HW 3: COS 597D, Thinking like a theorist

1. Show that a diagonal dominant matrix is positive semidefinite (i.e., all its eigenvalues are nonnegative). Exhibit a positive semidefinite matrix that is not diagonal dominant.

2. Compute all eigenvalues and eigenvectors of the boolean hypercube.

3. Suppose a d-regular graph on n vertices is an (α, β) expander where $\beta > d - 3$. Show that the subgraph induced on every subset of αn vertices is 3-colorable.