Radiometry and Light Transport

COS 526, Fall 2006

Overview

- Radiometry
- Local light transport
- Definition of BRDF
- BRDF properties and common BRDFs
- Rendering equation

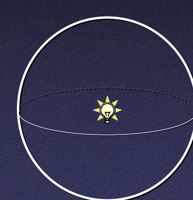
Radiometric Units

- Light is a form of energy
  - Measured in Joules (J)
- Power: energy per unit time
  - Measured in Joules/sec = Watts (W)
  - Also called Radiant Flux (Φ)

Isotropic Point Source

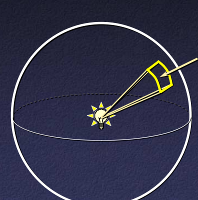
- Radiant flux leaves point source in all directions
- Flux distributed evenly over sphere

$$E = \frac{\Phi}{4\pi}$$



Point Light Source in a Direction

- How to define radiant flux for one direction?
  - Solid angle



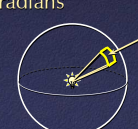
Solid angle $d\omega$

Digression – Solid Angle

- Angle in radians

Length l
Angle $\theta = l/r$

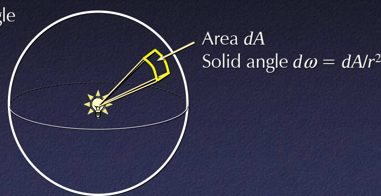
- Solid angle in steradians



Area A
Solid angle $\omega = A/r^2$

Point Light Source in a Direction

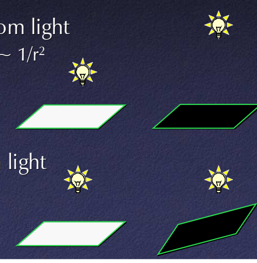
- How to define radiant flux for one direction?
  - Solid angle



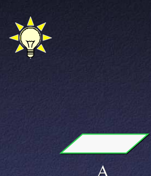
- *Irradiance* (E) = radiant flux per unit solid angle
  - Measured in Watts per steradian (W/sr)

Light Falling on a Surface from A Direction

- Power per unit area – *Irradiance* (E)
  - Measured in W/m²
- Move surface away from light
  - Inverse square law: $E \sim 1/r^2$
- Tilt surface away from light
  - Cosine law: $E \sim \mathbf{n} \cdot \mathbf{l}$

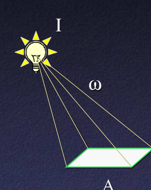


Light Falling on a Surface from A Direction



$$E = \frac{\Phi}{A}$$

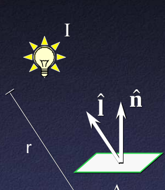
Light Falling on a Surface from A Direction



$$E = \frac{\Phi}{A}$$

$$\Phi = I\omega$$

Light Falling on a Surface from A Direction



$$E = \frac{\Phi}{A}$$

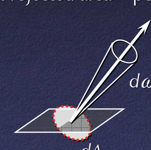
$$\Phi = I\omega$$

$$\omega = \frac{A(\hat{\mathbf{n}} \cdot \hat{\mathbf{l}})}{r^2}$$

$$\Rightarrow E = \frac{I\,(\hat{\mathbf{n}} \cdot \hat{\mathbf{l}})}{r^2}$$

Light Emitted from a Surface in A Direction

- Power per unit area per unit solid angle – *Radiance* (L)
  - Measured in W/m²/sr
  - *Projected area* – perpendicular to given direction



$$L = \frac{d\Phi}{dA_p\, d\omega}$$

Radiance

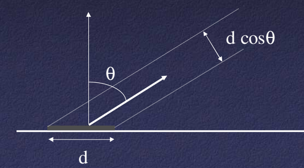
- <u>Flux per unit projected area per unit solid angle</u>.

$$L = \frac{d\Phi}{dA_p \; d\vec{\omega}}$$

- Units – watts per steradian m^2
- We have now introduced *projected area*, a cosine term. $L = \dfrac{d\Phi}{dA\cos\theta \; d\vec{\omega}}$

Why the Cosine Term?

