Global Illumination

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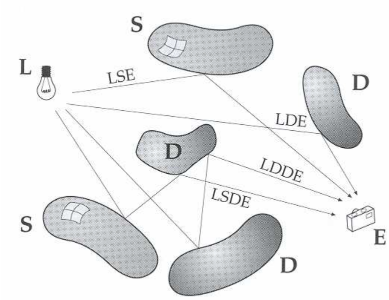
Princeton University
COS 526, Fall 2006

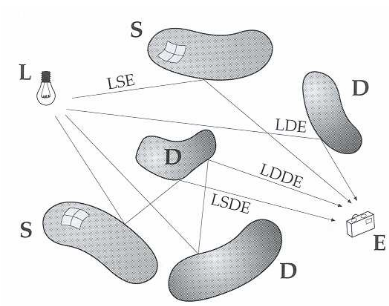
Global Illuminaton



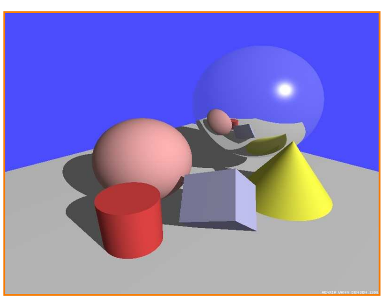
Overview

- Path types
  - L(S|D)*E
- Rendering is integration
  - Rendering equation
- Solution methods
  - OpenGL
  - Ray tracing
  - Path tracing
  - Radiosity

Path Types



Path Types?



Henrik Wann Jensen

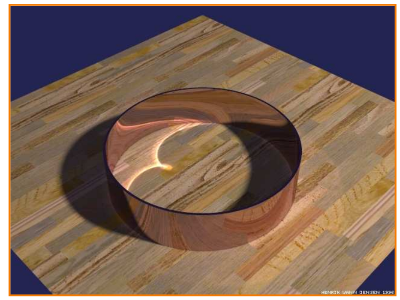
Path Types?

Herf

Path Types?

Henrik Wann Jensen

Path Types?



Henrik Wann Jensen

Path Types?

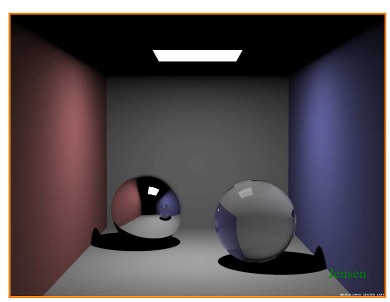
RenderPark

Path Types?

Paul Debevec

Path Types?

Path Types



Ray tracing

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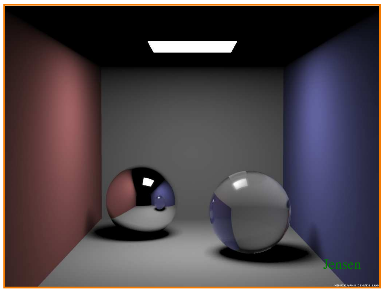
Path Types



+ soft shadows

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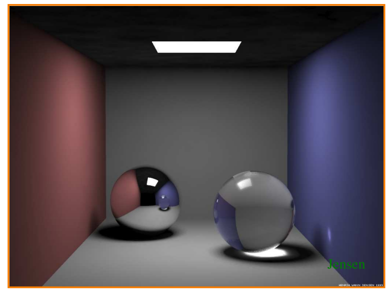
Path Types



+ caustics

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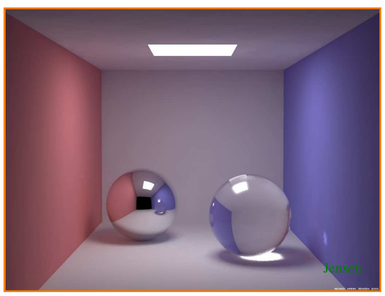
Path Types

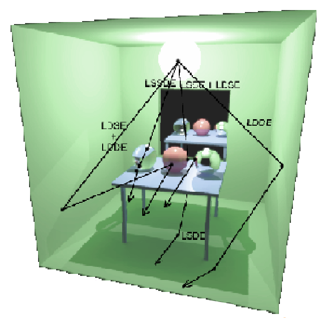


+ indirect diffuse illumination

Henrik Wann Jensen

Rendering Methods – Path Types

- OpenGL
  - LDE
- Ray tracing
  - LDS*E
- Path tracing
  - L(D|S)*E
- Radiosity
  - LD*E



