COS 341, December 6, 2000

Due: December 13, 2000

# Homework Set 9

**Reading Assignments** Read Chapter 6.

**Written Assignments** Do Exercises 5, 19, and 30 in Chapter 13.6; do Exercise 2 in Chapter 6.6 (you should use Inclusion-Exclusion for this problem).

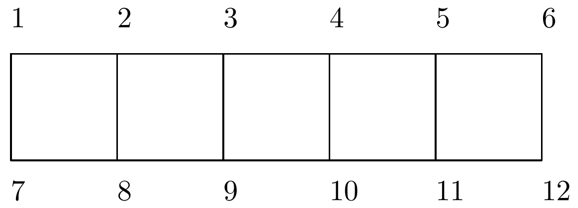
*Remark* In your solution to Exercise 5 in Chapter 13.6, be sure to give a rigorous justification of your claim.

*Special Problem 1* [Counted as 1 exercise] For any $n > 0$, let $G_n$ be the ladder graph with $n$ rungs, which has $2n$ vertices and $3n - 2$ edges. (The ladder graph $G_6$ is shown below.) Let $t_n$ be the number of spanning trees of $G_n$. Clearly, $t_1 = 1, t_2 = 4$. Let $T(x) = \sum_{n \geq 1} t_n x^n$.

(a) Determine the value of $t_3$ and $t_4$.

(b) Derive a closed-form formula for $T(x)$.

*Hint:* You may want to set up recurrences involving two sequences, one of which being $t_1, t_2, t_3, \cdots$.




*Special Problem 2* [Counted as 2 exercises] Let $G = (V, E)$ where $V = \{v_1, v_2, v_3, v_4, v_5, w_1, w_2, w_3\}$ and $E$ consists of the edges $\{v_i, v_j\}$, $1 \leq i, j \leq 5$ and $\{v_i, w_j\}$, $1 \leq i \leq 3$, $1 \leq j \leq 3$. Answer each of the following questions; justify your answers.

(a) What is $\omega(G)$, the size of the largest clique?

(b) What is $\alpha(G)$, the size of the largest independent set?

(c) What is $\chi(G)$, the chromatic number?

(d) Does $G$ have a (closed) Eulerian trail?

(e) Does $G$ contain a Hamiltonian cycle?

*Special Problem 3* (counted as 1 exercise) An ancient DNA fragment of a dinosaur has just been found. It is known that this critical fragment $\sigma$ contains some critical information. If the string $ACGAACT$ appears in $\bar{\sigma}$, then it can fly; if the string $CTCACG$ appears in $\bar{\sigma}$, then it is vegetarian; if the string $TGACCT$ appears in $\bar{\sigma}$, then it is a timid dinosaur.

The fragment has length 17, and you have subjected it to the hybridization procedure with $\ell = 4$. The spectrum $S$ you get consists of the strings $ACGA, AACT, ACTC, ACGT$, $GACT, CTCA, CGAA, CTGA, TGAC$, $GAAC, GACG, ACTG, CACG, TCAC$. What is $G_S$? Find all the $\sigma$ which has $S$ as its spectrum. What kind of attributes can you definitely infer about this dinosaur (ie, does it fly, is it a vegetarian, is it timid)?