Ray Casting

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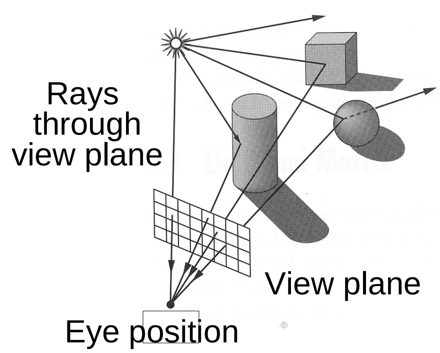
3D Rendering

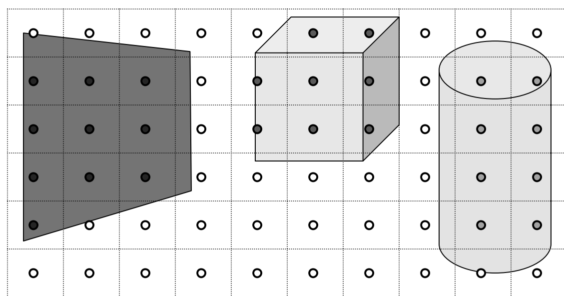
- The color of each pixel on the view plane depends on the radiance emanating from visible surfaces

Simplest method is ray casting



Ray Casting

- For each sample …
  - Construct ray from eye position through view plane
  - Find first surface intersected by ray through pixel
  - Compute color sample based on surface radiance



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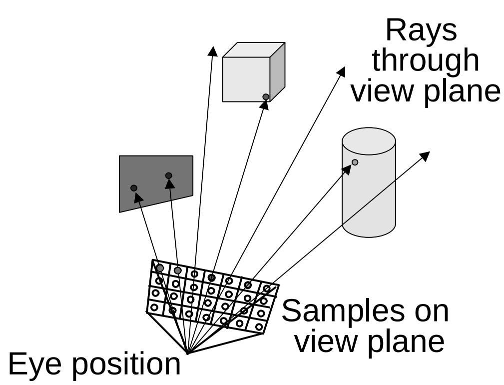

