COS 528 notes

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1 Open Problems

1. top down approach for lowerbounds?
2. The cost of a find estimated to inverse akerman function. Could we have the cost of a find depending on the size of the step using top down?
3. What is the tight analysis
4. Can we simulate path compression on a balaned forest without using union by rank?

2 Segmented Path Compression

(Rodney Farrow 1976)

2.1 Rank Redefined

Rank is redefined in the following way: Let $rank(x) = r$ and $rank(y) = s$ We want to link node x to node y The rank of y after the link is s'

1. initial ranks are zero
2. if $r < s$ then $s' = s$
3. if $r > s$ then $s' = r$
4. if $r = s$ then $s' = s + 1 = r + 1$

In the segmented path probme any sequence of equal rank nodes constitute a path, not a tree.

2.2 Path Compression

Let the *position* of x be $t(x)$ Let the *steppe* of x be $s(x)$

Given a path of equal rank nodes, we number the nodes consecutively to get the *position* of the nodes. If we number the nodes along the path in the pattern $0, 1, 0, 2, 0, 1, 0, 3, 0, 1, ...$ this will give us the *steppe*.

A more formal way of defining *position*: if $r > s$ then $s' = r$ and $t(y) = t(x) + 1$ if $r = s$ then $s' = s + 1$ and $t(y) = 1$

A more formal way of defining *steppe*: $s(x) = \max i$ s.t. 2^i evenly divides $t(x)$

There is an implicit binary tree. The steppe gives us the level in this binary tree.

Let x be a node and y be a root. In path compression, we define the new parent of x as follows:

if $r(x) < r(y)$ OR $r(x) = r(y)$ AND all nodes after x have smaller steppe, then $p(x) = y$ else $p(x) =$ max steppe node after x

(prove $log * n$ amortized time for arbitrary sequence of links)