COS 487: Theory of Computation **Fall 2000**

Assignment #2

Due: Monday, October 16 *Sanjeev Arora*

Suggested reading (for lectures 4,5,6): Sipser Chapter 2.

Note: For all problems except Problem 1, it is OK to describe a PDA in English (possibly with some pictures thrown in).

Problems (from lectures 4, 5, 6):

1. Show that the language $\{0^m 1^n : m \leq n \leq 2m\}$ is context-free by giving a PDA that accepts it. Indicate in words why your construction works.

2. Problem 2.17 in the text.

3. Let $G = (V, \Sigma, P, <\text{stmt}>)$ be the following grammar:

$$\begin{aligned}
<\text{stmt}> &\rightarrow <\text{assignment}> \mid <\text{if-then}> \mid <\text{if-then-else}> \mid \\
&\quad <\text{begin-end}> \\
<\text{if-then}> &\rightarrow \textbf{if condition then } <\text{stmt}> \\
<\text{if-then-else}> &\rightarrow \textbf{if condition then } <\text{stmt}> \textbf{ else } <\text{stmt}> \\
<\text{begin-end}> &\rightarrow \textbf{begin } <\text{stmt-list}> \textbf{ end} \\
<\text{stmt-list}> &\rightarrow <\text{stmt-list}> \ <\text{stmt}> \mid <\text{stmt}> \\
<\text{assignment}> &\rightarrow \mathbf{a = 1}
\end{aligned}$$

   $\Sigma = \{\textbf{if}, \textbf{condition}, \textbf{then}, \textbf{else}, \textbf{begin}, \textbf{end}, \mathbf{a = 1}\}$.
   $V = \{<\text{stmt}>, <\text{assignment}>, <\text{if-then}>, <\text{if-then-else}>, <\text{begin-end}>, <\text{stmt-list}>\}$.

   G is a natural-looking grammar for a fragment of a programming language.

   a Show that G is ambiguous.

   b Give a new unambiguous grammar for the same language.

   c Let symbols like **if**, **then** etc. be assigned their usual meanings. Re-state (in words) your modification in part (b) with respect to this meaning. (*Note: Compiler designers have to worry about such semantic effects —sometimes unintended— of the way they define a CFG.*)

4. Let $\Sigma = \{0, 1, \sharp\}$.

   a Show that $\{x\sharp y : x, y \in \{0,1\}^*, x \neq y\}$ is context free.

   b Show that $\{x\sharp y : x, y \in \{0,1\}^*, x = y\}$ is not context-free.

c Using parts (a) and (b), argue (carefully!) that the set of context-free languages is not closed under complementation.

5. Let $\Sigma = \{0, 1, \sharp\}$.

   a Show that $\left\{w\sharp w^R\sharp w : w \in \{0,1\}^*\right\}$ is not context-free.

   b Express the language in part (a) as $A \cap B$, where A and B are context-free. Note that you have shown that the set of CFL's is not closed under intersection.

6. Let $G = (V, \Sigma, S, R)$ be a context-free grammar in Chomsky Normal Form. We define a directed graph whose set of vertices is V (the set of variables) and whose set of edges is

$$\{(A, B) : A, B \in V \text{ and for some } C \in V, \text{ either } A \to BC \text{ or } A \to CB \text{ is in } R\}.$$

   We say that this graph is *acyclic* if it has no (directed) cycle (and no self-loops either).

   Show that if this graph is acyclic then grammar G generates a regular language.

7. (*Optional*) Show that the context-free language $\{0^m1^n2^s : \text{either } m = n \text{ or } n = s\}$ is inherently ambiguous (i.e., every grammar for it is ambiguous).