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Image Processing

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(covering for Finkelstein 9/18)
Princeton University
COS 426, Fall 2001

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Overview

- Image representation
  - What is an image?
- Halftoning and dithering
  - Trade spatial resolution for intensity resolution
  - Reduce visual artifacts due to quantization
- Sampling and reconstruction
  - Key steps in image processing
  - Avoid visual artifacts due to aliasing

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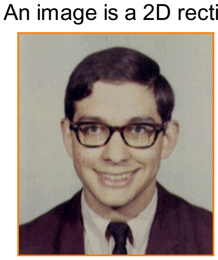
What is an Image?

- An image is a 2D rectilinear array of pixels


Continuous image


Digital image

4

What is an Image?

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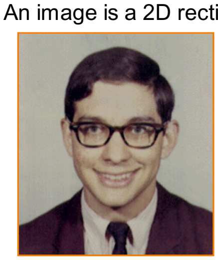
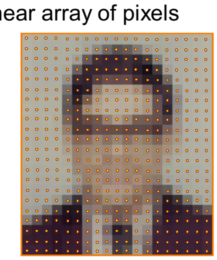

Continuous image


Digital image

A pixel is a sample, not a little square!

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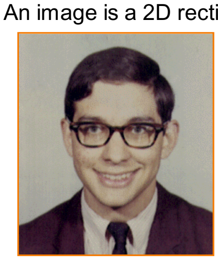
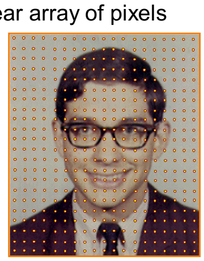
What is an Image?

- An image is a 2D rectilinear array of pixels


Continuous image


Digital image

A pixel is a sample, not a little square!

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Image Acquisition

- Pixels are samples from continuous function
  - Photoreceptors in eye
  - CCD cells in digital camera
  - Rays in virtual camera

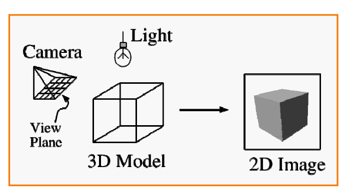


Image Display

- Re-create continuous function from samples
  - Example: cathode ray tube

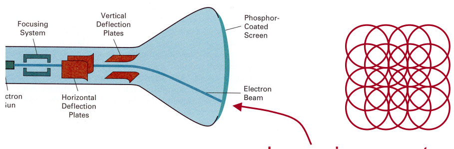


Image is reconstructed by displaying pixels with finite area (Gaussian)

Image Resolution

- Intensity resolution
  - Each pixel has only "Depth" bits for colors/intensities
- Spatial resolution
  - Image has only "Width" x "Height" pixels
- Temporal resolution
  - Monitor refreshes images at only "Rate" Hz

Typical Resolutions

	Width x Height	Depth	Rate
NTSC	640 x 480	8	30
Workstation	1280 x 1024	24	75
Film	3000 x 2000	12	24
Laser Printer	6600 x 5100	1	-

Sources of Error

- Intensity quantization
  - Not enough intensity resolution
- Spatial aliasing
  - Not enough spatial resolution
- Temporal aliasing
  - Not enough temporal resolution

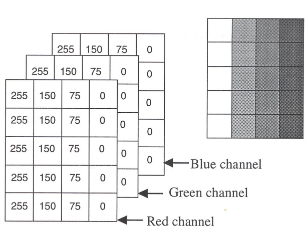
$$E^2 = \sum_{(x,y)} \left(I(x,y) - P(x,y)\right)^2$$

Overview

- Image representation
  - What is an image?
- Halftoning and dithering
  - Reduce visual artifacts due to quantization
- Sampling and reconstruction
  - Reduce visual artifacts due to aliasing

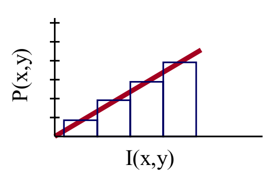
Quantization

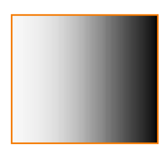
- Artifacts due to limited intensity resolution
  - Frame buffers have limited number of bits per pixel
  - Physical devices have limited dynamic range



