# Symmetry Transform

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## Motivation

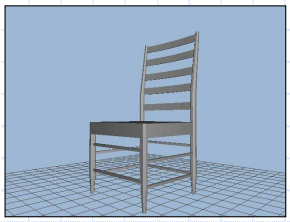
- Symmetry is related to psychophysical recognition/classification of objects.
- Symmety corresponds to "interesting" region of an image.
- Symmetry can separate texture (noise) from objects (signal).
- Can symmetry describe shape?

# Symmetry

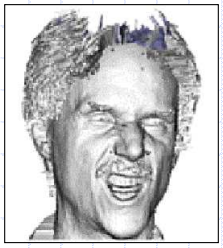
- Ø Bilateral
- Radial

Gesu, et. al.

Bruce



Tecfa



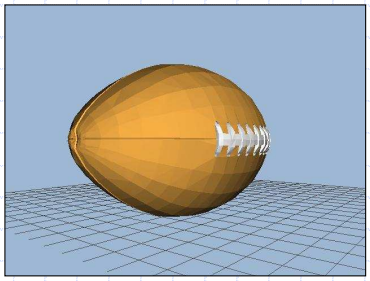
Erturk, et. al.

# Symmetry

- Bilateral
- Ø Radial

Talmi, et. al.

Gesu, et. al.



Avatara

## Talk Outline

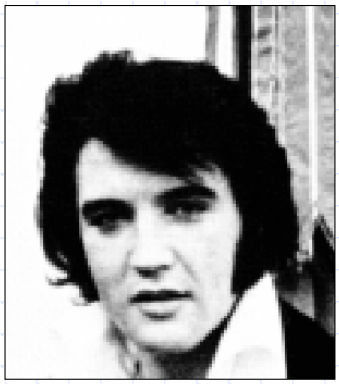
- Introduction/Motivation
- Techniques
  - Generalized Symmetry Transform
  - Discrete Symmetry Transform
- Results
- Conclusion

## Generalized Symmetry XForm

- Continuous definition of 2D symmetry.
- Low-level vision cue (context-free).
- Variants for both bilateral and radial symmetry.
- Think of Hough transform!

## Generalized Symmetry XForm

- 2D Image
- Gradient
- Perform transform on gradient.



Elvis, et. al.

## Generalized Symmetry XForm

- 2D Image
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Elvis, et. al.

# Generalized Symmetry XForm

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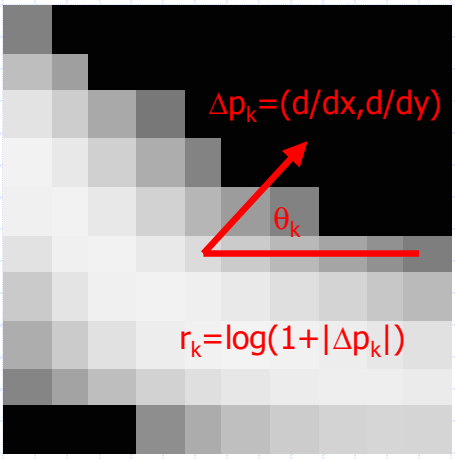



# Generalized Symmetry XForm

$$\Gamma(p) = \left\{(i,j) \middle| \frac{p_i + p_j}{2} = p\right\}$$

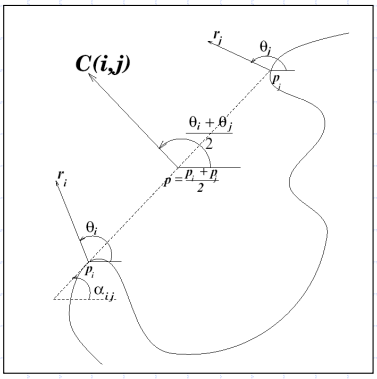
$$D_\sigma(i,j) = \frac{1}{\sqrt{2\pi}\sigma} e^{-\frac{\|p_i - p_j\|}{2\sigma}}$$

$$P(i,j) = (1 - \cos(\theta_i + \theta_j - 2\alpha_{ij}))(1 - \cos(\theta_i - \theta_j))$$

Reisfeld, et. al.

# Generalized Symmetry XForm

$$C(i,j) = D_\sigma(i,j)P(i,j)r_i r_j$$

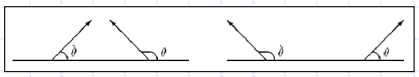



Reisfeld, et. al.

# Generalized Symmetry XForm

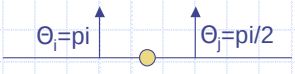
- Phase function penalizes/favors (a)symmetrically **oriented** edges.

