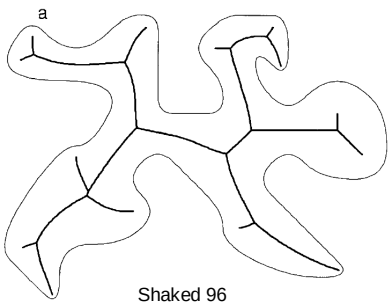
Medial Axis

Diego Nehab

Blum 67

- "A Transformation For Extracting New Descriptors of Shape"
- Locus of points equidistant from contour
- Medial Axis
- Symmetric Axis
- Skeleton
- Shock Graph



Shaked 96

Applications

- Shape matching
- Animation
- Dimension reduction
- Solid modeling
- Smoothing or sharpening of shape
- Motion planning
- Mesh generation

Outline of talk

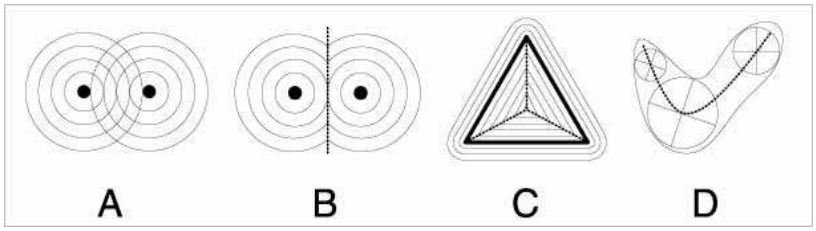
- Definitions, properties, and examples
- Application examples
- How to compute
- Hierarchic Voronoi Skeletons
- No conclusions

Different definitions

- Locus of points equidistant from contour
- Grass-fire, prairie-fire, wave-front collision
- Locus of centers of maximal circles
- Local maxima in distance transform
- Result of topology preserving thinning
- Ridges in envelope of cones (apexes on contour)

Pictorial definitions

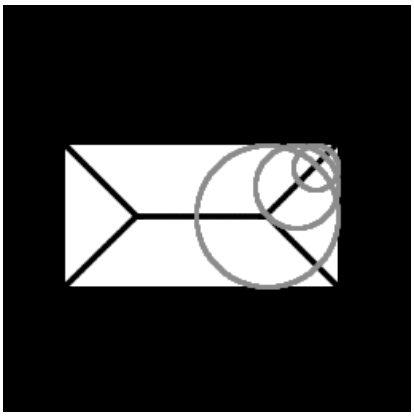
- Grass-fire, prairie-fire, wave-front collision
- Locus of centers of maximal circles



van Tonder

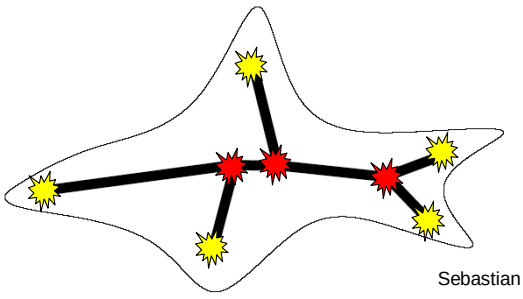
Medial Axis Transform

- Medial Axis augmented by radius function
- Transformation is invertible



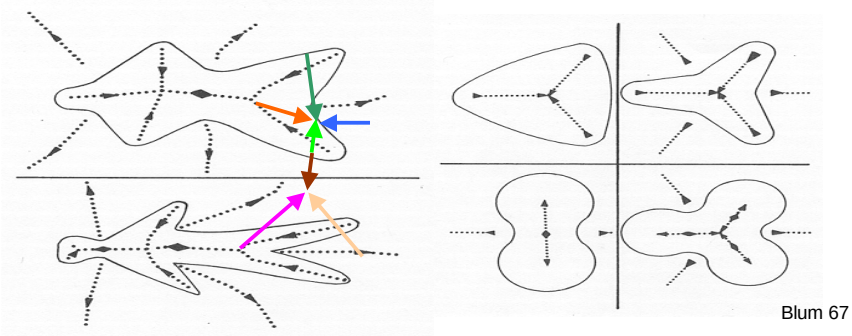
Classes of points in skeleton

- Equidistant from 1 joint pint set (end points)
- Equidistant from 2 disjoint sets (normal points)
- Equidistant from 3 or more disjoint sets (branch points)