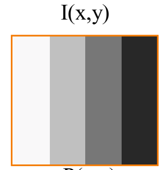
Uniform Quantization

P(x, y) = trunc(I(x, y) + 0.5)





I(x,y)



P(x,y)
(4 bits per pixel)

Uniform Quantization

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- Images with decreasing bits per pixel:

8 bits | 4 bits | 2 bits | 1 bit

Notice contouring

Reducing Effects of Quantization

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- Halftoning
  - Classical halftoning
- Dithering
  - Random dither
  - Ordered dither
  - Error diffusion dither

Classical Halftoning

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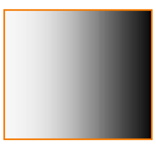
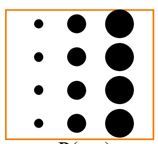
- Use dots of varying size to represent intensities
  - Area of dots proportional to intensity in image



I(x,y)



P(x,y)

Classical Halftoning

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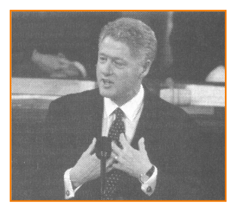
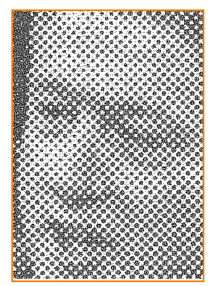


Newspaper Image



From New York Times, 9/21/99

Halftone patterns

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- Use cluster of pixels to represent intensity
  - Trade spatial resolution for intensity resolution

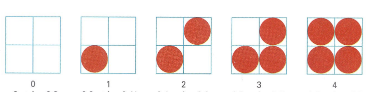

Figure 14.37 from H&B

Dithering

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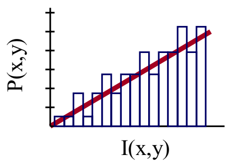
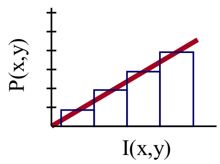
- Distribute errors among pixels
  - Exploit spatial integration in our eye
  - Display greater range of perceptible intensities

Original (8 bits) | Uniform Quantization (1 bit) | Floyd-Steinberg Dither (1 bit)

Random Dither

- Randomize quantization errors
  - Errors appear as noise



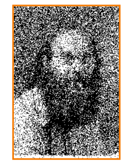


P(x, y) = trunc(I(x, y) + noise(x,y) + 0.5)

Random Dither

Original
(8 bits)

Uniform
Quantization
(1 bit)



Random
Dither
(1 bit)

Ordered Dither

- Pseudo-random quantization errors
  - Matrix stores pattern of threshholds

i = x mod n
j = y mod n
e = I(x,y) - trunc(I(x,y))
if (e > D(i,j))
    P(x,y) = ceil(I(x, y))
else
    P(x,y) = floor(I(x,y))

$$D_2 = \begin{bmatrix} 3 & 1 \\ 0 & 2 \end{bmatrix}$$

Ordered Dither

- Bayer's ordered dither matrices

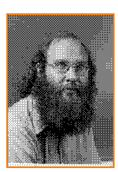
$$D_n = \begin{bmatrix} 4D_{n/2} + D_2(1,1)U_{n/2} & 4D_{n/2} + D_2(1,2)U_{n/2} \\ 4D_{n/2} + D_2(2,1)U_{n/2} & 4D_{n/2} + D_2(2,2)U_{n/2} \end{bmatrix}$$

$$D_2 = \begin{bmatrix} 3 & 1 \\ 0 & 2 \end{bmatrix} \qquad D_4 = \begin{bmatrix} 15 & 7 & 13 & 5 \\ 3 & 11 & 1 & 9 \\ 12 & 4 & 14 & 6 \\ 0 & 8 & 2 & 10 \end{bmatrix}$$

Ordered Dither

Original
(8 bits)

