PRINCETON UNIVERSITY — FALL 2002

COS 487: THEORY OF COMPUTATION — Prof. Amit Sahai

Mid-Term Examination

November 4 1:30-2:50 PM — *TA: Manoj M P*

This exam has four questions, with many sub-parts. A total of 90 points are distributed among the questions. You have 80 minutes to take this exam.

Please write your name and year below:

Name:

Year:

- Write out and sign the Honor Code pledge before turning in the test.

"I pledge my honor that I have not violated the Honor Code during this examination."

1. (a) Recall that the Kolmogorov complexity of a string x is defined as

$$K(x) = \min\{|d| \mid d = \langle M, w\rangle \text{ and } M(w) = x\}$$

Show that $L_1 = \{(x, k) | x \in \{0, 1\}^*, k \text{ an integer}, K(x) \leq k\}$ is recursively enumerable.(3)

(b) Define *time-bounded Kolmogorov Complexity*, $K_t(x)$ as follows:

$$K_t(x) = \min\{|d| + \lfloor \log t \rfloor \mid d = \langle M, w\rangle \text{ and } M(w) \text{ halts in at most } t \text{ steps outputting } x\}$$

Show that $L_2 = \{(x, k) | x \in \{0, 1\}^*, k \text{ an integer}, K_t(x) = k\}$ is recursive. (3)

2. (a) Give a *regular expression* (not a DFA, though it may be useful to think of one first) for the set of *ternary* strings (strings over the alphabet $\{0, 1, 2\}$) which are multiples of two, when interpreted as numbers in base 3 written with the least significant digit first. (For example, the number 7 would correspond to the input 12.) (8)

(b) Give a CFG for the set of strings over alphabet $\{a, b\}$ with twice as many a's as b's. Do NOT give a PDA (though it may be useful to think of one first). (16)

3. A *two dimensional finite automaton* or 2D-DFA is defined as follows. The input is an $m \times n$ rectangle for any $m, n \geq 2$. The squares along the boundary of the rectangle contain the symbol # and the internal squares contain symbols over the input alphabet Σ. The transition function is a mapping $Q \times \Sigma \rightarrow Q \times \{L, R, U, D\}$ to indicate the next state and the new head position (Left, Right, Up or Down). The machine starts with its head on the top left corner of the input rectangle. The machine accepts when it enters one of the designated accept states. It rejects if it tries to move off the input rectangle or if it never halts.

   (a) Let $\mathsf{A}_{\mathsf{2D\text{-}DFA}} = \{\langle M, x\rangle \,|\, M \text{ is a } \mathsf{2D\text{-}DFA} \text{ and } M \text{ accepts } x\}$. Show that $\mathsf{A}_{\mathsf{2D\text{-}DFA}}$ is decidable. (10)

   (b) Let $\mathsf{E}_{\mathsf{2D\text{-}DFA}} = \{\langle M\rangle \,|\, M \text{ is a } \mathsf{2D\text{-}DFA} \text{ and } L_M = \phi\}$ where L_M is the language accepted by M. Show that $\mathsf{E}_{\mathsf{2D\text{-}DFA}}$ is undecidable. (20)

   (Hint: Think of computation histories of a TM.)

4. A Deterministic Counter Automaton (DCA) is a deterministic PDA, whose stack alphabet consists of just one symbol. Just like a PDA, at every transition a DCA can push or pop a string of symbols from the stack, and it can test for the stack being empty.

(a) Answer the following with a brief explanation.
(Hint: Compare with the case of CFG/PDA)

i. Is the class of languages accepted by DCA closed under complement? (3)

ii. Is the class of languages accepted by DCA closed under intersection? (6)

(b) i. Give a non-regular language accepted by a DCA. (3)

ii. Is the language $L = \{w\#w^R | w \in \{0,1\}^*\}$ accepted by a DCA? Prove. (18)
(Hint: How many configurations are possible for a DCA when it reaches #?)