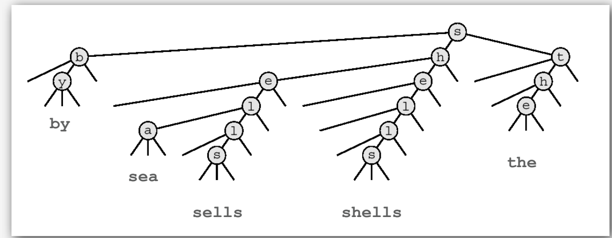
- Use the characters of the key to guide the search.
- Each node has three children: smaller (left), equal (middle), larger (right).

Ex. **sells sea shells by the sea**



Observation. Only three null links in leaves!

Ternary search tries

TST. [Bentley-Sedgewick, 1997]

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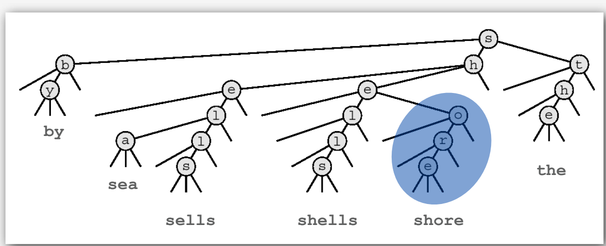
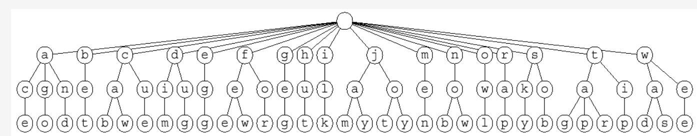


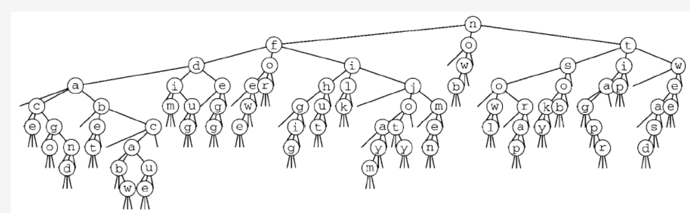
Observation. Only three null links in leaves!

26-Way Trie vs. TST

TST. Collapses empty links in 26-way trie.



26-way trie (1035 null links, not shown)



TST (155 null links)

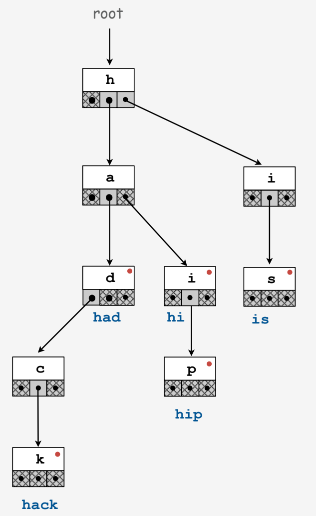
now
for
tip
ilk
dim
tag
jot
sob
nob
sky
hut
ace
bet
men
egg
few
jay
owl
joy
rap
gig
wee
was
cab
wad
caw
cue
fee
tap
ago
tar
jam
dug
and

TST representation in Java

A TST node is five fields:

- A character c.
- A reference to a left TST. [smaller]
- A reference to a middle TST. [equal]
- A reference to a right TST. [larger]
- A bit to indicate whether this node is the last character in some key.

```
private class Node
{
   private char c;
   private Node left, mid, right;
   private boolean end;
}
```



TST that, represents { **hack**, **had**, **hi**, **hip**, **is** }

TST: Java implementation

```
public class TST
{
   private class Node
   {  /* see previous slide */  }

   public boolean contains(String s)
   {  return contains(root, s, 0);  }

   private boolean contains(Node x, String s, int d)
   {
      if (x == null) return false;
      char c = s.charAt(d);
      if      (c < x.c)            return contains(x.left,  s, d);
      else if (c > x.c)            return contains(x.right, s, d);
      else if (d < s.length()-1)   return contains(x.mid,   s, d+1);
      else                         return x.end;
   }

   public void add(String s)
   {  /* see next slide */  }
}
```


TST: Java implementation (cont)

```
public void add(String s)
{  root = add(root, s, 0);  }

private Node add(Node x, String s, int d)
{
   char c = s.charAt(d);
   if (x == null) x = new Node(c);
   if      (c < x.c)            x.left  = add(x.left,  s, d);
   else if (c > x.c)            x.right = add(x.right, s, d);
   else if (d < s.length()-1)   x.mid   = add(x.mid,   s, d+1);
   else                         x.end = true;
   return x;
}
```


String set implementation cost summary

	typical case			dedup	
implementation	search hit	insert	space	moby.txt	actors.txt
red-black	L + log N	log N	C	1.40	97.4
hashing	L	L	C	0.76	40.6
R-way trie	L	L	RN + C	1.12	out of memory
TST	L + log N	L + log N	3N + C	0.72	38.7

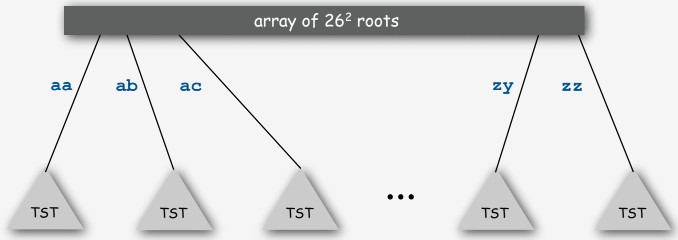
Remark. Can build balanced TSTs via rotations to achieve L + log N worst-case guarantees.

