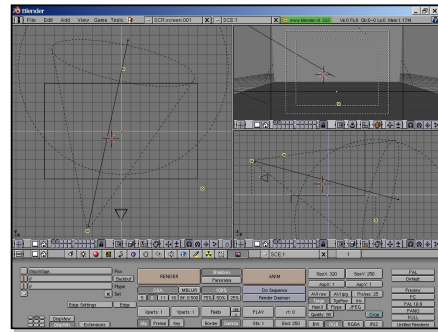


# Computer Animation

Adam Finkelstein  
Princeton University  
COS 426, Fall 2001

## Advertisement

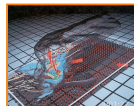


## Computer Animation

- What is animation?
  - Make objects change over time according to scripted actions
- What is simulation?
  - Predict how objects change over time according to physical laws

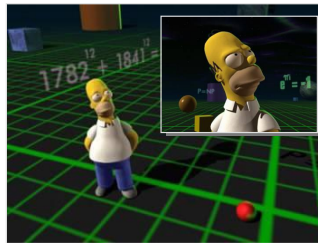


Pixar



University of Illinois

## 3-D and 2-D animation



Homer 3-D



Homer 2-D

## Outline

- Principles of animation
- Keyframe animation
- Articulated figures
- Kinematics
- Dynamics



Angel Plate 1

## Principles of Traditional Animation

- Squash and stretch
- Slow In and out
- Anticipation
- Exaggeration
- Follow through and overlapping action
- Timing
- Staging
- Straight ahead action and pose-to-pose action
- Arcs
- Secondary action
- Appeal

Disney

**Principles of Traditional Animation** 7

- Squash and stretch

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**Principles of Traditional Animation** 8

- Slow In and Out

Watt Figure 13.5

**Principles of Traditional Animation** 9

- Anticipation (and squash & stretch)

Lasseter '87

**Principles of Traditional Animation** 10

- Squash and stretch
- Slow In and out
- Anticipation
- Exaggeration
- Follow through and overlapping action
- Timing
- Staging
- Straight ahead action and pose-to-pose action
- Arcs
- Secondary action
- Appeal

Disney

**Computer Animation** 11

- Animation pipeline
  - 3D modeling
  - Motion specification
  - Motion simulation
  - Shading, lighting, & rendering
  - Postprocessing

Pixar

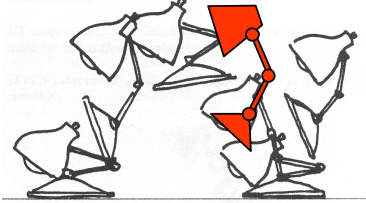
**Keyframe Animation** 12

- Define character poses at specific time steps called "keyframes"

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**Keyframe Animation** 13

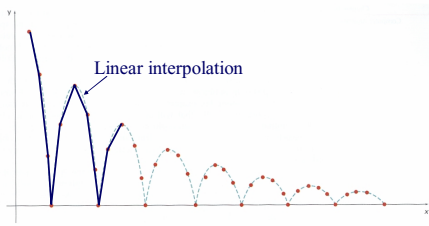
- Interpolate variables describing keyframes to determine poses for character "in-between"



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**Keyframe Animation** 14

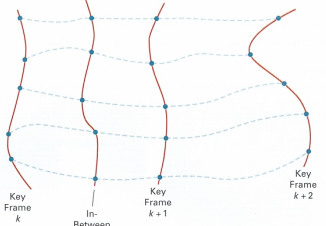
- Inbetweening:
  - Linear interpolation - usually not enough continuity



H&B Figure 16.16

**Keyframe Animation** 15

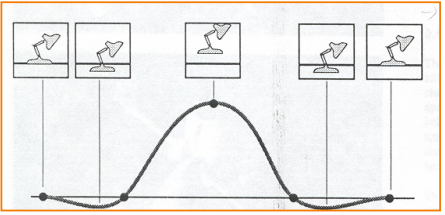
- Inbetweening:
  - Spline interpolation - maybe good enough