Shock graph

- Analyze skeleton evolution in time, flow of shocks
- Split skeleton into monotonic segments
- More refined than MA



Blum 67

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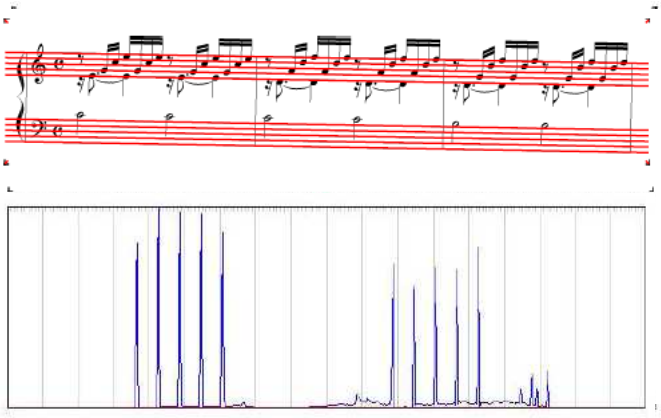
Nehab 03

- "Staff-line detection of music scores"
- Compute medial axis of score
- Extract and simplify polygonal graph
- Compute histogram of segment directions
- Project segments in the most popular direction
- Detect peaks in resulting histogram

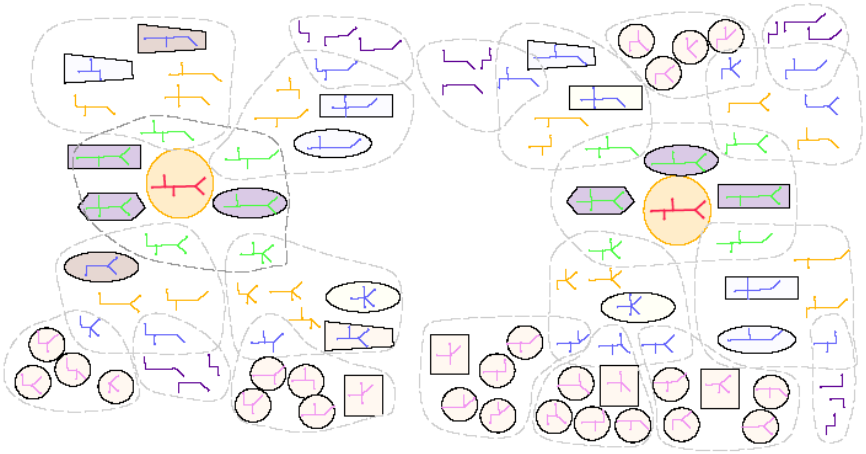
Nehab 03



Sebastian 01

- "Recognition of Shapes by Editting Shock Graphs"
- Compute Shock Graphs for each shape
- Shapes whose shock graphs have same topology are clustered into equivalent classes
- Editing operations are transitions between classes
- Associate a cost to each edit operation
- Find minimum edit cost path between shapes

Sebastian 01



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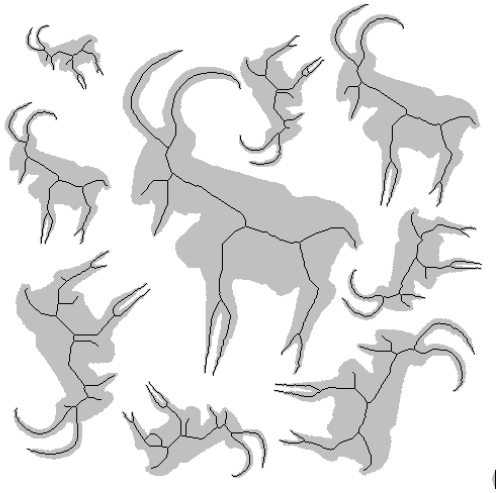
Hilditch 69

- “Linear Skeletons from Square Cupboards”
- Work inwards from the boundary
- Remove all points except for skeleton points
- Preserve topology by a number of tests
  - **Is in edge, is not singled out, is not tip, doesn’t disconnect**
- Repeat until no point can be removed

