COS 341, December 6, 2000
Due: December 13, 2000

Homework Set 9

Reading Assignments Read Chapter 6.

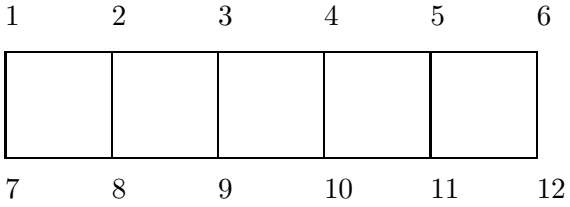
Written Assignments Do Exercises 5, 19, and 30 in Chapter 13.6; do Exercise 2 in Chapter 6.6 (you should use Inclusion-Exclusion for this problem).

Remark In your solution to Exercise 5 in Chapter 13.6, be sure to give a rigorous justification of your claim.

Special Problem 1 [Counted as 1 exercise] For any $n > 0$, let G_n be the ladder graph with n rungs, which has $2n$ vertices and $3n - 2$ edges. (The ladder graph G_6 is shown below.) Let t_n be the number of spanning trees of G_n. Clearly, $t_1 = 1, t_2 = 4$. Let $T(x) = \sum_{n \geq 1} t_n x^n$.

(a) Determine the value of t_3 and t_4.
(b) Derive a closed-form formula for $T(x)$.

Hint: You may want to set up recurrences involving two sequences, one of which being $t_1, t_2, t_3, \cdots$.



Special Problem 2 [Counted as 2 exercises] Let $G = (V, E)$ where $V = \{v_1, v_2, v_3, v_4, v_5, w_1, w_2, w_3\}$ and E consists of the edges $\{v_i, v_j\}$, $1 \leq i, j \leq 5$ and $\{v_i, w_j\}, 1 \leq i \leq 3, 1 \leq j \leq 3$. Answer each of the following questions; justify your answers.

(a) What is $\omega(G)$, the size of the largest clique?
(b) What is $\alpha(G)$, the size of the largest independent set?
(c) What is $\chi(G)$, the chromatic number?
(d) Does G have a (closed) Eulerian trail?
(e) Does G contain a Hamiltonian cycle?

Special Problem 3 (counted as 1 exercise) An ancient DNA fragment of a dinosaur has just

been found. It is known that this critical fragment σ contains some critical information. If the string $ACGAACT$ appears in $\bar{\sigma}$, then it can fly; if the string $CTCACG$ appears in $\bar{\sigma}$, then it is vegetarian; if the string $TGACCT$ appears in $\bar{\sigma}$, then it is a timid dinosaur.

The fragment has length 17, and you have subjected it to the hybridization procedure with $\ell = 4$. The spectrum S you get consists of the strings $ACGA, AACT, ACTC, ACGT$, $GACT, CTCA, CGAA, CTGA, TGAC$, $GAAC, GACG, ACTG, CACG, TCAC$. What is G_S? Find all the σ which has S as its spectrum. What kind of attributes can you definitely infer about this dinosaur (ie, does it fly, is it a vegetarian, is it timid)?