

## Solid Modeling

Adam Finkelstein  
Princeton University  
COS 426, Fall 2001

### 3D Object Representations

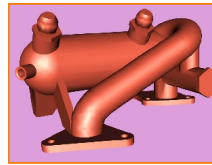
- Raw data
  - Point cloud
  - Range image
  - Polygon soup
- Surfaces
  - Mesh
  - Subdivision
  - Parametric
  - Implicit
- Solids
  - Voxels
  - BSP tree
  - CSG
  - Sweep
- High-level structures
  - Scene graph
  - Skeleton
  - Application specific

### 3D Object Representations

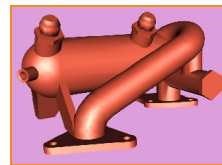
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### Implicit Surfaces

- Points satisfying:  $F(x,y,z) = 0$



Polygonal Model

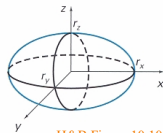


Implicit Model

Bill Lorensen  
SIGGRAPH 99  
Course #4 Notes

### Implicit Surfaces

- Example: quadric
  - $f(x,y,z) = ax^2 + by^2 + cz^2 + 2dxy + 2eyz + 2fzx + 2gx + 2hy + 2jz + k$
- Common quadric surfaces:
  - Sphere
  - Ellipsoid  $\rightarrow \left(\frac{x}{r_x}\right)^2 + \left(\frac{y}{r_y}\right)^2 + \left(\frac{z}{r_z}\right)^2 - 1 = 0$
  - Torus
  - Paraboloid
  - Hyperboloid



H&B Figure 10.10

### Implicit Surfaces

- Advantages:
  - Very concise
  - Guaranteed validity
  - Easy to test if point is on surface
  - Easy to intersect two surfaces
- Disadvantages:
  - Hard to describe complex shapes
  - Hard to enumerate points on surface
  - Hard to draw

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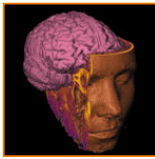
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### Solid Modeling

- Represent solid interiors of objects
  - Surface may not be described explicitly



Visible Human  
*(National Library of Medicine)*




SUNY Stony Brook

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### Motivation 1

- Some acquisition methods generate solids
  - Example: CAT scan



Stanford University

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### Motivation 2

- Some applications require solids
  - Example: CAD/CAM

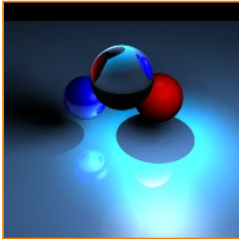


Intergraph Corporation

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### Motivation 3

- Some algorithms require solids
  - Example: ray tracing with refraction

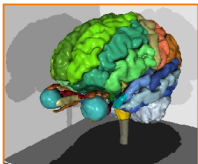


Addy Ngan and Zaijin Guan  
COS 426, 1998  
Princeton University

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### Solid Modeling Representations

- What makes a good solid representation?
  - Accurate
  - Concise
  - Affine invariant
  - Easy acquisition
  - Guaranteed validity
  - Efficient boolean operations
  - Efficient display



Lorenzen

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### Solid Modeling Representations

- Voxels
- Quadtrees & Octrees
- Binary space partitions
- Constructive solid geometry

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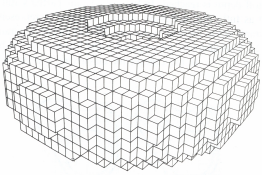
### Solid Modeling Representations

- **Voxels**
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### Voxels

- Partition space into uniform grid
  - Grid cells are called a *voxels* (like pixels)
- Store properties of solid object with each voxel
  - Occupancy
  - Color
  - Density
  - Temperature
  - etc.

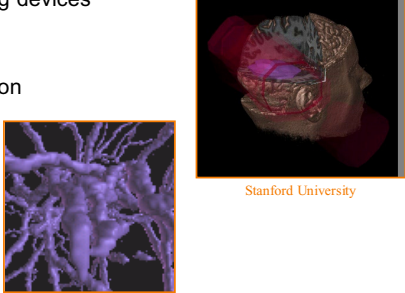


FvDFH Figure 12.20

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### Voxel Acquisition

- Scanning devices
  - MRI
  - CAT
- Simulation
  - FEM



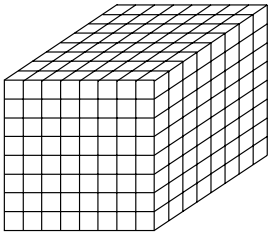
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### Voxel Storage

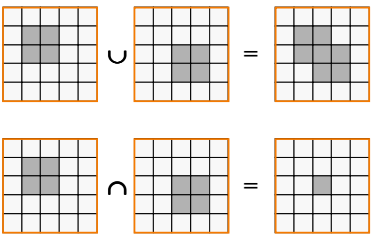
- $O(n^3)$  storage for  $n \times n \times n$  grid
  - 1 billion voxels for  $1000 \times 1000 \times 1000$



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### Voxel Boolean Operations

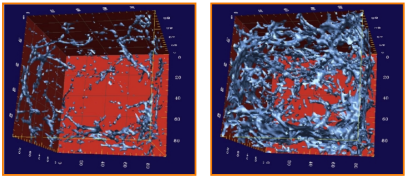
- Compare objects voxel by voxel
  - Trivial



