Computer Science 341
Discrete Mathematics

Problem Session 11
Mon, Dec 9, 2002

Problem 1

Let $G(n,p)$, $0 \leq p \leq 1$, be an undirected graph on n nodes where each edge e (among the possible $n(n-1)/2$ edges on n nodes) is included in the graph with probability p independent of any other edge. If $p = \frac{1}{2}$ show that:
(a) With high probability (*i.e.* with probability tending to 1 as $n \to \infty$) the maximum size of an independent set in $G(n, \frac{1}{2})$ is no more than $(4+\varepsilon)\log n$, for any $\varepsilon > 0$.
(b) Deduce then, that for some constant $c > 0$, with probability tending to 1 as $n \to \infty$,

$$\frac{cn}{\log n} \leq \chi(G),$$

where $\chi(G)$ is the node chromatic number of G.

Problem 2

Given a random sequence of r_1 alphas and r_2 betas, find the expected value of the number of alpha runs.

Problem 3

Show that any bipartite graph $G(X \cup Y, E)$ ($|X| = |Y| = n$) with edge probability $p = \frac{1}{2}$ has a perfect matching with high probability (i.e. with probability tending to 1 as $n \to \infty$).

Problem 4

You have k dollars that you have decided to use for gambling. At each round a coin (assumed to be fair) is tossed and if it lands heads, you win a dollar, otherwise you lose a dollar. You've decided that you'll play until you have n dollars, where $n > k$ or until you have lost all your money.
(a) What is the probability that you will walk home with n dollars?
(b) What is the expected number of rounds you'll play until the game is over? (i.e. you decide to go home with 0 or n dollars)