- Foreshortening is by cosine of angle.
- Radiance gives energy by *effective* surface area.



Irradiance from Radiance

$$E = \int_{\Omega} L\cos\theta \; d\omega$$

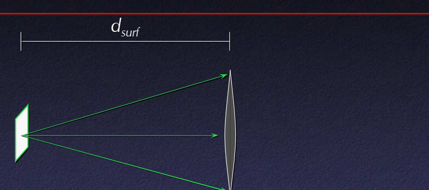
- $\cos\theta \; d\omega$ is projection of a differential area

Radiance as a unit of measure

- Radiance doesn't change with distance
  - Therefore it's the quantity we want to measure in a ray tracer.
- Radiance proportional to what a sensor (camera, eye) measures.
  - Therefore it's what we want to output.

Imaging

Imaging



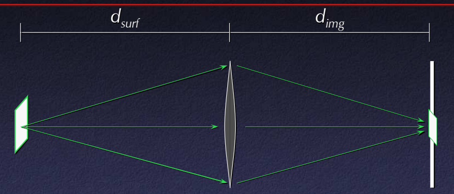
Area A_{surf}
Radiance L

$\Rightarrow I = L \, A_{surf}$

Area $A_{aperture}$

$\Rightarrow \Phi = L \, A_{surf} \dfrac{A_{aperture}}{d_{surf}^2}$

Imaging



Area A_{surf}
Radiance L

$\Rightarrow I = L\,A_{surf}$

Area $A_{aperture}$

$\Rightarrow \Phi = L\,A_{surf}\,\frac{A_{aperture}}{d_{surf}^2}$

Area A_{img}

$\Rightarrow E = \frac{\Phi}{A_{img}}$

Imaging

$I = L\,A_{surf}$ $\quad \Phi = L\,A_{surf}\,\frac{A_{aperture}}{d_{surf}^2}$ $\quad E = \frac{\Phi}{A_{img}}$

$$E = L\,\frac{A_{aperture}}{d_{surf}^2}\,\frac{A_{surf}}{A_{img}}$$

$$\frac{A_{surf}}{A_{img}} = \left(\frac{d_{surf}}{d_{img}}\right)^2$$

$$E = L\,\frac{A_{aperture}}{d_{img}^2} \longleftarrow \text{Depends only on camera}$$

- Punch line: cameras "see" radiance

Surface Reflectance – BRDF

- Reflected radiance is proportional to incoming flux and to irradiance (incident power per unit area).

$$dL_r(\vec{\omega}_r) \propto dE(\vec{\omega}_i)$$

Surface Reflectance – BRDF

- Bidirectional Reflectance Distribution Function

$$f_r(\omega_i \to \omega_o) = \frac{L_o(\omega_o)}{E_i(\omega_i)}$$

- 4-dimensional function: also written as

$$f_r(\theta_i, \varphi_i, \theta_o, \varphi_o) = \frac{L_o(\theta_o, \varphi_o)}{E_i(\theta_i, \varphi_i)}$$

(the symbol ρ is also used sometimes)

Surface Reflectance – BRDF

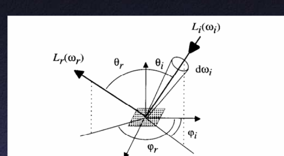


Figure 2.9: *Bidirectional reflection distribution function.*

$$f_r(\vec{\omega}_i \to \vec{\omega}_r) \equiv \frac{L_r(\vec{\omega}_r)}{L_i(\vec{\omega}_i)\cos\theta_i d\omega_i}$$

Properties of the BRDF

- Energy conservation:

$$\int_\Omega f_r(\theta_i, \varphi_i, \theta_o, \varphi_o)\cos\theta_o d\omega_o \le 1$$

- Helmholtz reciprocity:

$$f_r(\omega_i \to \omega_o) = f_r(\omega_o \to \omega_i)$$

(not always obeyed by "BRDFs" used in graphics)