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## Voxel Display

- Isosurface rendering
  - Render surfaces bounding volumetric regions of constant value (e.g., density)

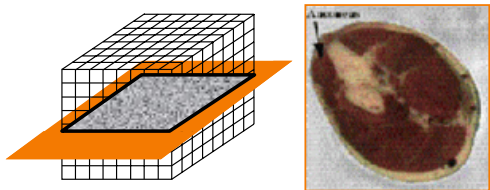


Isosurface Visualization  
Princeton University

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## Voxel Display

- Slicing
  - Draw 2D image resulting from intersecting voxels with a plane

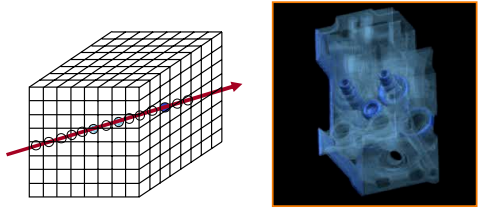


Visible Human  
*(National Library of Medicine)*

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## Voxel Display

- Ray casting
  - Integrate density along rays through pixels



Engine Block  
Stanford University

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## Voxels

- Advantages
  - Simple, intuitive, unambiguous
  - Same complexity for all objects
  - Natural acquisition for some applications
  - Trivial boolean operations
- Disadvantages
  - Approximate
  - Not affine invariant
  - Large storage requirements
  - Expensive display

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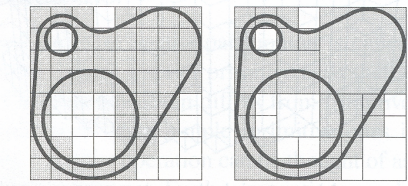
## Solid Modeling Representations

- Voxels
- **Quadtrees & Octrees**
- Binary space partitions
- Constructive solid geometry

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## Quadtrees & Octrees

- Refine resolution of voxels hierarchically
  - More concise and efficient for non-uniform objects



Uniform Voxels                  Quadtree

FvDFH Figure 12.21

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### Quadtree Boolean Operations

FvDFH Figure 12.24

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### Quadtree Display

- Extend voxel methods
  - Slicing
  - Isosurface extraction
  - Ray casting

Finding neighbor cell requires traversal of hierarchy ( $O(1)$ )

FvDFH Figure 12.25

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### Solid Modeling Representations

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### Binary Space Partitions (BSPs)

- Recursive partition of space by planes
  - Mark leaf cells as inside or outside object

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### BSP Fundamentals

- Single geometric operation
  - Partition a convex region by a hyperplane
- Single combinatorial operation
  - Two child nodes added as leaf nodes

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### BSP is a Search Structure

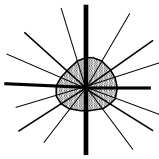
- Exploit hierarchy of convex regions
  - Regions decrease in size along any tree path
  - Regions converge in the limit to the surface

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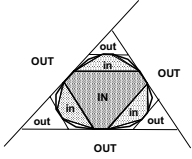
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### BSP Acquisition

- Must construct a "good" binary search structure
  - Efficiency comes from logarithmic tree depth



"Bad"



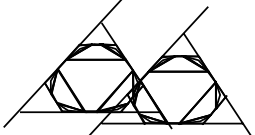
"Good"

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### BSP Boolean Operations

- Divide and conquer
  - Each node  $V$  corresponds to a convex region containing all geometry in the subtree rooted at  $V$
  - No intersection with bounding volume of  $V$  means no intersection with subtree rooted at  $V$
  - Do detail work only in regions required
  - Boolean operations grow with  $O(\log n)$  if "good" tree

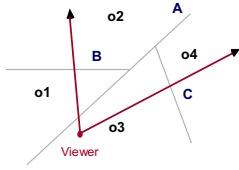


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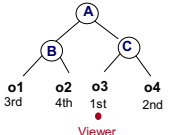
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### BSP Display

- Visibility ordering
  - Determine on which side of plane the viewer lies
    - » near-subtree -> polygons on split -> far-subtree



Partitioning Tree



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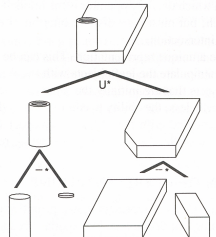
### Solid Modeling Representations

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### Constructive Solid Geometry (CSG)

- Represent solid object as hierarchy of boolean operations
  - Union
  - Intersection
  - Difference

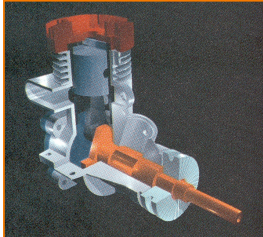


FvDFH Figure 12.27

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### CSG Acquisition

- Interactive modeling programs
  - CAD/CAM



H&B Figure 9.9

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### CSG Boolean Operations

- Create a new CSG node joining subtrees
  - Union
  - Intersection
  - Difference

FvDPH Figure 12.27

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### CSG Display & Analysis

- Ray casting

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### Summary

	Voxels	Octree	BSP	CSG
Accurate	No	No	Some	Some
Concise	No	No	No	Yes
Affine invariant	No	No	Yes	Yes
Easy acquisition	Some	Some	No	Some
Guaranteed validity	Yes	Yes	Yes	No
Efficient boolean operations	Yes	Yes	Yes	Yes
Efficient display	No	No	Yes	No

