FINAL EXAM

Name:

"I pledge my honor that I have not violated the Honor Code during this examination"

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There are eighteen questions, of equal weight. This exam is open book, open notes. As usual, if there is something you find ambiguous or incorrect in the question, state your assumptions and then answer.

1. What is the maximum number of times during the execution of quicksort that the largest element can be moved, for a file of N distinct elements?

2. Suppose that mergesort is implemented to split the file at a *random* position, rather than exactly in the middle. Which of the following best describes the average running time as a function of the input size?

(A) linear
(B) slower than standard mergesort, by a constant factor
(C) same as standard mergesort
(D) faster than standard mergesort, by a constant factor
(E) quadratic

3. Suppose that a heap contains a permutation of the integers `1` through `1023`. What is the maximum number of numbers greater than `1000` that can appear on the bottom level?

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4. Give the binomial queue that results when the keys `E A S Y` are inserted into an intially empty binomial queue, and give the binomial queue that results when the keys `Q U E S T I O N` are inserted into an intially empty binomial queue. Then give the result when the *join* operation is performed on the resulting queues.

5. Give the number of bytes examined by MSD radix sort and three-way radix Quicksort for a file of N keys `A`, `AA`, `AAA`, `AAAA`, `AAAAA`, `AAAAAA`, . . .,

6. Give the binary search tree that results when the keys E A S Y Q U E S T I O N are inserted into an initially empty tree. Give the result of deleting the S, then give the result of reinserting it using the top-down insertion method.

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7. Give the KMP FSA for the string 011011.

8. Draw a NDFSA for the string a*b*c + a*.

9. Draw the trie that distinguishes the keys 000001, 10, 000000, 00101, and 111.

Do your work on this sheet (use the back if you must).

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10. What is the value of 3^{195} mod 10?

11. Consider the following recursive program:

```
int F(int N)
{
    if (N < 1) return 0;
    if (N == 1) return 1;
    return 3*F(N/2) - 2*F(N - N/2) + N;
}
```

Give an estimate of its running time in the worst case and in the best case when called with argument N, assuming that the implementation does not recompute known answers.

12. Draw the Euclidean MST and the Voronoi diagram of the points $(0, 0)$, $(0, 5)$, $(5, 0)$, and $(1, 1)$.

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13. Draw the 2D tree that is constructed when the points $(0, 0)$, $(1, 5)$, $(5, 1)$, $(2, 2)$, $(3, 4)$, and $(4, 3)$ are inserted in that order into an initially empty tree.

14. Describe how you would find the MST of a graph with N nodes and $5N$ edges, with all edge weights equal to 1. Give the worst-case running time for your method.

15. Consider the graph on six nodes, labelled A through F, with ten directed edges AB EB AE AC CD DC EF FG BC FB. Give the maximum and minimum possible values for the maximum depth of the recursion when this graph is searched with depth-first search, starting at A.

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16. Suppose that you have a very large disk file with $2^{32}-1$ records, each with a unique 32-bit identifier. Since there are 2^{32} different possible identifier values, there is one value that is not present on any record. How would you find out which value is missing?

17. Which of the following best describes the asymptotic growth (within a constant factor) of the length of the code produced by the two versions of LZW encoding for a string of $2N$ alternating `a`'s and `z`'s ? Give two answers, one for each version.

(A) $\log N$
(B) $(\log N)^2$
(C) $\sqrt{N}$
(D) $N/\log N$
(E) N

18. Use Horner's method to convert the number 3101321 from base-4 to base-10 representation.