Isotropy

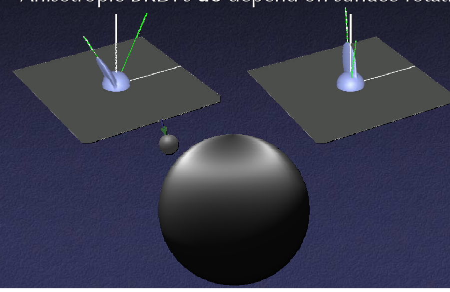
- A BRDF is isotropic if it stays the same when surface is rotated around normal

- Isotropic BRDFs are 3-dimesional functions:

$$f_r(\theta_i, \theta_o, \varphi_i - \varphi_o)$$

Anisotropy

- Anisotropic BRDFs **do** depend on surface rotation



BRDF Representations

- Physically-based vs. phenomenological models
- Measured data
- Desired characteristics:
  - Fast to evaluate
  - Maintain reciprocity, energy conservation
  - For global illumination: easy to importance sample

Diffuse

- The simplest BRDF is "ideal diffuse" or *Lambertian*: just a constant

$$f_r(\omega_i \to \omega_o) = k_d$$

- Note: does *not* include $\cos(\theta_i)$
  - Remember definition of irradiance

Diffuse BRDF

- Assume BRDF reflects a fraction ρ of light

$$\int_\Omega f_{r,Lambertian}(\omega_i \to \omega_o) \cos\theta_o \, d\omega_o = \rho$$

$$\int_{\substack{\theta \in [0..\pi/2] \\ \varphi \in [0..2\pi]}} k_d \cos\theta_o \sin\theta_o \, d\theta_o d\varphi_o = \rho$$

$$2\pi k_d \int_{\theta \in [0..\pi/2]} \sin\theta_o \cos\theta_o \, d\theta_o = \rho$$

$$\pi k_d = \rho$$

$$\therefore f_{r,Lambertian} = \frac{\rho}{\pi}$$

- The quantity ρ is called the albedo

Ideal Mirror

- All light incident from one direction is reflected into another

- BRDF is zero everywhere except where

$$\theta_o = \theta_i$$

$$\varphi_o = \varphi_i + \pi$$

Ideal Mirror

- To conserve energy,

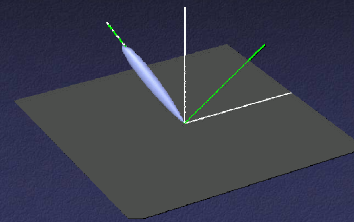
$$\int_\Omega f_{r,Mirror}(\omega_i \to \omega_o)\cos\theta_o \, d\omega_o = \rho$$

- So, BRDF is a delta function at direction of ideal mirror reflection

$$f_{r,Mirror} = \frac{\delta(\theta_i - \theta_o)\,\delta(\varphi_i + \pi - \varphi_o)}{\cos(\theta_i)}$$

Glossy Reflection

- Non-ideal specular reflection
- Most light reflected *near* ideal mirror direction



Phong BRDF

- Phenomenological model for glossy reflection

$$f_{r,Phong} = k_s(\hat{l} \cdot \hat{r})^n$$

l is a vector to the light source
r is the direction of mirror reflection

- Exponent *n* determines width of specular lobe
- Constant k_s determines size of lobe

Torrance-Sparrow BRDF

- Physically-based BRDF model
  - Originally used in the physics community
  - Adapted by Cook & Torrance and Blinn for graphics

$$f_{r,T-S} = \frac{DGF}{\pi \cos\theta_i \cos\theta_o}$$

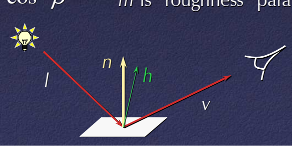
- Assume surface consists of tiny "microfacets" with mirror reflection off each

Torrance-Sparrow BRDF

- *D* term is distribution of microfacets (i.e., how many are pointing in each direction)
- Beckmann distribution

$$D = \frac{e^{-[(\tan\beta)/m]^2}}{4m^2\cos^4\beta}$$

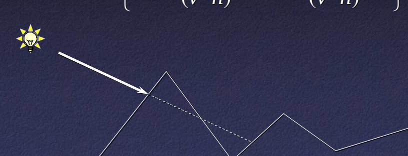
β is angle between *n* and *h*
h is halfway between *l* and *v*
m is "roughness" parameter