Hilditch skeletonization

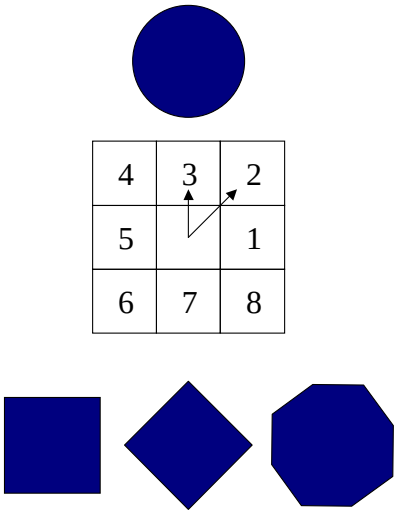


Distance Transform

- Distance in shape with respect to it's complement
- Copy of shape, each point is labeled with distance
- Each point represents disk centered at the point
- Disk size is given by point label
- Disk shape is given by distance metric
- Can be computed in two passes over image

Chamfer Distance

- Chamfer distances are easier to use than Euclidean
  - **(W_{odd}, W_{even})**
  - **Chessboard (1, 1)**
  - **Cityblock (1, oo)**
- But unstable with rotation
- (3,4)-DT is good compromise



Distance Transform Examples

di Baja 94

- "Well-Shaped, Stable and Reversible Skeletons from the (3,4)-Distance Transform
- Compute (3,4) DT
- Identify local maxima and saddle points
- Grow connecting paths in direction of maximal gradient
- Hole fill, final thin

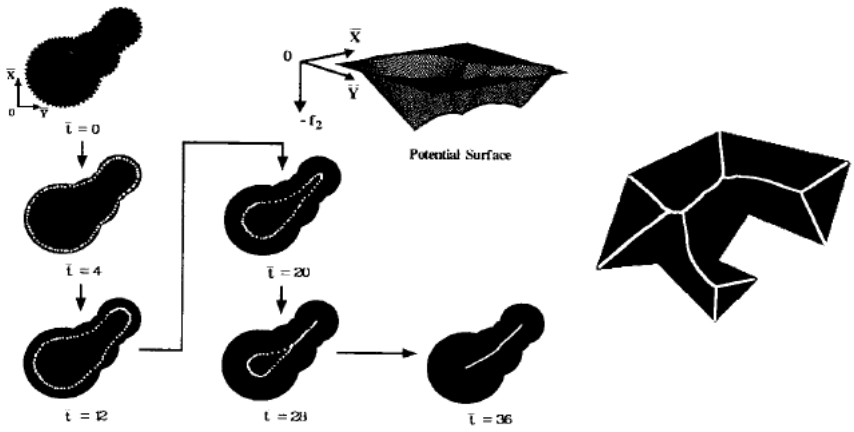
Finding maximal disks

- Disk is maximal if not contained by any other
- Center of MD is a local maximum of DT
- Label comparison between neighbors is enough to determine "containment"
- Disk is maximal if it is not the smaller neighbor of any of its neighbors

Leymarie 92

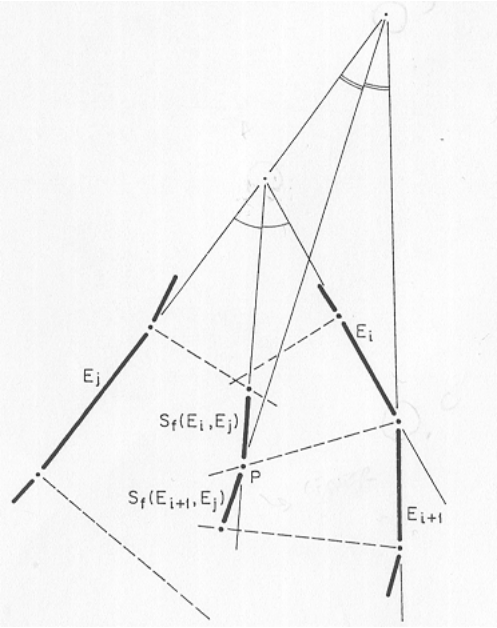
- "Simulating the Grassfire Transform using an Active Contour Model"
- Compute the Distance Transform
- Define a potential function equal to -DT
- Place a snake over the iso-contour -1
- Iterate based on gradient and internal constraints
- Skeleton points correspond to Snake meeting points

Leymarie 92



Bookstein 79

- "The Line Skeleton'



