1. Let $A/B = \{w \mid wx \in A \text{ for some } x \in B\}$. Show that, if A is context free and B is regular, then A/B is context free.

2. For any language A, let $SUFFIX(A) = \{v \mid uv \in A \text{ for some string } u\}$. Show that the class of context-free languages is closed under the $SUFFIX$ operation.

3. Show that, if G is a CFG in Chomsky normal form, then for any string $w \in L(G)$ of length $n > 1$, exactly $2n - 1$ steps are required for any derivation of w.

4. Let $G = (V, \Sigma, R, \langle \textbf{STMT} \rangle)$ be the following grammar.

$$
\begin{aligned}
\langle \textbf{STMT} \rangle &\rightarrow \langle \textbf{ASSIGN} \rangle \mid \langle \textbf{IF} - \textbf{THEN} \rangle \mid \langle \textbf{IF} - \textbf{THEN} - \textbf{ELSE} \rangle \\
\langle \textbf{IF} - \textbf{THEN} \rangle &\rightarrow \textbf{if condition then } \langle \textbf{STMT} \rangle \\
\langle \textbf{IF} - \textbf{THEN} - \textbf{ELSE} \rangle &\rightarrow \textbf{if condition then } \langle \textbf{STMT} \rangle \textbf{ else } \langle \textbf{STMT} \rangle \\
\langle \textbf{ASSIGN} \rangle &\rightarrow \textbf{a} := \textbf{1}
\end{aligned}
$$

$$
\begin{aligned}
\Sigma &= \{\textbf{if, condition, then, else, a} := \textbf{1}\} \\
V &= \{\langle \textbf{STMT} \rangle, \langle \textbf{ASSIGN} \rangle, \langle \textbf{IF} - \textbf{THEN} \rangle, \langle \textbf{IF} - \textbf{THEN} - \textbf{ELSE} \rangle\}
\end{aligned}
$$

   G is a natural-looking grammar for a fragment of a programming language, but G is ambiguous.

   (a) Show that G is ambiguous.

   (b) Give a new unambiguous grammar for the same language.

5. Let B be the language of all palindromes over $\{0, 1\}$ containing an equal number of 0s and 1s. Show that B is not context free.

6. (Optional) Let $C = \{x \# y \mid x, y \in \{0, 1\}^* \text{ and } x \neq y\}$. Show that C is a context-free language.