

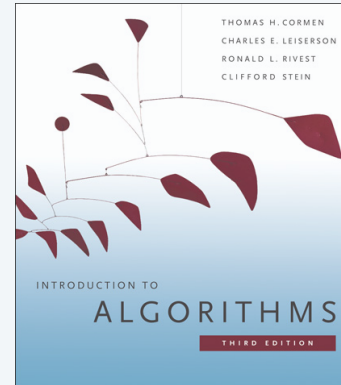
AMORTIZED ANALYSIS

- ▶ *dynamic table demo: insert*
- ▶ *dynamic table demo: insert and delete*

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AMORTIZED ANALYSIS

- ▶ *dynamic table demo: insert*
- ▶ *dynamic table demo: insert and delete*

CHAPTER 17

Dynamic table demo: insert only (accounting method)

Insert. Charge 3 credits (use 1 credit to insert; save 2 with new item).

Invariant. 2 credits with each item in right half of table; none in left half.

insert A

capacity = 1

A



Dynamic table demo: insert only (accounting method)

Insert. Charge 3 credits (use 1 credit to insert; save 2 with new item).

Invariant. 2 credits with each item in right half of table; none in left half.

insert B

capacity = 1

A



← destroy extra credit
(not needed)

capacity = 2

A

B



Dynamic table demo: insert only (accounting method)

Insert. Charge 3 credits (use 1 credit to insert; save 2 with new item).

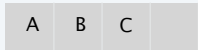
Invariant. 2 credits with each item in right half of table; none in left half.

insert C

capacity = 2



capacity = 4



5

Dynamic table demo: insert only (accounting method)

Insert. Charge 3 credits (use 1 credit to insert; save 2 with new item).

Invariant. 2 credits with each item in right half of table; none in left half.

insert D

capacity = 4



6

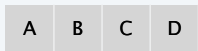
Dynamic table demo: insert only (accounting method)

Insert. Charge 3 credits (use 1 credit to insert; save 2 with new item).

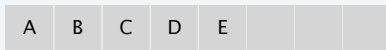
Invariant. 2 credits with each item in right half of table; none in left half.

insert E

capacity = 4



capacity = 8



7

Dynamic table demo: insert only (accounting method)

Insert. Charge 3 credits (use 1 credit to insert; save 2 with new item).

Invariant. 2 credits with each item in right half of table; none in left half.

insert F

capacity = 8



8

Dynamic table demo: insert only (accounting method)

Insert. Charge 3 credits (use 1 credit to insert; save 2 with new item).

Invariant. 2 credits with each item in right half of table; none in left half.

insert G

capacity = 8



9

Dynamic table demo: insert only (accounting method)

Insert. Charge 3 credits (use 1 credit to insert; save 2 with new item).

Invariant. 2 credits with each item in right half of table; none in left half.

insert H

capacity = 8



10

Dynamic table demo: insert only (accounting method)

Insert. Charge 3 credits (use 1 credit to insert; save 2 with new item).

Invariant. 2 credits with each item in right half of table; none in left half.

insert I

capacity = 8



capacity = 16



11

Dynamic table demo: insert only (accounting method)

Insert. Charge 3 credits (use 1 credit to insert; save 2 with new item).

Invariant. 2 credits with each item in right half of table; none in left half.

insert J

capacity = 16



12

Dynamic table demo: insert only (accounting method)

Insert. Charge 3 credits (use 1 credit to insert; save 2 with new item).

Invariant. 2 credits with each item in right half of table; none in left half.

insert K

capacity = 16

A	B	C	D	E	F	G	H	I	J	K				
---	---	---	---	---	---	---	---	---	---	---	--	--	--	--



13

Dynamic table demo: insert only (accounting method)

Insert. Charge 3 credits (use 1 credit to insert; save 2 with new item).

Invariant. 2 credits with each item in right half of table; none in left half.

insert L

capacity = 16

A	B	C	D	E	F	G	H	I	J	K	L			
---	---	---	---	---	---	---	---	---	---	---	---	--	--	--



14

Dynamic table demo: insert only (accounting method)

Insert. Charge 3 credits (use 1 credit to insert; save 2 with new item).

Invariant. 2 credits with each item in right half of table; none in left half.

insert M

capacity = 16

A	B	C	D	E	F	G	H	I	J	K	L	M		
---	---	---	---	---	---	---	---	---	---	---	---	---	--	--



15

Dynamic table demo: insert only (accounting method)

Insert. Charge 3 credits (use 1 credit to insert; save 2 with new item).

