COS487 Theory of Computation **Fall 2006**

Assignment #6

Due: Tuesday November 7 *Sean Hallgren*

This HomeWork is due Tuesday, Nov 7 in class, i.e. in the week after the break. You will recieve the corrected homeworks as well as the Take-Home midterm on Thursday, Nov 9 in class. The midterm answers are to be submitted on Tuesday, Nov 14 in class.

1. Show that EQ_{CFG} is co-Turing-recognizable. (Look at the definition of EQ_{CFG} in the book.)

2. If $A \leq_m B$ and B is a regular language, does that imply that A is a regular language? Why or why not?

3. Let $\Gamma = \{a, 1, \sqcup\}$ be the tape alphabet for all TMs in this problem. Define the *busy beaver function* $BB : \mathcal{N} \rightarrow \mathcal{N}$ as follows. For each value of k, consider all k-state TMs that halt when started with a blank tape. Let $BB(k)$ be the maximum number of 1s that remain on the tape among all of these machines. Show that BB is not a computable function.

4. Let
$$f(x) = \{ \begin{array}{ll} 3x+1 & \text{for odd } x \\ x/2 & \text{for even } x \end{array}$$
for any natural number x. If you start with an integer x and iterate f, you obtain a sequence, x, $f(x)$, $f(f(x)), \ldots$. Stop if you ever hit 1. For example, if $x = 17$, you get the sequence 17, 52, 26, 13, 40, 20, 10, 5, 16, 8, 4, 2, 1. Extensive computer tests have shown that every starting point between 1 and a large positive integer gives a sequence that ends in 1. But, the question of whether all positive starting points end up at 1 is unsolved; it is called the $3x + 1$ problem. Suppose that A_{TM} were decidable by a TM H. Use H to describe a TM that is guaranteed to state the answer to the $3x + 1$ problem.

5. Let $S = \{\langle M \rangle \mid M$ is a TM and $L(M) = \{\langle M \rangle\}\}$. Show that neither S nor $\bar{S}$ is Turing-recognizable.

6. (Optional) Prove that there exist two languages A and B that are Turing-incomparable, i.e. where $A \not\leq_T B$ and $B \not\leq_T A$.