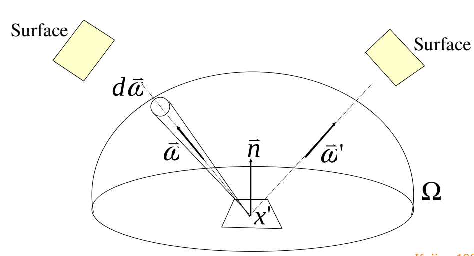
John Hart

Overview

- Path types
  - L(S|D)*E
- Ø Rendering is integration
  - Rendering equation
- Solution methods
  - OpenGL
  - Ray tracing
  - Path tracing
  - Radiosity

Rendering Equation

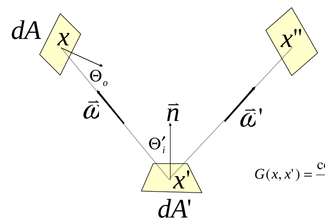
$$L_o(x', \vec{\omega}') = L_e(x', \vec{\omega}') + \int_\Omega f_r(x', \vec{\omega}, \vec{\omega}') L_i(x', \vec{\omega})(\vec{\omega} \bullet \vec{n}) d\vec{\omega}$$



Kajiya 1986

Rendering Equation (2)

$$L(x' \to x'') = L_e(x' \to x'') + \int_S f_r(x \to x' \to x'') L(x \to x') V(x,x') G(x,x') dA$$



$$G(x,x') = \frac{\cos\Theta_i' \cos\Theta_o}{\|x - x'\|^2}$$

Kajiya 1986

Overview

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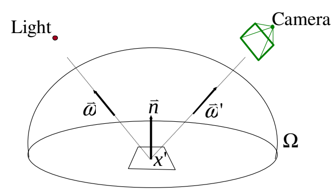
OpenGL

$$L_o(x',\vec{\omega}') = L_e(x',\vec{\omega}') + \int_\Omega f_r(x',\vec{\omega},\vec{\omega}') L_i(x',\vec{\omega})(\vec{\omega} \bullet \vec{n}) d\vec{\omega}$$

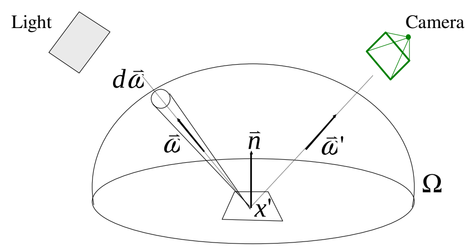
Assume direct illumination from point lights and ignore visibility



$$L_o(x',\vec{\omega}') = L_e(x',\vec{\omega}') + \sum_{i=1}^{nlights} f_r(x',\vec{\omega},\vec{\omega}') L_i(x',\vec{\omega})(\vec{\omega} \bullet \vec{n})$$

Direct Illumination

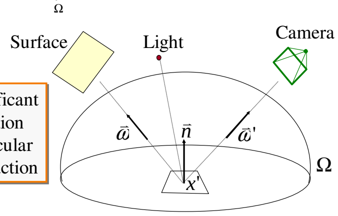
$$L_o(x',\vec{\omega}') = L_e(x',\vec{\omega}') + \int_{\Omega_L} f_r(x',\vec{\omega},\vec{\omega}') L_i(x',\vec{\omega})(\vec{\omega} \bullet \vec{n}) d\vec{\omega}$$



Ray Tracing

$$L_o(x',\vec{\omega}') = L_e(x',\vec{\omega}') + \int_\Omega f_r(x',\vec{\omega},\vec{\omega}') L_i(x',\vec{\omega})(\vec{\omega} \bullet \vec{n}) d\vec{\omega}$$

Assume only significant indirect illumination due to perfect specular reflection and refraction

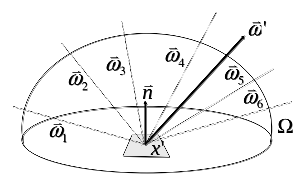


$$L_o(x',\vec{\omega}') = L_e(x',\vec{\omega}') + \sum_{i=1}^{nlights} f_r(x',\vec{\omega},\vec{\omega}') L_i(x',\vec{\omega})(\vec{\omega} \bullet \vec{n}) + specular$$

Distribution Ray Tracing

$$L_o(x',\vec{\omega}') = L_e(x',\vec{\omega}') + \int_\Omega f_r(x',\vec{\omega},\vec{\omega}') L_i(x',\vec{\omega})(\vec{\omega} \bullet \vec{n}) d\vec{\omega}$$

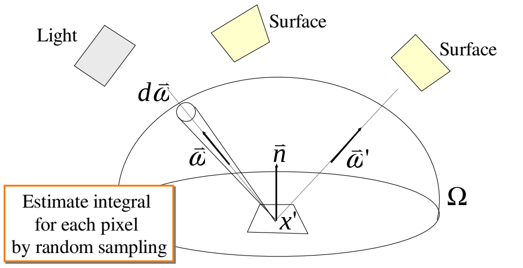
Estimate integral for each reflection by random sampling



Also:
- Depth of field
- Motion blur
- etc.