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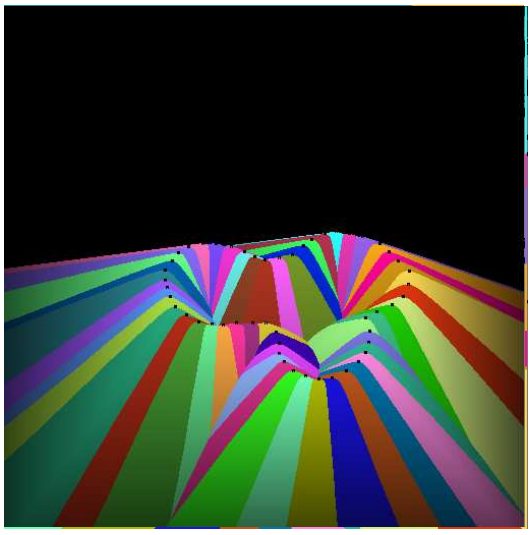
Voronoi Skeleton



Regularizing skeletons

- Skeletons are highly sensitive to noise in boundary
- Regularization can be performed in shape (smoothing) or in MAT (prunning)
- Pruning affects the inverse MAT and smoothing affects the MAT

Shape smoothing

- Discrete case, apply hole-filling before
  - **Weird loops in MA are hard to get rid of later**
- Curvature flow
- Blur image
- Usually, pruning skeleton is better idea
  - **Shaked 96, "Pruning Medial Axis"**
  - **Smoothing might change skeleton topology**

Skeleton prunning

- Small perturbations in the axis or the associated function may result in an axis-like description that does not correspond to a planar shape
- Simple transformations known to be allowed are
  - **Uniform reduction of radius function values**
  - **Prunning of branches, preserving connectivity**

Ogniewicz 95

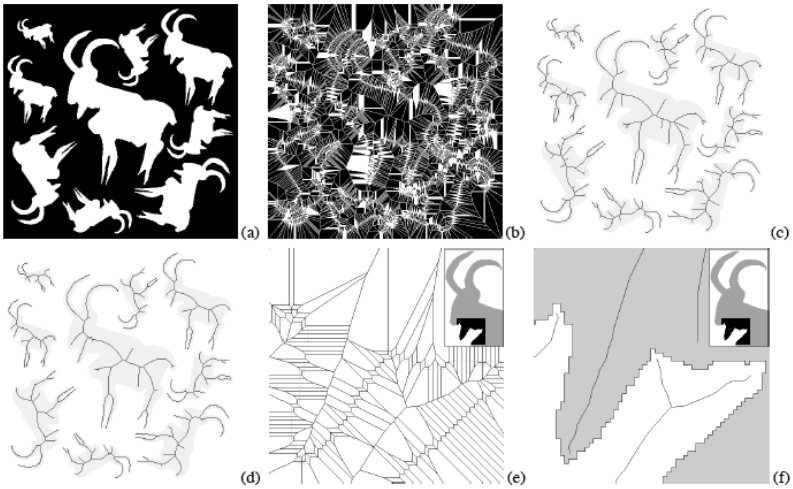
- "Hierarchic Voronoi Skeletons"
- Compute VD of shape's boundary points
- Associate topology preserving importance metric with each edge of VD
- Hierarchic clustering of skeleton edges creates a skeleton pyramid

Anchor distance

- "Geodesic" distance between generators of a given edge
- Assign to each edge their corresponding anchor distance
- For any edge, there is a monotonic path until max
- Thresholding mantains connectivity

Ogniewicz 95

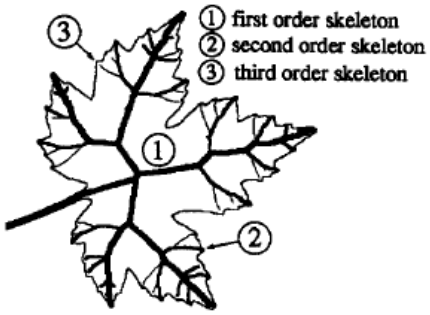
Discrete Voronoi Skeleton



Ogniewicz 95

Hierarchic Voronoi Skeleton

- Clean with anchor distance threshold
- Start with first level edges
  - **Parameter controls how much to include**
- Follow skeleton in order of least steep descent
  - **Parameter controls how much change starts a new level**



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- Sebastian 01