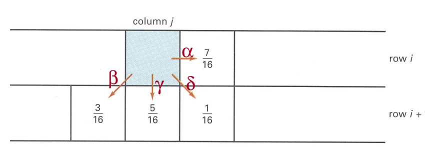
Random
Dither
(1 bit)



Ordered
Dither
(1 bit)

Error Diffusion Dither

- Spread quantization error over neighbor pixels
  - Error dispersed to pixels right and below



$$\alpha + \beta + \gamma + \delta = 1.0$$

Figure 14.42 from H&B

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Error Diffusion Dither

Original (8 bits) | Random Dither (1 bit) | Ordered Dither (1 bit) | Floyd-Steinberg Dither (1 bit)

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Overview

- Image representation
  - What is an image?
- Halftoning and dithering
  - Reduce visual artifacts due to quantization
- Sampling and reconstruction
  - Reduce visual artifacts due to aliasing

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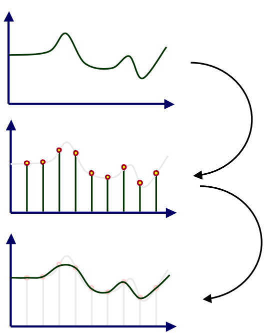
Sampling and Reconstruction



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Sampling and Reconstruction

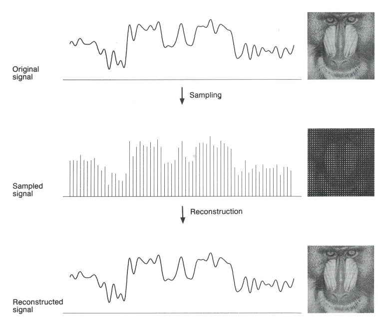

Figure 19.9 FvDFH

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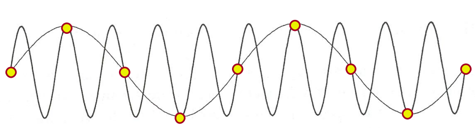
Aliasing

- In general:
  - Artifacts due to under-sampling or poor reconstruction
- Specifically, in graphics:
  - Spatial aliasing
  - Temporal aliasing



Under-sampling

Figure 14.17 FvDFH

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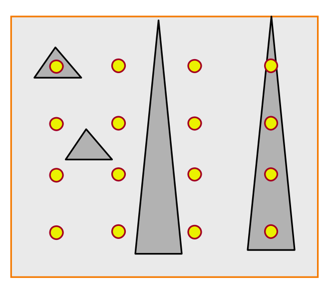
Spatial Aliasing

- Artifacts due to limited spatial resolution



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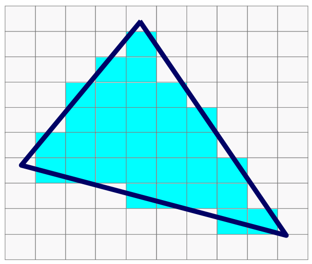
Spatial Aliasing

- Artifacts due to limited spatial resolution



"Jaggies"

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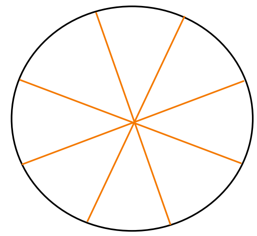
Temporal Aliasing

- Artifacts due to limited temporal resolution
  - Strobing
  - Flickering



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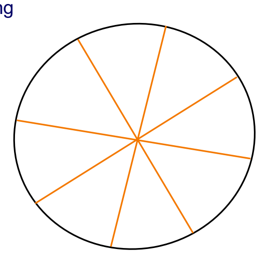
Temporal Aliasing

- Artifacts due to limited temporal resolution
  - Strobing
  - Flickering



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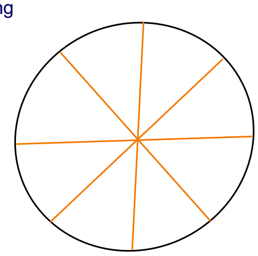
Temporal Aliasing

