Computer Science 341
Discrete Mathematics

Final Exam
Due 24 hours from time obtained

Problem 1:
Let G be a simple path of length n. A valid coloring of the path is an assignment of colors to the vertices such that no edge is monochromatic. How many ways are there to color the path with five colors (red, green, blue, yellow and violet) if:

1. There are no more restrictions.
2. For every color there is at least one node colored with that color.
3. The colors red and green do not appear one next to the other.

Problem 2:
A bin contains $\frac{n}{3}$ white balls and $\frac{2n}{3}$ red balls. Consider the following process: a blindfolded person picks a ball from the bin, then check the color. If the ball was white, he returns the ball to the bin and adds c more white balls. If the ball was red, he returns the ball and this time adds c red balls.
If this process is repeated k times, what is the probability of taking out a red ball in the $k+1$ iteration ?

Problem 3:
Scientists at the Princeton Genomics Institute have discovered the following process: Given a test tube filled with DNA strands, it is possible to insert an enzyme that will connect two end points of stands (possibly of the same strand), and then dissolve. Experiments show every pair of endpoints is equally likely to be joined. If two endpoints of the same strand connect, then a "DNA cycle" is formed.
Suppose that N enzymes are inserted into a test tube with N strands. What is the expected number of DNA cycles that will be formed?

Problem 4:
Prove that for every $k \geq 3$ there is a number $N(k)$ such that for every set $|S| \geq N$ of points on the plane there is a subset $T \subseteq S$ of size $|T| = k$, such that all points in T lie on line, or no three points in T lie on the same line.