COS 341 Discrete Mathematics

Administrative Information

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Discussion Sessions

- Discussion Session on Monday 5-6:00pm, 105 CS building
- Mailing list for class:
- Send mail to majordomo@cs.princeton.edu
- subscribe cs341 in the body

Proof Techniques: the pigeonhole principle

- n+1 pigeons in n holes
- There exists one hole with at least 2 pigeons
- Generalization: a pigeons in b holes
- There exists one hole with at least $\lceil a/b \rceil$ pigeons

A nontrivial proof

Consider the numbers 1,2 … 1000. Show that amongst any 501 of them there exist two numbers such that one divides the other.

A nontrivial proof

Consider the numbers 1,2 … 1000. Show that amongst any 501 of them there exist two numbers such that one divides the other.

Write each number in the form

$2^k(2m+1), \quad k, m \geq 0$

Since m takes at most 500 distinct values, the set contains two numbers of the form

$2^k(2m+1)$ and $2^{k'}(2m+1)$

Building blocks of logic

- Proposition: declarative sentence that is true or false (but not both).

  x + y = z
  2 + 2 = 3
  Today is Wednesday

- Basic building blocks of logic
- Usually denoted by lowecase letters: *p, q, r, s*
- Truth value of proposition denoted by T or F

Building New Propositions

Negation

p	¬p
T	F
F	T

Truth table

Conjunction (AND)

p	q	p q
T	T	T
T	F	F
F	T	F
F	F	F

Disjunction (OR)

p	q	p∨q
T	T	T
T	F	T
F	T	T
F	F	F

Exclusive OR

p	q	p⊕q
T	T	F
T	F	T
F	T	T
F	F	F

Disjunction (OR)

p	q	p∨q
T	T	T
T	F	T
F	T	T
F	F	F

Implication

p	q	p→q
T	T	T
T	F	F
F	T	T
F	F	T

p implies q
if p, then q
q if p
q when p
q whenever p
q follows from p
p is sufficient for q
a sufficient condition for q is p
q is necessary for p
a necessary condition for p is q
p only if q

Implication

p	q	p→q
T	T	T
T	F	F
F	T	T
F	F	T

Biconditional

p	q	p↔q
T	T	T
T	F	F
F	T	F
F	F	T

p if and only if q
p iff q
p is necessary and sufficient for q
if p then q and conversely

Translating from English

"You can access the internet from campus only if you are a computer science major or you are not a freshman"

a : You can access the internet from campus
c : You are a computer science major
f : You are a freshman

$a \rightarrow (c \vee \neg f)$

Precendence rules

Operator	Precedence
$\neg$	1
$\wedge$	2
$\vee$	3
$\rightarrow$	4
$\leftrightarrow$	5

Logical Equivalences

- A compound proposition that is always true is called a tautology.
- Propositions p and q are logically equivalent if they have the same truth values in all possible cases,
  i.e. if $p \leftrightarrow q$ is a tautology
- This is denoted by the notation $p \equiv q$
- Proved by verifying that truth tables agree or by using rules of logical equivalence

Equivalence by truth table

Show that $p \rightarrow q$ and $\neg p \vee q$ are logically equivalent

p	q	$\neg p$	$\neg p \vee q$	$p \rightarrow q$
T	T	F	T	T
T	F	F	F	F
F	T	T	T	T
F	F	T	T	T

Logical Equivalences

Equivalence	Name
$p \wedge T \equiv p$ $p \vee F \equiv p$	Identity laws
$p \vee T \equiv T$ $p \wedge F \equiv F$	Domination laws
$p \vee p \equiv p$ $p \wedge p \equiv p$	Idempotent laws
$\neg(\neg p) \equiv p$	Double negation law

Logical Equivalences

Equivalence	Name
$p \vee q \equiv q \vee p$ $p \wedge q \equiv q \wedge p$	Commutative laws
$(p \vee q) \vee r \equiv p \vee (q \vee r)$ $(p \wedge q) \wedge r \equiv p \wedge (q \wedge r)$	Associative laws
$p \vee (q \wedge r) \equiv (p \vee q) \wedge (p \vee r)$ $p \wedge (q \vee r) \equiv (p \wedge q) \vee (p \wedge r)$	Distributive laws

Logical Equivalences

Equivalence	Name
$\neg(p \wedge q) \equiv \neg p \vee \neg q$ $\neg(p \vee q) \equiv \neg p \wedge \neg q$	De Morgan's laws
$p \vee (p \wedge q) \equiv p$ $p \wedge (p \vee q) \equiv p$	Absorption laws
$p \vee \neg p \equiv T$ $p \wedge \neg p \equiv F$	Negation laws

Applying equivalence laws: example

Show that $(p \wedge q) \rightarrow (p \vee q)$ is a tautology

$(p \wedge q) \rightarrow (p \vee q)$

$\equiv \neg(p \wedge q) \vee (p \vee q)$ example

$\equiv (\neg p \vee \neg q) \vee (p \vee q)$ first De Morgan's Law

$\equiv (\neg p \vee p) \vee (\neg q \vee q)$ associative and commutative laws

$\equiv T \vee T$

$\equiv T$ domination law

Other Equivalences

Implication

$$p \to q \equiv \neg p \lor q$$
$$p \to q \equiv \neg q \to \neg p$$
$$(p \to q) \land (p \to r) \equiv p \to (q \land r)$$
$$(p \to r) \land (q \to r) \equiv (p \lor q) \to r$$

Biconditionals

$$p \leftrightarrow q \equiv (p \to q) \land (q \to p)$$
$$p \leftrightarrow q \equiv \neg q \leftrightarrow \neg p$$
$$p \leftrightarrow q \equiv (p \land q) \lor (\neg p \land \neg q)$$
$$\neg (p \leftrightarrow q) \equiv p \leftrightarrow \neg q$$

Quantifiers

- Universal quantifier $\forall$
  $\forall x\ P(x)$
- Existential quantifier $\exists$
  $\exists x\ P(x)$

Negation of quantifiers

$$\neg \forall x\ P(x) \equiv \exists x \neg P(x)$$

$$\neg \exists x\ Q(x) \equiv \forall x \neg Q(x)$$

Rules of inference

Justification of steps used to show conclusion follows logically from a set of hypothesis.

e.g. The tautology $(p \land (p \to q)) \to q$ gives the following rule of inference called modus ponens

$$\begin{array}{l} p \\ p \to q \\ \hline \therefore q \end{array}$$

Rule of inference	Name
p $\therefore p \lor q$	Addition
$p \land q$ $\therefore p$	Simplification
p q $\therefore p \land q$	Conjunction
p $p \to q$ $\therefore q$	Modus ponens
$\neg q$ $p \to q$ $\therefore \neg p$	Modus tollens
$p \to q$ $q \to r$ $\therefore p \to r$	Hypothetical syllogism
$p \lor q$ $\neg p$ $\therefore q$	Disjunctive syllogism