Computer Science 341
Discrete Mathematics

Problem Session 2
September 30, 2002

Problem 1
Suppose one starts in the lower left corner of an $n \times n$ chess board and makes a series of moves, where each move is to go either one square to the right or one square up, so that one ends up in the top right corner.

(a) How many different paths are there?

(b) How many different paths are there that avoid the deadly land mine planted at the square in row i and column j?

Problem 2
How many ways are there to split $2n+1$ places in a committee among 3 nonempty parties, such that a coalition of any two parties constitutes majority ?

Problem 3
In how many ways can you choose 3 different numbers i, j, k such that $1 \leq i, j, k \leq 300$ and $i + j + k$ is divisible by 3.

Problem 4
How many numbers between 1000 and 9999 have all distinct digits?

Problem 5
Given a regular deck of 52 cards, what is the number of 5-card hands such that there is at least one card of each suit?

Problem 6
Give a combinatorial interpretation of the following identity:

$$\binom{n+m}{r} = \binom{n}{0}\binom{m}{r} + \binom{n}{1}\binom{m}{r-1} + \ldots + \binom{n}{r}\binom{m}{0}$$

Problem 7
Give a combinatorial interpretation of the following identity:

$$\binom{n}{m}\binom{m}{k} = \binom{n}{k}\binom{n-k}{m-k}.$$