Path Tracing

$$L_o(x',\vec{\omega}') = L_e(x',\vec{\omega}') + \int_{\Omega} f_r(x',\vec{\omega},\vec{\omega}') L_i(x',\vec{\omega})(\vec{\omega}\bullet\vec{n})\,d\vec{\omega}$$



Path Tracing

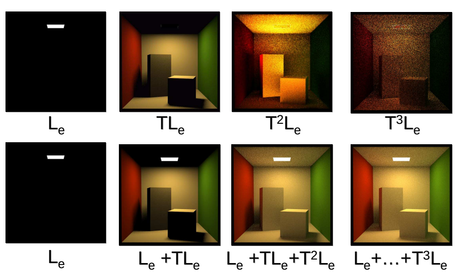
$$L_o(x',\vec{\omega}') = L_e(x',\vec{\omega}') + \int_{\Omega} f_r(x',\vec{\omega},\vec{\omega}') L_i(x',\vec{\omega})(\vec{\omega}\bullet\vec{n})\,d\vec{\omega}$$

Perform Neumann series expansion

$L = L_e + TL$ where $T(x,\vec{\omega}') = \int_{\Omega} f_r(x',\vec{\omega},\vec{\omega}') g(x,\vec{\omega})(\vec{\omega}\bullet\vec{n})\,d\vec{\omega}$

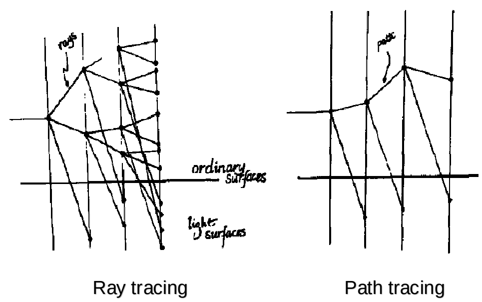
$$L = L_e + TL_e + T^2L_e + T^3L_e + \ldots$$

Path Tracing



Dutré

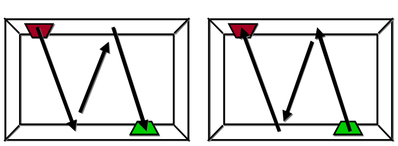
Ray Tracing vs. Path Tracing



Kajiya

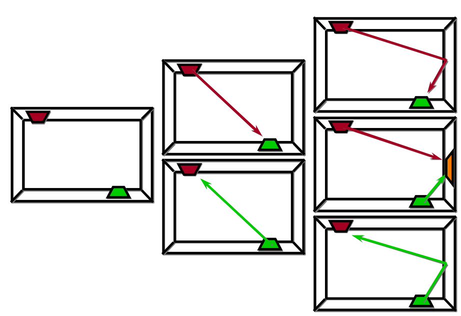
Bidirectional Path Tracing

- Role of source and receiver can be switched, flux does not change



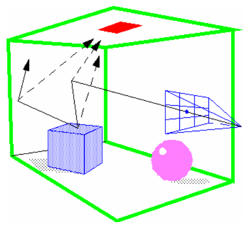
Dutré

Bidirectional Path Tracing



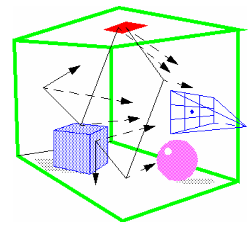
Dutré

Tracing From Eye



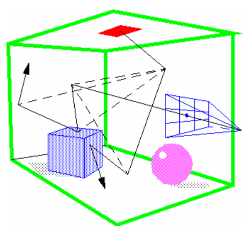
Dutré

Tracing from Lights



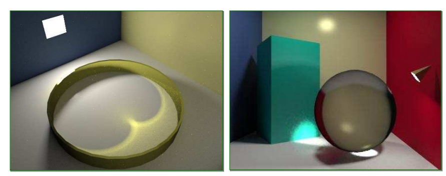
Dutré

Bidirectional Path Tracing



Dutré

Bidirectional Path Tracing



(RenderPark 98)