$(1-\cos(\theta_i+\theta_j-2\alpha_{ij}))$



Measures directional agreement.

$(1-\cos(\theta i-\theta j))$




Penalizes collinear edges.

# Isotropic Symmetry

$$M_\sigma(p) = \sum_{(i,j)\in\Gamma(p)} C(i,j)$$

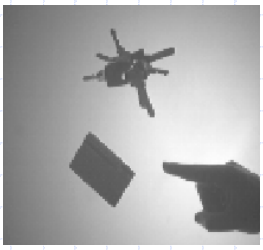
$$\varphi(i,j) = \frac{\theta_i + \theta_j}{2}$$

$$S_\sigma(p) = (M_\sigma(p), \phi(p))$$

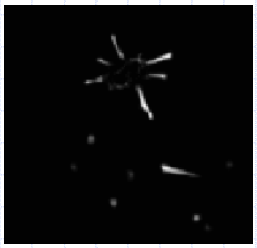
Reisfeld, et. al.

$\Phi(p) = \text{argmax}\{\varphi(i,j)\}$

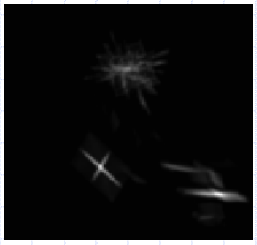
# Isotropic Symmetry



Original Image



small σ



large σ

Reisfeld, et. al.

# Radial Symmetry

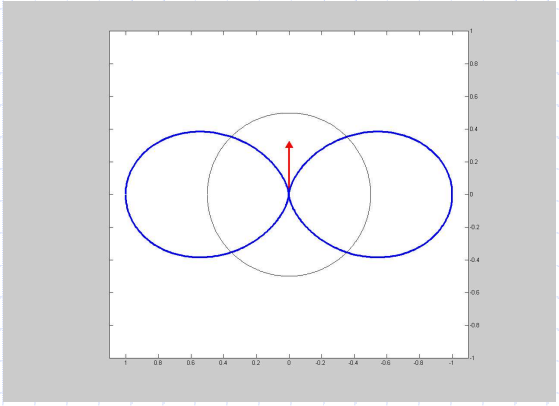
- Detect points highly symmetric in multiple distinct orientations.

$$RS_\sigma(p) = \sum_{(i,j)\in\Gamma(p)} C(i,j)\ \sin^2(\varphi(i,j)-\phi(p))$$

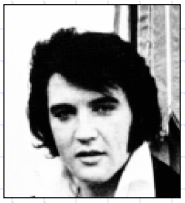
Reisfeld, et. al.

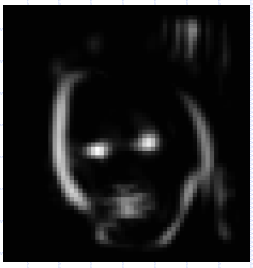
# Radial Symmetry

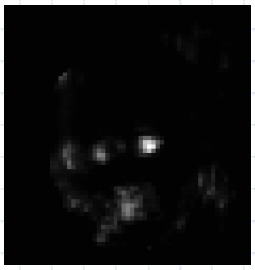
$\sin^2(\varphi(i,j) - pi/2)$



# Radial Symmetry







Isotropic

Radial

Points of maximum radial symmetry.

Reisfeld, et. al.

# Complexity

2D Image (input)

↓ O(n)

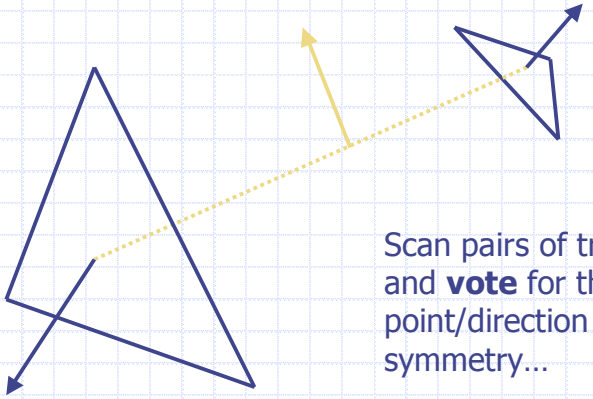
Gradient

↓ $O(n^2)$ or $O(nr^2)$

Symmetry Image

r can be derived from value of σ

# 3D Symmetry Transform?



Scan pairs of triangles and **vote** for the point/direction of symmetry…