COS 487: Theory of Computation	Fall 2000

Assignment #4

Due: Wed, November 22 — *Sanjeev Arora*

Suggested reading (for lectures 11–15): Sipser Chapters 6 and 7. Handout 2 on Gödel's Theorem.

Problems (from lectures upto Nov. 13)

1. Show that no infinite subset of MIN_{TM} is recognizable.

2. The axiomatic system given in class has a finite number of axioms and derivation rules. Argue briefly that Gödel's Theorem also holds for any alternative system whose axioms and derivation rules are enumerable.

3. A 2CNF formula is an AND of clauses, where each clause is an OR of at most two literals. Let 2SAT $= \{\varphi : \varphi$ is a satisfiable 2CNF formula$\}$. Show that 2SAT $\in$ P.

4. Point out the fallacy in the following proof that the language $\overline{CLIQUE} = \{< G, k >:$ every clique in graph G has size less than $k\}$ is in NP.

   In class we saw that $CLIQUE$ is in NP. Just take that NDTM and swap its accept and reject states.

5. Suppose P = NP. Show that then we can do the following in polynomial time (be very careful, since the problems listed below are not decision problems!). (i) Given a boolean formula, find a satisfying assignment for it (if one exists). (ii) Given a graph and a number k, find a clique of size k in the graph (if one exists). (iii) Given $(< T > 1^n)$, where T is a number-theoretic statement, find a proof of T in Peano Arithmetic of size n (if one exists)[1].

6. For each of the following languages, state whether it is one or more of the following: in P, in NP, NP-complete, NP-hard. If you can't classify some language in any of these four categories, mention if you think that the exact classification is an open problem. Justify your answer in each case.

   (i) $A_{TM} = \{\langle M, w\rangle : M$ is a deterministic TM that accepts $w\}$.

   (ii) $\{\langle M, w, k\rangle : M$ is an NDTM that accepts w in time $k\}$.

   (iii) $\left\{\left\langle M, w, 1^k\right\rangle : M \text{ is an NDTM that accepts } w \text{ in time } k\right\}$.

[1] In other words, if P = NP mathematical proofs can be mechanically discovered in time that is polynomial in the number of symbols in the proof. Ergo, mathematicians had better hope that P $\neq$ NP.

7. (*Just when you thought you would never see calculus ever again...*) Show that the following problem is NP-complete. Given integers $a_1, a_2, \ldots, a_n$, to decide if

$$\int_0^{2\pi} \left(\prod_{i=1}^{n} \cos a_i\theta\right) d\theta \neq 0.$$

(Hint: Generalize the formula $\cos A \cos B = (\cos(A+B) + \cos(A-B))/2$ and do a reduction from PARTITION.)

8. (*Extra credit*) Describe an algorithm and some $c < 1$ such that the algorithm can decide satisfiability for 3CNF formulae on n variables in 2^{cn} time.