H&B Figure 16.11

**Keyframe Animation** 16

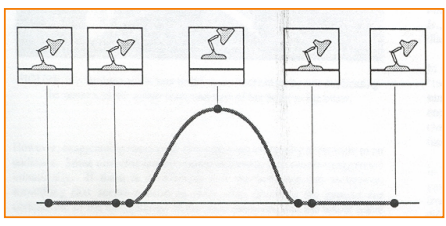
- Inbetweening:
  - Cubic spline interpolation - maybe good enough
    - » May not follow physical laws



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**Keyframe Animation** 17

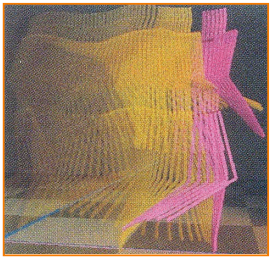
- Inbetweening:
  - Cubic spline interpolation - maybe good enough
    - » May not follow physical laws



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**Keyframe Animation** 18

- Inbetweening:
  - Inverse kinematics or dynamics




Rose et al. '96

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### Outline

- Principles of animation
- Keyframe animation
- Articulated figures
- Kinematics
- Dynamics

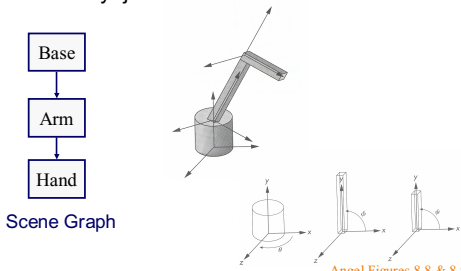


Angel Plate 1

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### Articulated Figures

- Character poses described by set of rigid bodies connected by "joints"



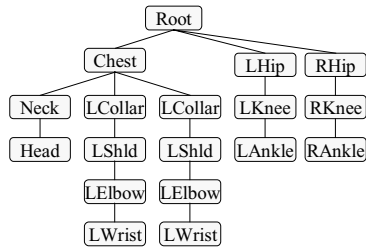
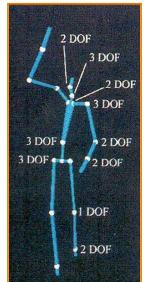
Scene Graph