- Artifacts due to limited temporal resolution
  - Strobing
  - Flickering



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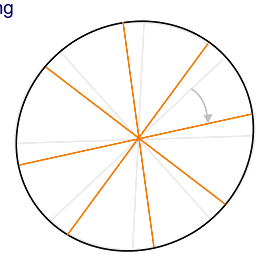
Temporal Aliasing

- Artifacts due to limited temporal resolution
  - Strobing
  - Flickering



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Antialiasing

- Sample at higher rate
  - Not always possible
  - Doesn't always solve problem
- Pre-filter to form bandlimited signal
  - Form bandlimited function (low-pass filter)
  - Trades aliasing for blurring

Must consider sampling theory!

Sampling Theory

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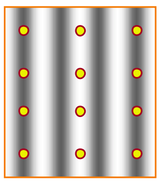
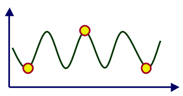
- How many samples are required to represent a given signal without loss of information?
- What signals can be reconstructed without loss for a given sampling rate?





Spectral Analysis

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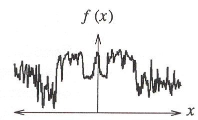
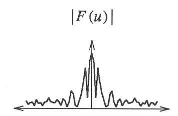
- Spatial domain:
  - Function: f(x)
  - Filtering: convolution
- Frequency domain:
  - Function: F(u)
  - Filtering: multiplication





Any signal can be written as a sum of periodic functions.

Fourier Transform

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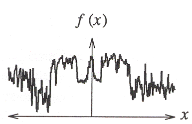

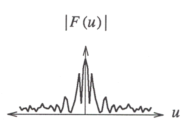

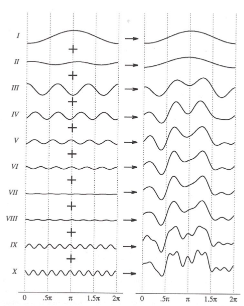

Figure 2.6 Wolberg

Fourier Transform

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- Fourier transform:

$$F(u) = \int_{-\infty}^{\infty} f(x) e^{-i2\pi xu} dx$$

- Inverse Fourier transform:

$$f(x) = \int_{-\infty}^{\infty} F(u) e^{+i2\pi ux} du$$

Sampling Theorem

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- A signal can be reconstructed from its samples, if the original signal has no frequencies above 1/2 the sampling frequency - Shannon
- The minimum sampling rate for bandlimited function is called “Nyquist rate”

A signal is bandlimited if its highest frequency is bounded. The frequency is called the bandwidth.

Convolution

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- Convolution of two functions (= filtering):

$$g(x) = f(x) \otimes h(x) = \int_{-\infty}^{\infty} f(\lambda) h(x-\lambda) d\lambda$$

- Convolution theorem
  - Convolution in frequency domain is same as multiplication in spatial domain, and vice-versa

Image Processing
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- Quantization
  - Uniform Quantization
  - Random dither
  - Ordered dither
  - Floyd-Steinberg dither
- Pixel operations
  - Add random noise
  - Add luminance
  - Add contrast
  - Add saturation
- Filtering
  - Blur
  - Detect edges
- Warping
  - Scale
  - Rotate
  - Warps
- Combining
  - Morphs
  - Composite

Image Processing
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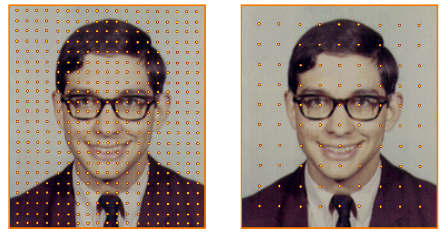
- Consider reducing the image resolution



Original image   1/4 resolution

Image Processing
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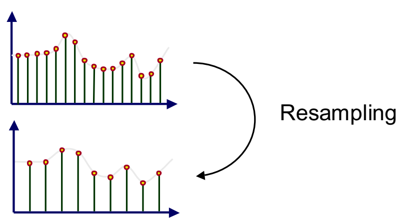
- Image processing is a resampling problem



Thou shalt avoid aliasing!

