COS487 Theory of Computation **Fall 2006**

Assignment #8

Due: Thursday December 7 *Sean Hallgren*

1. Show that **PSPACE** is closed under the operations union, complementation, and star.

2. The Japanese game *go-moku* is played by two players,"X" and "O" on a 19×19 grid. Players take turns placing markers, and the first player to achieve 5 of his markers consecutively in a row, column, or diagonal, is the winner. Consider this game generalized to an $n \times n$ board. Let

   $GM = \{\langle B \rangle \mid B \text{ is a position in generalized go-moku, where player "X" has a winning strategy}\}$.

   By a *position* we mean a board with markers placed on it, such as may occur in the middle of a play of the game, together with an indication of which player moves next. Show that $GM \in$ **PSPACE**.

3. Show that, if every **NP**-hard language is also **PSPACE**-hard, then **PSPACE = NP**.

4. Let A be the language of properly nested parentheses. For example, (()) and ((()(()))) are in A, but)(is not. Show that A is in **L**.

5. (a) Let $ADD = \{\langle x, y, z \rangle \mid x, y, z > 0 \text{ are binary integers and } x + y = z\}$. Show that $ADD \in$ **L**.

   (b) Let $PAL\text{-}ADD = \{\langle x, y \rangle \mid x, y > 0$ are binary integers where $x + y$ is an integer whose binary representation is a palindrome$\}$. (Note that the binary representation of the sum is assumed not to have leading zeros. A palindrome is a string that equals its reverse). Show that $PAL\text{-}ADD \in$ **L**.

6. (Optional) Show that *2SAT* is **NL**-complete.