COS487 Theory of Computation **Fall 2006**

Assignment #4

Due: Thursday October 19 *Sean Hallgren*

1. Let $\Sigma = \{1, 2, 3, 4\}$ and $C = \{w \in \Sigma^* \mid$ in w, the number of 1s equals the number of 2s, and the number of 3s equals the number of 4s$\}$. Show that C is not context free.

2. Give an example of a language that is not context free but that acts like a CFL in the pumping lemma. Prove that your example works. (See the analogous example for regular languages in **Problem 1.54**.)

3. Let $L = \{w \mid w$ does not contain twice as many 0s as 1s$\}$. Give an implementation-level description of Turing machine that decides L over the alphabet {0,1}. (Read 3.8(a), Page-163 for an example answer).

4. A *Turing machine with doubly infinite tape* is similar to an ordinary Turing machine, but its tape is infinite to the left as well as to the right. The tape is initially filled with blanks except for the portion that contains the input. Computation is defined as usual except that the head never encounters an end to the tape as it moves leftward. Show that this type of Turing machine recognizes the class of Turing-recognizable languages.

5. (a) Show that the collection of decidable languages is closed under the operation of concatenation.
   (b) Show that the collection of Turing-recognizable languages is closed under the operation of concatenation.

6. (Optional) Prove the following stronger form of the pumping lemma, wherein *both* pieces v and y must be nonempty when the string s is broken up.

   If A is a context-free language, then there is a number k where, if s is any string in A of length at least k, then s may be divided into five pieces, $s = uvxyz$, satisfying the conditions:

   (a) for each $i \geq 0$, $uv^ixy^iz \in A$,
   (b) $v \neq \epsilon$ and $y \neq \epsilon$, and
   (c) $|vxy| \leq k$.