

COS 521: Advanced Algorithm Design
Homework 2

Due: Wed, Oct 18 (in class)

Collaboration Policy: You may collaborate with other students on these problems. Collaboration is limited to discussion of ideas only, and you should write up the solutions entirely *on your own* and *list your collaborators* as well as *cite any references* you may have used.

1. Consider a variant of Move-To-Front for the list update problem. When an element at position k is accessed, Half-MTF moves it to location $\lfloor k/2 \rfloor$. Prove that Half-MTF is 4-competitive. As we did in class, you may consider only access operations in your analysis.

2. A conservative paging algorithm is one that satisfies the following property: *on any consecutive input subsequence containing at most k distinct page references, the algorithm incurs at most k page faults.* Here k is the size of the cache.
 - (a) Show that LRU and FIFO are conservative.
 - (b) Show that any conservative algorithm has competitive ratio k .

3. Consider the load balancing problem in the unrelated machines model. The greedy algorithm for this problem simply places each job on the machine that minimizes the resulting load (ties are broken arbitrarily). Show that the greedy algorithm has competitive ratio $\Theta(n)$ for this problem. You should show both an upper bound and a lower bound for full credit.

4. Consider a matrix M . Let $N(M)$ be the maximum number of nonzero entries in M , no two of which are in the same row or the same column. Let $L(M)$ be the minimum number of lines (a row or a column) that include all the nonzero entries. Prove that $N(M) = L(M)$.