COS 341, October 11, 2000
Due: October 18, 2000

Homework Set 4

Reading Assignments Start reading Chapter 7.

Written Assignments Do exercises 11, 12 and 20 in Section 5.8.

Special Problem 1 (to be counted as 1 exercise) In the *NBA Finals* between two teams A and B, the first team that wins 4 games gets to be the champion (ie, this is a best-of-7 series). Let $0 < r < 1$ be any fixed real number. Assume that, for each game team A has a probability r of winning over team B, independent of what has happened so far. Let X denote the random variable corresponding to the length of the series. Evaluate $E(X)$ and $Var(X)$. (Your answers should be expressions in terms of r.)

Remark In class we discussed the special case $r = 1/2$.

In the following problems, your answers must be given as closed-form expressions of n.

Special Problem 2 (to be counted as 1 exercise) Let $n > 0$. Evaluate

$$f(n) = \sum_{0 \le k \le n-1} 2^k \frac{\binom{n-1}{k}}{\binom{n}{k}}.$$

Special Problem 3 (to be counted as 1 exercise) Let $n > 0$. Evaluate

$$f(n) = \sum_{0 \le k \le n} \sum_{0 \le m \le n} (k+m)^2 \binom{n}{k} \binom{n}{m}.$$

Special Problem 4 (to be counted as 2 exercises) In Mathville, there are exactly n one-way streets from west to east (named as Streets 1 to n), and n one-way streets from south to north (named as Avenues 1 to n). Let A be the south-west corner and B be the north-east corner of town (thus A is at the intersection of 1st Street and 1st Avenue, and B is at the intersection of nth Street and nth Avenue). Assume that Alice wants to go from point A to point B, and she obeys the one-way traffic regulation.

Note that the last leg of any path must be either the n-th Street or the n-th Avenue. Let X be the number of blocks that Alice travels on this last leg.
Question: What is the expected value of X, assuming that all paths are equally likely to be taken? What is the variance of X?