Bottom line. TST is as fast as hashing (for string keys), space efficient.

TST with R^2 branching at root

Hybrid of R-way and TST.

- Do R^2-way branching at root.
- Each of R^2 root nodes points to a TST.



Q. What about one- and two-letter word?

String set implementation cost summary

	typical case			dedup	
implementation	search hit	insert	space	moby.txt	actors.txt
red-black	L + log N	log N	C	1.40	97.4
hashing	L	L	C	0.76	40.6
R-way trie	L	L	RN + C	1.12	out of memory
TST	L + log N	L + log N	3N + C	0.72	38.7
TST with R^2	L + log N	L + log N	$3N + C + R^2$	0.51	32.7

Bonus. TST performance even better with nonuniform keys.

Ex. 5 times faster than hashing for library call numbers.

TST vs. hashing

Hashing.

- Need to examine entire key.
- Hits and misses cost about the same.
- Need good hash function for every key type.
- No help for ordered-key APIs.

TSTs.

- Works only for digital keys.
- Only examines just enough key characters.
- Search miss may only involve a few characters.
- Can handle ordered-key APIs.

Bottom line. TSTs are faster than hashing (especially for search misses) and more flexible than red-black trees (stay tuned).

- tries
- TSTs
- **applications**

Extending the **StringSET** API

Add. Insert a key.
Contains. Check if given key in the set.
Delete. Delete key from the set.
} `equals()`

Sort. Iterate over keys in ascending order.
Select. Find the k^{th} largest key.
Range search. Find all keys between k_1 and k_2.
} `compareTo()`

Longest prefix match. Find key with longest prefix match.
Wildcard match. Find key allowing wildcard characters.
Near neighbor search. Find keys that differ in ≤ P chars.
Autocomplete. Find all keys that start with given prefix.
} `charAt()`

Longest prefix match

Find string in set that is the longest prefix of given key.

Ex. Search IP database for longest prefix matching destination IP, and route packets accordingly.

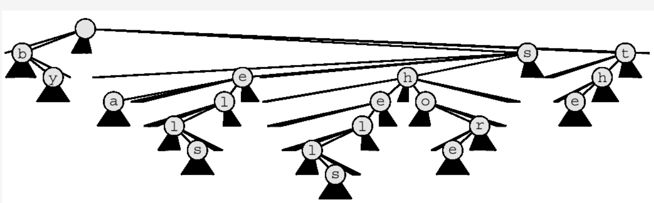
```
"128"
"128.112"          ← represented as 32-bit binary number
"128.112.055"        for IPv4 (instead of string)
"128.112.055.15"
"128.112.136"
"128.112.155.11"
"128.112.155.13"
"128.222"
"128.222.136"

prefix("128.112.136.11") = "128.112.136"
prefix("128.166.123.45") = "128"
```

Q. Why isn't longest prefix match the same as floor or ceiling?

Longest prefix match

Find string in set that is the longest prefix of given key.



R-way trie implementation of longest prefix match operation

Easy to implement for R-way trie (below) or TST (see book).

```
public String prefix(String s)
{
   int length = prefix(root, s, 0);
   return s.substring(0, length);
}

private int prefix(Node x, String s, int d)
{
   if (x == null) return 0;
   int length = 0;
   if (x.end) length = d;
   if (d == s.length()) return length;
   char c = s.charAt(d);
   return Math.max(length, prefix(x.next[c], s, d+1));
}
```

Wildcard match

Use wildcard . to match any character in alphabet.

```
coalizer
coberger
codifier
cofaster
cofather
cognizer
cohelper
colander
coleader
...
compiler
...
composer
computer
cowkeper
```

`co....er`

```
acresce
acroach
acuracy
octarch
science
scranch
scratch
scrauch
screich
scrinch
scritch
scrunch
scudick
scutock
```

`.c...c.`

Wildcard match: TST implementation

Search as usual if character is not a period;
go down all three branches if query character is a period.

```
public void wildcard(String s)
{  wildcard(root, s, 0, "");  }

private void wildcard(Node x, String s, int d, String prefix)
{
   if (x == null) return;
   char c = s.charAt(i);
   if (c == '.' || c <  x.c) wildcard(x.left, s, d, prefix);
   if (c == '.' || c == x.c)
   {
      if (i < s.length() - 1)
         wildcard(x.mid, s, d+1, prefix + x.c);
      else if (x.end)
         StdOut.println(prefix + x.c);
   }
   if (c == '.' || c >  x.c) wildcard(x.right, s, d, prefix);
}
```


T9 texting

Goal. Type text messages on a phone keypad.