Ray Casting

- Simple implementation:

```
Image RayCast(Camera camera, Scene scene, int width, int height)
{
    Image image = new Image(width, height);
    for (int i = 0; i < width; i++) {
        for (int j = 0; j < height; j++) {
            Ray ray = ConstructRayThroughPixel(camera, i, j);
            Intersection hit = FindIntersection(ray, scene);
            image[i][j] = GetColor(hit);
        }
    }
    return image;
}
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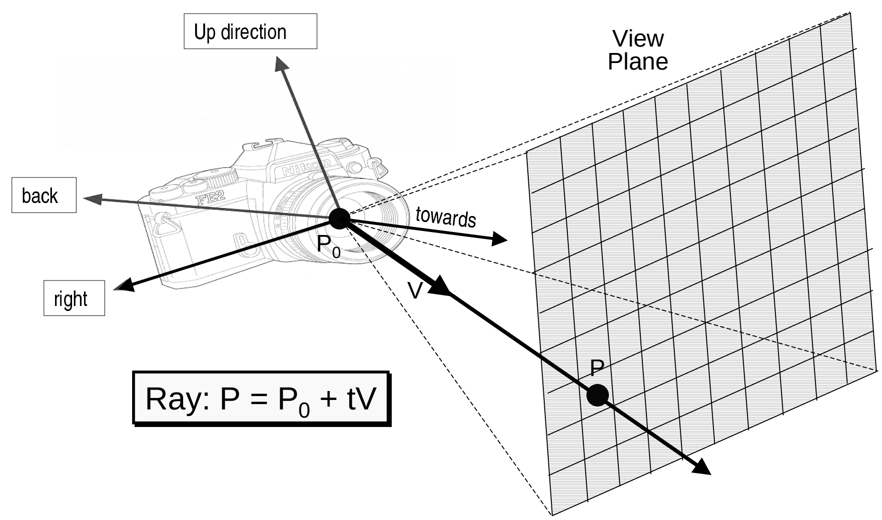
Constructing Ray Through a Pixel



Constructing Ray Through a Pixel

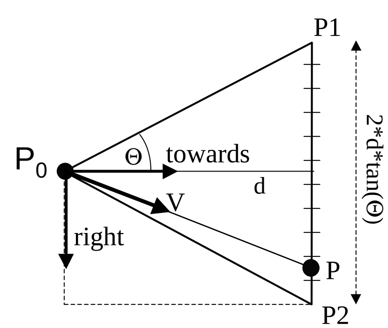
- 2D Example

Θ = frustum half-angle
d = distance to view plane

right = towards x up

$P1 = P_0 + d*towards - d*\tan(\Theta)*right$
$P2 = P_0 + d*towards + d*\tan(\Theta)*right$

$P = P1 + (i/width + 0.5) * 2*d*\tan(\Theta)*right$
$V = (P - P_0) / ||P - P_0||$



Ray: $P = P_0 + tV$

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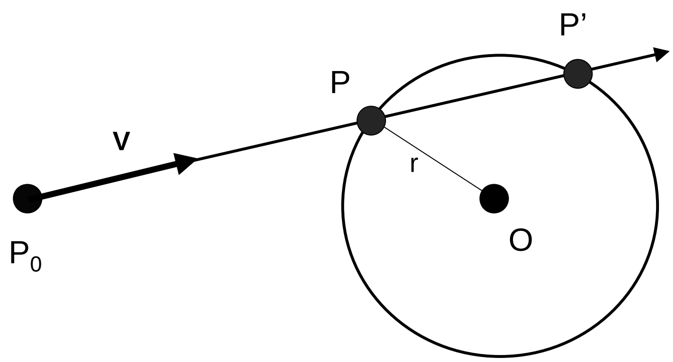
Ray-Scene Intersection

- Intersections with geometric primitives
  - Sphere
  - Triangle
  - Groups of primitives (scene)
- Acceleration techniques
  - Bounding volume hierarchies
  - Spatial partitions
    - » Uniform grids
    - » Octrees
    - » BSP trees

Ray-Sphere Intersection

Ray: $P = P_0 + tV$
Sphere: $|P - O|^2 - r^2 = 0$



Ray-Sphere Intersection I

Ray: $P = P_0 + tV$
Sphere: $|P - O|^2 - r^2 = 0$

Algebraic Method

Substituting for P, we get:

$$|\mathbf{P_0} + \mathbf{tV} - O|^2 - r^2 = 0$$

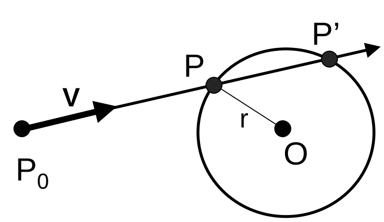
Solve quadratic equation:

$$at^2 + bt + c = 0$$

where:

$$a = 1$$
$$b = 2 V \bullet (P_0 - O)$$
$$c = |P_0 - C|^2 - r^2 = 0$$

$$P = P_0 + tV$$



Ray-Sphere Intersection II

Ray: $P = P_0 + tV$
Sphere: $|P - O|^2 - r^2 = 0$

Geometric Method

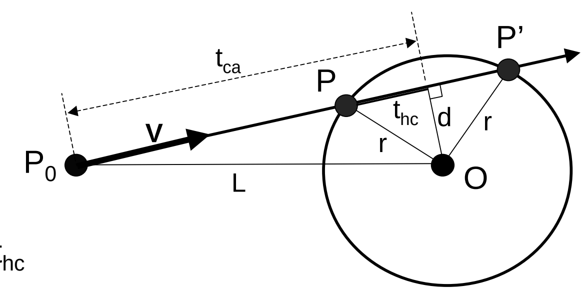
$L = O - P_0$

$t_{ca} = L \bullet V$
if ($t_{ca} < 0$) return 0

$d^2 = L \bullet L - t_{ca}^2$
if ($d^2 > r^2$) return 0

$t_{hc} = \mathrm{sqrt}(r^2 - d^2)$
$t = t_{ca} - t_{hc}$ and $t_{ca} + t_{hc}$

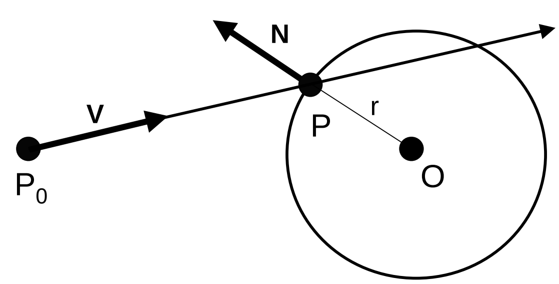
$P = P_0 + tV$



Ray-Sphere Intersection

- Need normal vector at intersection for lighting calculations

$$N = (P - O) / ||P - O||$$