Antialiasing in Image Processing
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- General Strategy
  - Pre-filter transformed image via convolution with low-pass filter to form bandlimited signal
- Rationale
  - Prefer blurring over aliasing

Image Processing
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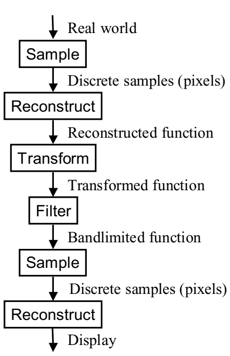

Image Processing
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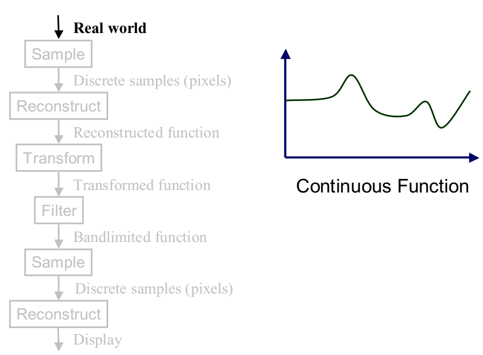

Image Processing

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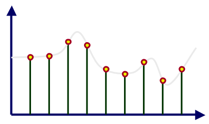
Real world
Sample
Discrete samples (pixels)
Reconstruct
Reconstructed function
Transform
Transformed function
Filter
Bandlimited function
Sample
Discrete samples (pixels)
Reconstruct
Display



Discrete Samples

Image Processing

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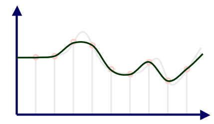
Real world
Sample
Discrete samples (pixels)
Reconstruct
Reconstructed function
Transform
Transformed function
Filter
Bandlimited function
Sample
Discrete samples (pixels)
Reconstruct
Display



Reconstructed Function

Image Processing

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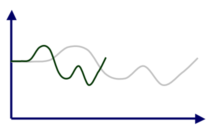
Real world
Sample
Discrete samples (pixels)
Reconstruct
Reconstructed function
Transform
Transformed function
Filter
Bandlimited function
Sample
Discrete samples (pixels)
Reconstruct
Display



Transformed Function

Image Processing

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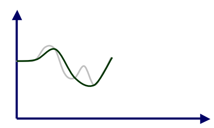
Real world
Sample
Discrete samples (pixels)
Reconstruct
Reconstructed function
Transform
Transformed function
Filter
Bandlimited function
Sample
Discrete samples (pixels)
Reconstruct
Display



Bandlimited Function

Image Processing

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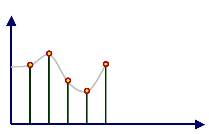
Real world
Sample
Discrete samples (pixels)
Reconstruct
Reconstructed function
Transform
Transformed function
Filter
Bandlimited function
Sample
Discrete samples (pixels)
Reconstruct
Display



Discrete samples

Image Processing

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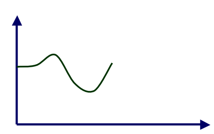
Real world
Sample
Discrete samples (pixels)
Reconstruct
Reconstructed function
Transform
Transformed function
Filter
Bandlimited function
Sample
Discrete samples (pixels)
Reconstruct
Display



Display

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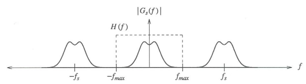
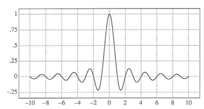
Ideal Low-Pass Filter

- Frequency domain



- Spatial domain



$$Sinc(x) = \frac{\sin \pi x}{\pi x}$$

Figure 4.5 Wolberg

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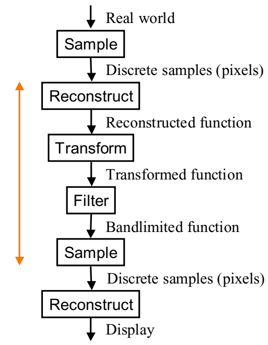
Practical Image Processing

- Finite low-pass filters
  - Point sampling (bad)
  - Triangle filter
  - Gaussian filter