Multi-tap input. Enter a letter by repeatedly pressing a key until the desired letter appears.

T9 text input. ["A much faster and more fun way to enter text."]

- Find all words that correspond to given sequence of numbers.
- Press 0 to see all completion options.

Ex. **hello**

- Multi-tap: 4 4 3 3 5 5 5 5 5 5 6 6 6
- T9: 4 3 5 5 6

www.t9.com

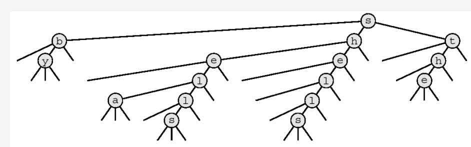
TST: collapsing 1-way branches

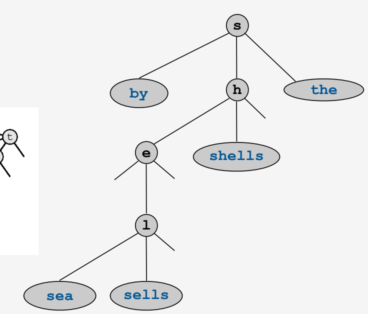
Collapsing 1-way branches at bottom.

- Internal node stores character; external node stores full key.
- Append sentinel character '\0' to every key.
- Search hit ends at leaf with given key.
- Search miss ends at null link or leaf with different key.

Collapsing interior 1-way branches.

- Keep char position in nodes.
- Need full compare at leaf.





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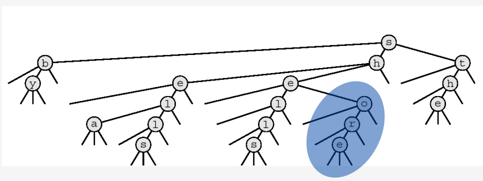

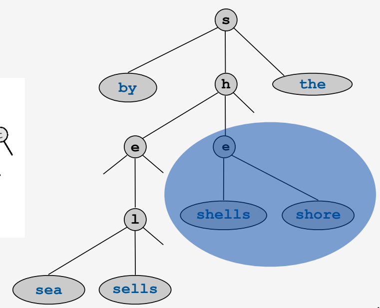

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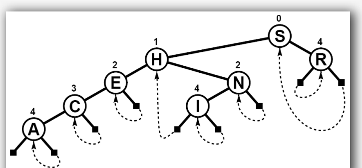
hashing	L	L	C
R-way trie	L	L	RN + C
TST	L + log N	L + log N	3N + C
TST with R^2	L + log N	L + log N	$3N + C + R^2$
R-way with no 1-way	L + $\log_R$ N	$\log_R$ N	RN + C
TST with no 1-way	L + log N	log N	C

Challenge met.
- Efficient performance for arbitrarily long keys.
- Search time is independent of key length!

A classic algorithm

Patricia tries. [Practical Algorithm to Retrieve Information Coded in Alphanumeric]
- Collapse one-way branches in binary trie.
- Thread trie to eliminate multiple node types.

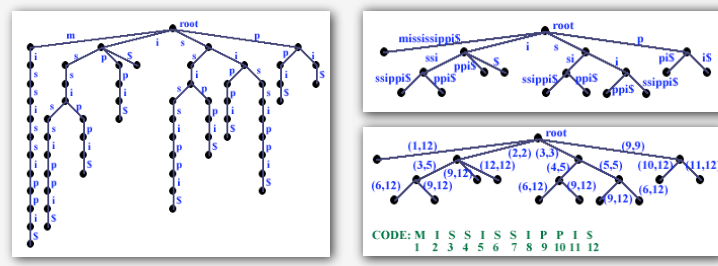


Applications.
- Database search.
- P2P network search.
- IP routing tables: find longest prefix match.
- Compressed quad-tree for N-body simulation.
- Efficiently storing and querying XML documents.

Beyond the scope of COS 226 (see Program 15.7).

Suffix tree

Suffix tree. Threaded trie with collapsed 1-way branching for string suffixes.



Applications.
- Linear-time longest repeated substring.
- Computational biology databases (BLAST, FASTA).

Beyond the scope of COS 226.

Symbol tables summary

A success story in algorithm design and analysis.

Red-black tree.
- Performance guarantee: log N key compares.
- Supports ordered symbol table API.

Hash tables.
- Performance guarantee: constant number of probes.
- Requires good hash function for key type.
- Enjoys systems support.

Tries. R-way, TST.
- Performance guarantee: log N characters accessed.
- Supports extensions to API based on partial keys.

Bottom line. You can get at anything by examining 50-100 bits (!!!)