Ray-Scene Intersection

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  - » **Triangle**
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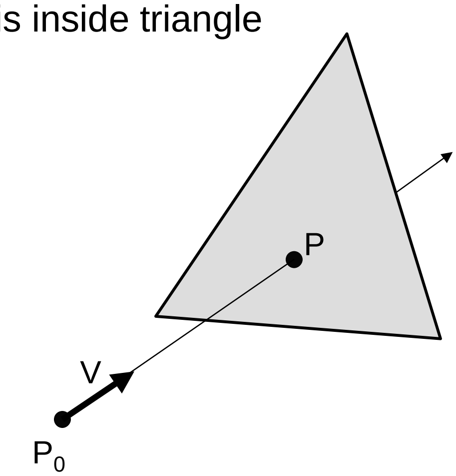
Ray-Triangle Intersection

- First, intersect ray with plane
- Then, check if point is inside triangle



Ray-Plane Intersection

Ray: $P = P_0 + tV$
Plane: $P \cdot N + d = 0$

Algebraic Method

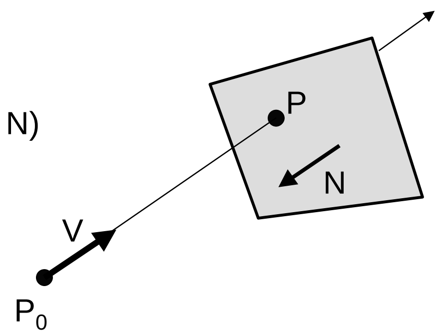
Substituting for P, we get:

$(\mathbf{P_0 + tV}) \cdot N + d = 0$

Solution:

$t = -(P_0 \cdot N + d) / (V \cdot N)$

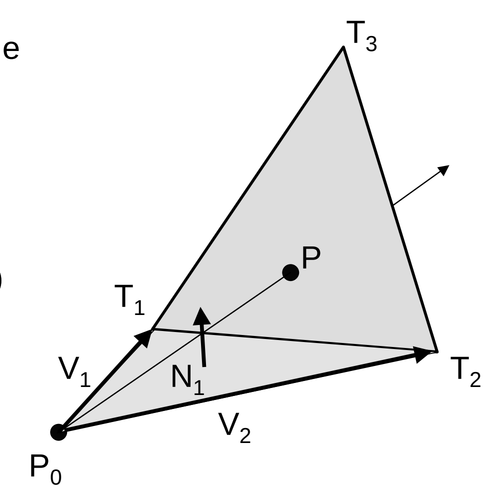
$P = P_0 + tV$



Ray-Triangle Intersection I

- Check if point is inside triangle algebraically

For each side of triangle
    $V_1 = T_1 - P$
    $V_2 = T_2 - P$
    $N_1 = V_2 \times V_1$
    Normalize N_1
    $d_1 = -P_0 \cdot N_1$
    if $((P \cdot N_1 + d_1) < 0)$
        return FALSE;
end



Ray-Triangle Intersection II

- Check if point is inside triangle parametrically

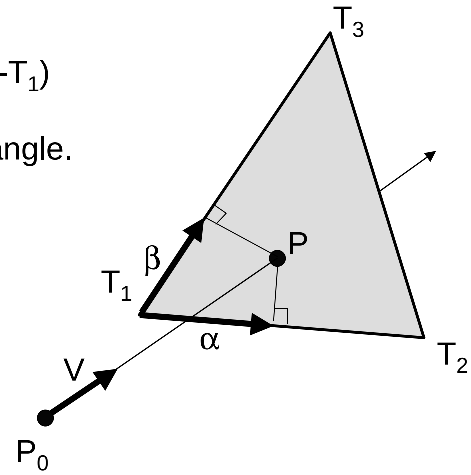
Compute α, β:

$P = \alpha (T_2-T_1) + \beta (T_3-T_1)$

Check if point inside triangle.

$0 \le \alpha \le 1$ and

$0 \le \beta \le 1$



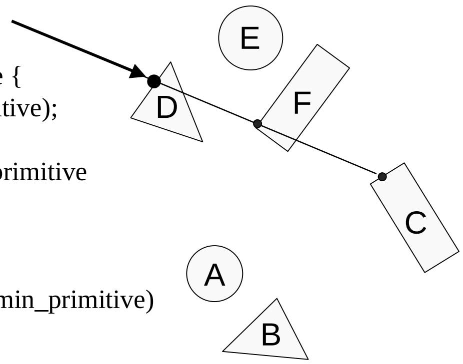
Other Ray-Primitive Intersections

- Cone, cylinder, ellipsoid:
  - Similar to sphere
- Box
  - Intersect 3 front-facing planes, return closest
- Convex polygon
  - Same as triangle (check point-in-polygon algebraically)
- Concave polygon
  - Same plane intersection
  - More complex point-in-polygon test

Ray-Scene Intersection

- Find intersection with front-most primitive in group

```
Intersection FindIntersection(Ray ray, Scene scene)
{
    min_t = infinity
    min_primitive = NULL
    For each primitive in scene {
        t = Intersect(ray, primitive);
        if (t < min_t) then
            min_primitive = primitive
            min_t = t
        }
    }
    return Intersection(min_t, min_primitive)
}
```



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Next Time!

Summary

- Writing a simple ray casting renderer is easy
  - Generate rays
  - Intersection tests
  - Lighting calculations

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			image[i][j] = GetColor(hit);
		}
	}
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