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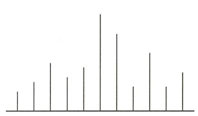
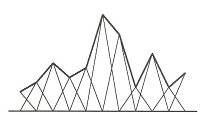
Triangle Filter

- Convolution with triangle filter



Input



Output

Figure 2.4 Wolberg

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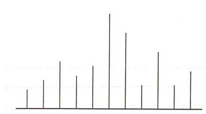
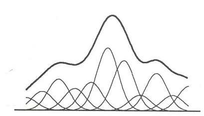
Gaussian Filter

- Convolution with Gaussian filter



Input



Output

Figure 2.4 Wolberg

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Image Processing

- Quantization
  - Uniform Quantization
  - Random dither
  - Ordered dither
  - Floyd-Steinberg dither
- Pixel operations
  - Add random noise
  - Add luminance
  - Add contrast
  - Add saturation
- Filtering
  - Blur
  - Detect edges
- Warping
  - Scale
  - Rotate
  - Warps
- Combining
  - Morphs
  - Composite

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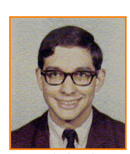
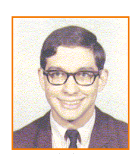
Adjusting Brightness

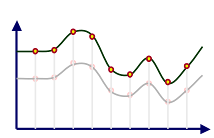
- Simply scale pixel components
  - Must clamp to range (e.g., 0 to 255)



Original



Brighter



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Adjusting Contrast

- Compute mean luminance $\mathcal{L}$ for all pixels
  - luminance = 0.30*r + 0.59*g + 0.11*b
- Scale deviation from $\mathcal{L}$ for each pixel component
  - Must clamp to range (e.g., 0 to 255)


Original


More Contrast

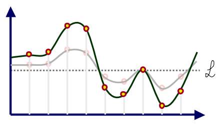

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Image Processing

- Quantization
  - Uniform Quantization
  - Random dither
  - Ordered dither
  - Floyd-Steinberg dither
- Pixel operations
  - Add random noise
  - Add luminance
  - Add contrast
  - Add saturation
- Filtering
  - Blur
  - Detect edges
- Warping
  - Scale
  - Rotate
  - Warps
- Combining
  - Morphs
  - Composite

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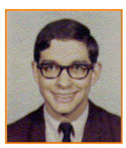
Adjust Blurriness

- Convolve with a filter whose entries sum to one
  - Each pixel becomes a weighted average of its neighbors


Original


Blur

$$\text{Filter} = \begin{bmatrix} 1/16 & 2/16 & 1/16 \\ 2/16 & 4/16 & 2/16 \\ 1/16 & 2/16 & 1/16 \end{bmatrix}$$

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Edge Detection

- Convolve with a filter that finds differences between neighbor pixels


Original


Detect edges

$$\text{Filter} = \begin{bmatrix} -1 & -1 & -1 \\ -1 & +8 & -1 \\ -1 & -1 & -1 \end{bmatrix}$$

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Image Processing

- Quantization
  - Uniform Quantization
  - Random dither
  - Ordered dither
  - Floyd-Steinberg dither
- Pixel operations
  - Add random noise
  - Add luminance
  - Add contrast
  - Add saturation
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  - Rotate
  - Warps
- Combining
  - Morphs
  - Composite

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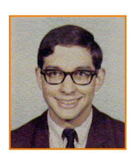
Scaling

- Resample with triangle or Gaussian filter

More on this next lecture!


Original


1/4X resolution


4X resolution

Image Processing

- Image processing is a resampling problem
  - Avoid aliasing
  - Use filtering

Summary

- Image representation
  - A pixel is a sample, not a little square
  - Images have limited resolution
- Halftoning and dithering
  - Reduce visual artifacts due to quantization
  - Distribute errors among pixels
    - » Exploit spatial integration in our eye
- Sampling and reconstruction
  - Reduce visual artifacts due to aliasing
  - Filter to avoid undersampling
    - » Blurring is better than aliasing