Torrance-Sparrow BRDF

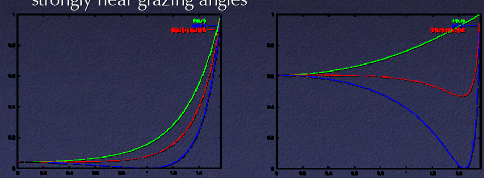
- *G* term accounts for self-shadowing

$$G = \min\left\{1, \frac{2(n\cdot h)(n\cdot v)}{(v\cdot h)}, \frac{2(n\cdot h)(n\cdot l)}{(v\cdot h)}\right\}$$



Torrance-Sparrow BRDF

- *F* term is Fresnel term – reflection from an ideal smooth surface (solution of Maxwell's equations)
- Consequence: most surfaces reflect (much) more strongly near grazing angles



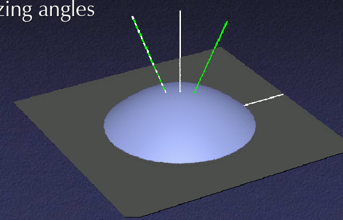
Dielectric

Metal

(note behavior at Brewster's angle)

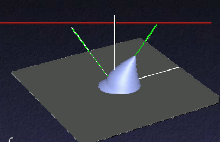
Other BRDF Features

- BRDFs for dusty surfaces scatter light towards grazing angles



Other BRDF Features

- Retroreflection: strong reflection back towards the light source



- Can arise from bumpy diffuse surfaces

- … or from corner reflectors

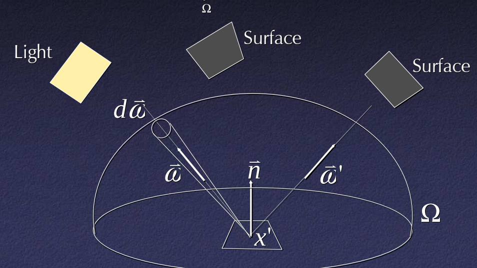
Beyond BRDFs

- So far, have assumed 4D BRDF
- Function of wavelength: 5D
- Fluorescence (absorb at one wavelength, emit at another): 6D
- Phosphorescence (absorb now, emit later): 7D
- Temporal dependence: 8D
- Spatial dependence: 10D
- Subsurface scattering: 12D
- Polarization
- Wave optics effects (diffraction, interference)
- …

"Cross product" of two plenoptic functions

Rendering Equation

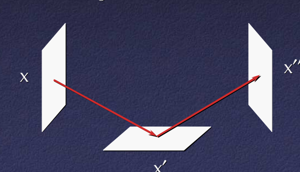
$$L_o(x',\vec{\omega}') = L_e(x',\vec{\omega}') + \int_{\Omega} f_r(x',\vec{\omega},\vec{\omega}') L_i(x',\vec{\omega})(\vec{\omega}\bullet\vec{n})\,d\vec{\omega}$$



Rendering Equation

- Originally expressed by [Kajiya 1986] as

$$I(x' \to x'') = I_e(x' \to x'') + G(x',x'')\int_S f_r(x \to x' \to x'')\, I(x \to x')\, V(x,x')\, dA$$



Rendering Equation

- Next 3-4 weeks in the course: ways to solve the rendering equation

Radiometric and Photometric Units

Radiant energy Joule (J)	Luminous energy Talbot
Radiant flux or power (F) Watt (W) = J / sec	Luminous power Lumen (lm) = talbot / sec = cd · sr
Radiant intensity (I) W / sr	Luminous intensity Candela (cd)
Irradiance (E) W / m^2	Illuminance Lux = lm / m^2
Radiance (L) W / m^2 / sr	Luminance Nit = lm / m^2 / sr
Radiosity (B) W / m^2	Luminosity Lux = lm / m^2