Angel Figures 8.8 & 8.9

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### Articulated Figures

- Well-suited for humanoid characters





Rose et al. '96

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### Articulated Figures

- Joints provide handles for moving articulated figure

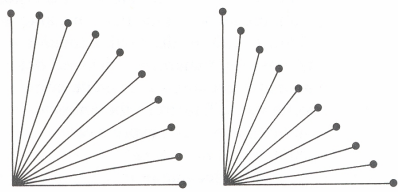


Mike Marr, COS 426, Princeton University, 1995

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### Articulated Figures

- Inbetweening
  - Compute joint angles between keyframes

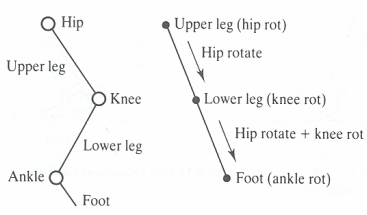


Watt & Watt

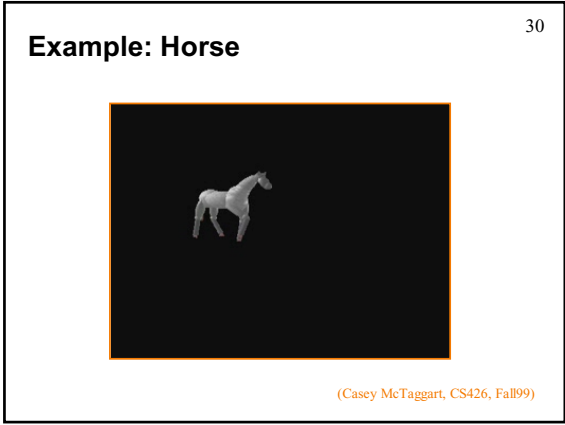
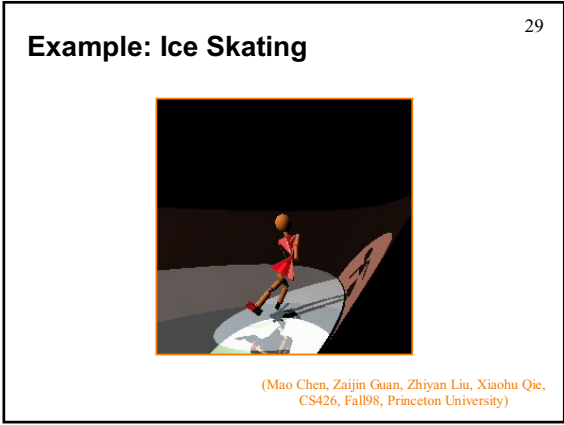
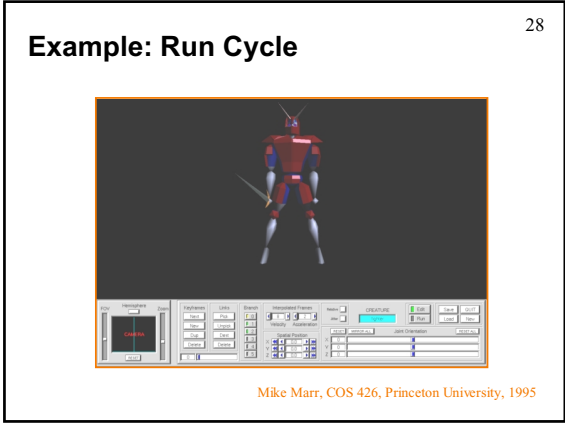
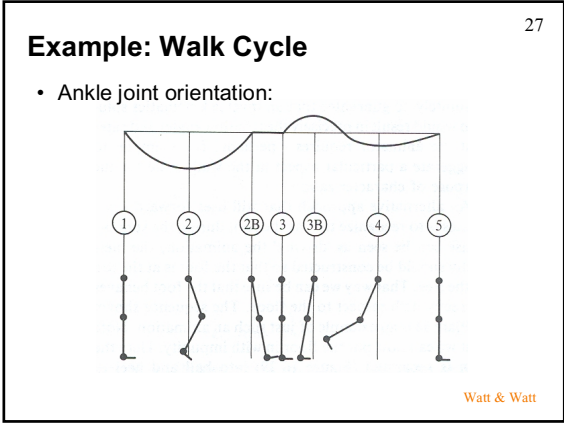
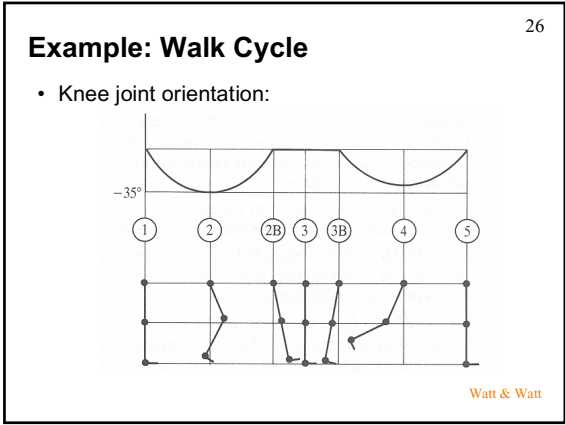
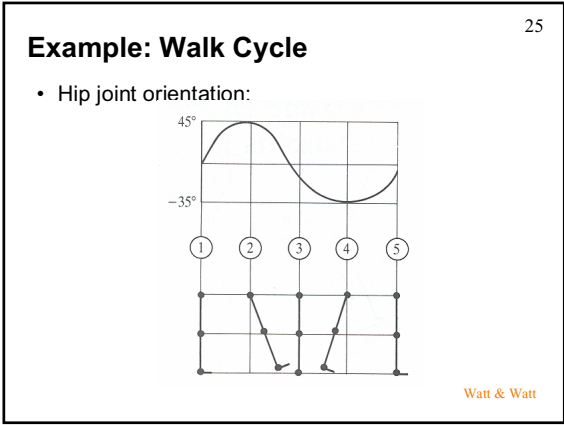
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### Example: Walk Cycle

- Articulated figure:




Watt & Watt



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### Outline

- Principles of animation
- Keyframe animation
- Articulated figures
- Kinematics**
- Dynamics**