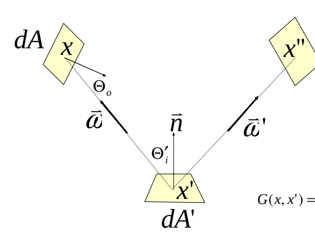
Dutré

Radiosity

- Indirect diffuse illumination – LD*E

Rendering Equation (2)

$$L(x' \to x'') = L_e(x' \to x'') + \int_S f_r(x \to x' \to x'') L(x \to x') V(x, x') G(x, x') dA$$



$$G(x, x') = \frac{\cos \Theta_i' \cos \Theta_o}{\|x - x'\|^2}$$

Kajiya 1986

Radiosity Equation

$$L(x'\to x'') = L_e(x'\to x'') + \int_S f_r(x\to x'\to x'')L(x\to x')V(x,x')G(x,x')dA$$

Assume everything is Lambertian

$$\rho(x') = f_r(x\to x'\to x'')\pi$$

$$L(x') = L_e(x') + \frac{\rho(x')}{\pi}\int_S L(x)V(x,x')G(x,x')dA$$

Convert to Radiosities

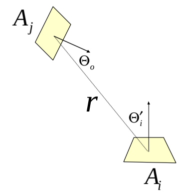
$$B = \int_\Omega L_o \cos\theta d\omega \qquad L = \frac{B}{\pi}$$

$$B(x') = B_e(x') + \frac{\rho(x')}{\pi}\int_S B(x)V(x,x')G(x,x')dA$$

Radiosity Approximation

$$B(x') = B_e(x') + \frac{\rho(x')}{\pi}\int_S B(x)V(x,x')G(x,x')dA$$

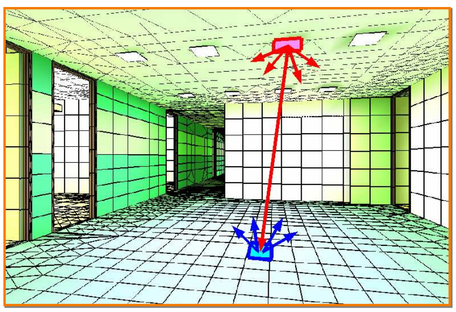
Discretize the surfaces into "elements"



$$B_i = E_i + \rho_i \sum_{j=1}^{N} B_j F_{ij}$$

where $F_{ij} = \frac{1}{A_i}\int_{A_i}\int_{A_j}\frac{V_{ij}\cos\Theta_i'\cos\Theta_o}{\pi r^2}dA_j dA_i$

Radiosity Approximation



System of Equations

$$B_i = E_i + \rho_i \sum_{j=1}^{N} B_j F_{ij}$$

$$E_i = B_i - \rho_i \sum_{j=1}^{N} B_j F_{ij}$$

$$B_i - \rho_i \sum_{j=1}^{N} B_j F_{ij} = E_i$$

$$\begin{bmatrix} 1-\rho_1 F_{1,1} & . & . \; . & -\rho_1 F_{1,n} \\ -\rho_2 F_{2,1} & 1-\rho_2 F_{2,2} & . \; . & -\rho_2 F_{2,n} \\ . & . & . \; . & . \\ . & . & . \; . & . \\ -\rho_{n-1} F_{n-1,1} & . & . \; . & -\rho_{n-1} F_{n-1,n} \\ -\rho_n F_{n,1} & . & . \; . & 1-\rho_n F_{n,n} \end{bmatrix} \begin{bmatrix} B_1 \\ B_2 \\ . \\ . \\ . \\ B_n \end{bmatrix} = \begin{bmatrix} E_1 \\ E_2 \\ . \\ . \\ . \\ E_n \end{bmatrix}$$

$$(1-\rho_i \sum_{j=1}^{N} F_{ii})B_i - \rho_i \sum_{j=1}^{N} F_{ij}B_j = E_i$$

$$B_i A_i = E_i A_i + \rho_i \sum_{j=1}^{N} F_{ji} B_j A_j$$

← This is an energy balance equation

Summary

- Rendering is integration
  - Rendering equation
- Different solution methods for different path types
  - OpenGL - LDE
  - Ray tracing – LDS*E
  - Path tracing – L(SD)*E
  - Radiosity - LD*E