Invariant. 2 credits with each item in right half of table; none in left half.

insert N

capacity = 16

A	B	C	D	E	F	G	H	I	J	K	L	M	N	
---	---	---	---	---	---	---	---	---	---	---	---	---	---	--



16

Dynamic table demo: insert only (accounting method)

Insert. Charge 3 credits (use 1 credit to insert; save 2 with new item).
Invariant. 2 credits with each item in right half of table; none in left half.

insert O

capacity = 16



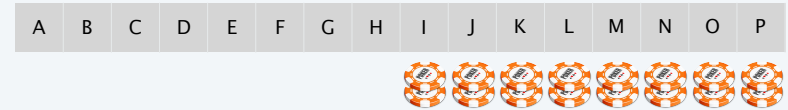
17

Dynamic table demo: insert only (accounting method)

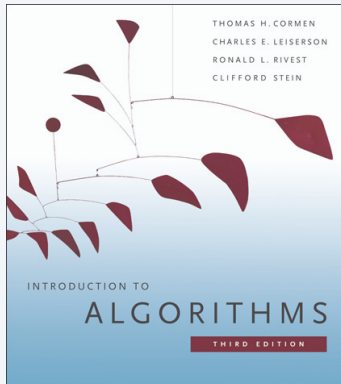
Insert. Charge 3 credits (use 1 credit to insert; save 2 with new item).
Invariant. 2 credits with each item in right half of table; none in left half.

insert P

capacity = 16



18



CHAPTER 17

AMORTIZED ANALYSIS

- ▶ *dynamic table demo: insert*
- ▶ *dynamic table demo: insert and delete*

Dynamic table demo: insert and delete (accounting method)

Insert. Charge 3 credits (1 to insert; save 2 with item if in right half).
Delete. Charge 2 credits (1 to delete; save 1 in empty slot if in left half).

Invariant 1. 2 credits with each item in right half of table.
Invariant 2. 1 credit with each empty slot in left half of table.

delete M

capacity = 16



20

Dynamic table demo: insert and delete (accounting method)

Insert. Charge 3 credits (1 to insert; save 2 with item if in right half).

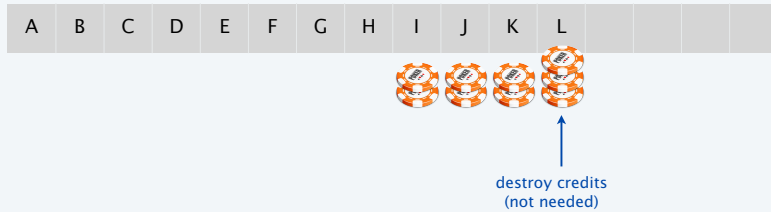
Delete. Charge 2 credits (1 to delete; save 1 in empty slot if in left half).

Invariant 1. 2 credits with each item in right half of table.

Invariant 2. 1 credit with each empty slot in left half of table.

delete L

capacity = 16



21

Dynamic table demo: insert and delete (accounting method)

Insert. Charge 3 credits (1 to insert; save 2 with item if in right half).

Delete. Charge 2 credits (1 to delete; save 1 in empty slot if in left half).

Invariant 1. 2 credits with each item in right half of table.

Invariant 2. 1 credit with each empty slot in left half of table.

delete K

capacity = 16



22

Dynamic table demo: insert and delete (accounting method)

Insert. Charge 3 credits (1 to insert; save 2 with item if in right half).

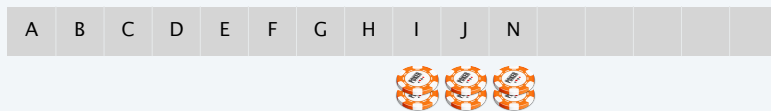
Delete. Charge 2 credits (1 to delete; save 1 in empty slot if in left half).

Invariant 1. 2 credits with each item in right half of table.

Invariant 2. 1 credit with each empty slot in left half of table.

insert N

capacity = 16



23

Dynamic table demo: insert and delete (accounting method)

Insert. Charge 3 credits (1 to insert; save 2 with item if in right half).

Delete. Charge 2 credits (1 to delete; save 1 in empty slot if in left half).

Invariant 1. 2 credits with each item in right half of table.

Invariant 2. 1 credit with each empty slot in left half of table.

delete N

capacity = 16



24

Dynamic table demo: insert and delete (accounting method)

Insert. Charge 3 credits (1 to insert; save 2 with item if in right half).

Delete. Charge 2 credits (1 to delete; save 1 in empty slot if in left half).

Invariant 1. 2 credits with each item in right half of table.

Invariant 2. 1 credit with each empty slot in left half of table.

delete J

capacity = 16



25

Dynamic table demo: insert and delete (accounting method)

Insert. Charge 3 credits (1 to insert; save 2 with item if in right half).