Angel Plate 1

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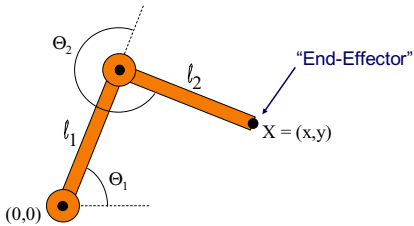
### Kinematics and Dynamics

- Kinematics**
  - Considers only motion
  - Determined by positions, velocities, accelerations
- Dynamics**
  - Considers underlying forces
  - Compute motion from initial conditions and physics

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### Example: 2-Link Structure

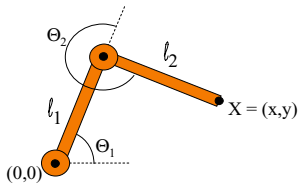
- Two links connected by rotational joints



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### Forward Kinematics

- Animator specifies joint angles:  $\Theta_1$  and  $\Theta_2$
- Computer finds positions of end-effector: X

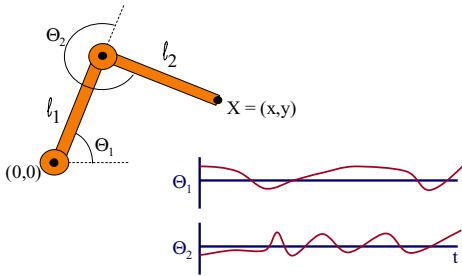


$$X = (l_1 \cos \Theta_1 + l_2 \cos(\Theta_1 + \Theta_2), l_1 \sin \Theta_1 + l_2 \sin(\Theta_1 + \Theta_2))$$

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### Forward Kinematics

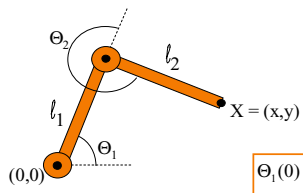
- Joint motions can be specified by spline curves



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### Forward Kinematics

- Joint motions can be specified by initial conditions and velocities



$$\Theta_1(0) = 60^\circ \quad \Theta_2(0) = 250^\circ$$

$$\frac{d\Theta_1}{dt} = 1.2 \quad \frac{d\Theta_2}{dt} = -0.1$$

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### Example: 2-Link Structure

- What if animator knows position of "end-effector"

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### Inverse Kinematics

- Animator specifies end-effector positions: X
- Computer finds joint angles:  $\Theta_1$  and  $\Theta_2$ :

$$\Theta_2 = \cos^{-1} \left( \frac{x^2 + y^2 - l_1^2 - l_2^2}{2l_1 l_2} \right)$$

$$\Theta_1 = \frac{-(l_2 \sin(\Theta_2)x + (l_1 + l_2 \cos(\Theta_2))y}{(l_2 \sin(\Theta_2))y + (l_1 + l_2 \cos(\Theta_2))x}$$

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### Inverse Kinematics

- End-effector positions can be specified by spline curves

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### Inverse Kinematics

- Problem for more complex structures
  - System of equations is usually under-defined
  - Multiple solutions

Three unknowns:  $\Theta_1, \Theta_2, \Theta_3$   
Two equations: x, y

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### Inverse Kinematics

- Solution for more complex structures:
  - Find best solution (e.g., minimize energy in motion)
  - Non-linear optimization

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### Inverse Kinematics


"Ballboy"

Fujito, Milliron, Ngan, & Sanocki  
Princeton University

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## Summary of Kinematics

- Forward kinematics
  - Specify conditions (joint angles)
  - Compute positions of end-effectors
- Inverse kinematics
  - "Goal-directed" motion
  - Specify goal positions of end effectors
  - Compute conditions required to achieve goals



Inverse kinematics provides easier specification for many animation tasks, but it is computationally more difficult

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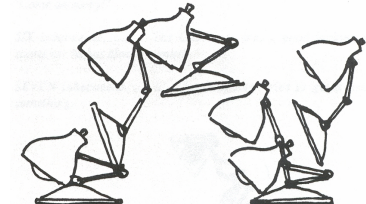
## Overview

- Kinematics
  - Considers only motion
  - Determined by positions, velocities, accelerations
- Dynamics
  - Considers underlying forces
  - Compute motion from initial conditions and physics

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## Dynamics

- Simulation of physics insures realism of motion




Lasseter '87

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## Spacetime Constraints


- Animator specifies constraints:
  - What the character's physical structure is
    - » e.g., articulated figure
  - What the character has to do
    - » e.g., jump from here to there within time t
  - What other physical structures are present
    - » e.g., floor to push off and land
  - How the motion should be performed
    - » e.g., minimize energy



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## Spacetime Constraints

- Computer finds the "best" physical motion satisfying constraints
- Example: particle with jet propulsion
  - $\mathbf{x}(t)$  is position of particle at time t
  - $\mathbf{f}(t)$  is force of jet propulsion at time t
  - Particle's equation of motion is:
 
$$m\mathbf{x}'' - \mathbf{f} - m\mathbf{g} = 0$$
  - Suppose we want to move from a to b within  $t_0$  to  $t_1$  with minimum jet fuel:
 
$$\text{Minimize } \int_{t_0}^{t_1} |f(t)|^2 dt \text{ subject to } x(t_0)=a \text{ and } x(t_1)=b$$



Witkin & Kass '88

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## Spacetime Constraints

- Discretize time steps:
 
$$x'_i = \frac{x_i - x_{i-1}}{h}$$

$$x''_i = \frac{x_{i+1} - 2x_i + x_{i-1}}{h^2}$$

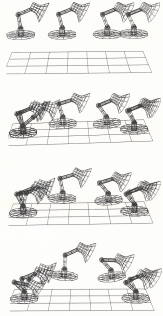
$$m \left( x''_i = \frac{x_{i+1} - 2x_i + x_{i-1}}{h^2} \right) - f_i - mg = 0$$
- Minimize  $h \sum_i |f_i|^2$  subject to  $x_0=a$  and  $x_i=b$

Witkin & Kass '88



**Spacetime Constraints** 49

- Solve with iterative optimization methods



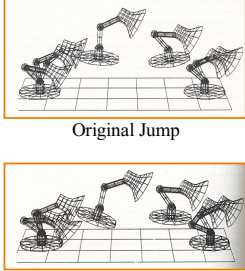
Witkin & Kass '88

**Spacetime Constraints** 50

- Advantages:
  - Free animator from having to specify details of physically realistic motion with spline curves
  - Easy to vary motions due to new parameters and/or new constraints
- Challenges:
  - Specifying constraints and objective functions
  - Avoiding local minima during optimization

**Spacetime Constraints** 51

- Adapting motion:



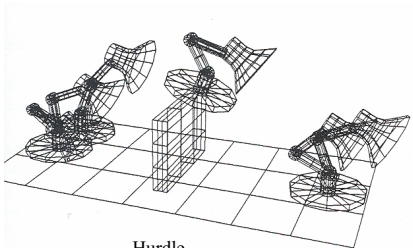
Original Jump

Heavier Base

Witkin & Kass '88

**Spacetime Constraints** 52

- Adapting motion:

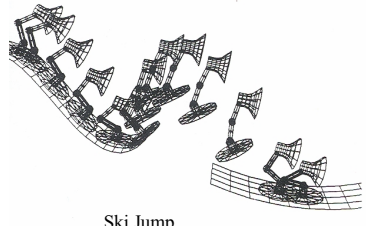


Hurdle

Witkin & Kass '88

**Spacetime Constraints** 53

- Adapting motion:




Ski Jump

Witkin & Kass '88

**Spacetime Constraints** 54

- Editing motion:




Li et al. '99

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## Spacetime Constraints

- Morphing motion:



*Gleicher '98*

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## Spacetime Constraints

- Advantages:
  - Free animator from having to specify details of physically realistic motion with spline curves
  - Easy to vary motions due to new parameters and/or new constraints
- Challenges:
  - Specifying constraints and objective functions
  - Avoiding local minima during optimization


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## Dynamics

- Other physical simulations:
  - Rigid bodies
  - Soft bodies
  - Cloth
  - Liquids
  - Gases
  - etc.



**Hot Gases**  
*(Foster & Metaxas '97)*



**Cloth**  
*(Baraff & Witkin '98)*

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## Summary

- Principles of animation
- Keyframe animation
- Articulated figures
- Kinematics
  - Forward kinematics
  - Inverse kinematics
- Dynamics
  - Space-time constraints
  - Also other physical simulations