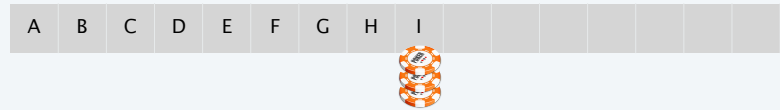
Delete. Charge 2 credits (1 to delete; save 1 in empty slot if in left half).

Invariant 1. 2 credits with each item in right half of table.

Invariant 2. 1 credit with each empty slot in left half of table.

delete I

capacity = 16



26

Dynamic table demo: insert and delete (accounting method)

Insert. Charge 3 credits (1 to insert; save 2 with item if in right half).

Delete. Charge 2 credits (1 to delete; save 1 in empty slot if in left half).

Invariant 1. 2 credits with each item in right half of table.

Invariant 2. 1 credit with each empty slot in left half of table.

delete H

capacity = 16



27

Dynamic table demo: insert and delete (accounting method)

Insert. Charge 3 credits (1 to insert; save 2 with item if in right half).

Delete. Charge 2 credits (1 to delete; save 1 in empty slot if in left half).

Invariant 1. 2 credits with each item in right half of table.

Invariant 2. 1 credit with each empty slot in left half of table.

delete G

capacity = 16



28

Dynamic table demo: insert and delete (accounting method)

Insert. Charge 3 credits (1 to insert; save 2 with item if in right half).

Delete. Charge 2 credits (1 to delete; save 1 in empty slot if in left half).

Invariant 1. 2 credits with each item in right half of table.

Invariant 2. 1 credit with each empty slot in left half of table.

delete F

capacity = 16



29

Dynamic table demo: insert and delete (accounting method)

Insert. Charge 3 credits (1 to insert; save 2 with item if in right half).

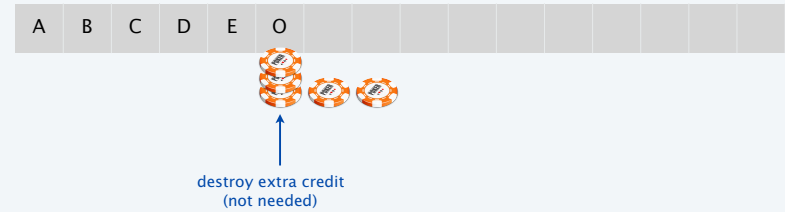
Delete. Charge 2 credits (1 to delete; save 1 in empty slot if in left half).

Invariant 1. 2 credits with each item in right half of table.

Invariant 2. 1 credit with each empty slot in left half of table.

insert O

capacity = 16



30

Dynamic table demo: insert and delete (accounting method)

Insert. Charge 3 credits (1 to insert; save 2 with item if in right half).

Delete. Charge 2 credits (1 to delete; save 1 in empty slot if in left half).

Invariant 1. 2 credits with each item in right half of table.

Invariant 2. 1 credit with each empty slot in left half of table.

delete O

capacity = 16



31

Dynamic table demo: insert and delete (accounting method)

Insert. Charge 3 credits (1 to insert; save 2 with item if in right half).

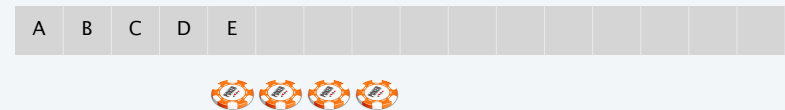
Delete. Charge 2 credits (1 to delete; save 1 in empty slot if in left half).

Invariant 1. 2 credits with each item in right half of table.

Invariant 2. 1 credit with each empty slot in left half of table.

delete E

capacity = 16



32

Dynamic table demo: insert and delete (accounting method)

Insert. Charge 3 credits (1 to insert; save 2 with item if in right half).

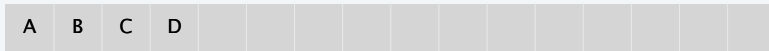
Delete. Charge 2 credits (1 to delete; save 1 in empty slot if in left half).

Invariant 1. 2 credits with each item in right half of table.

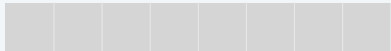
Invariant 2. 1 credit with each empty slot in left half of table.

delete D

capacity = 16



capacity = 8



33

Dynamic table demo: insert and delete (accounting method)

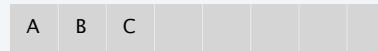
Insert. Charge 3 credits (1 to insert; save 2 with item if in right half).

Delete. Charge 2 credits (1 to delete; save 1 in empty slot if in left half).

Invariant 1. 2 credits with each item in right half of table.

Invariant 2. 1 credit with each empty slot in left half of table.

capacity = 8



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