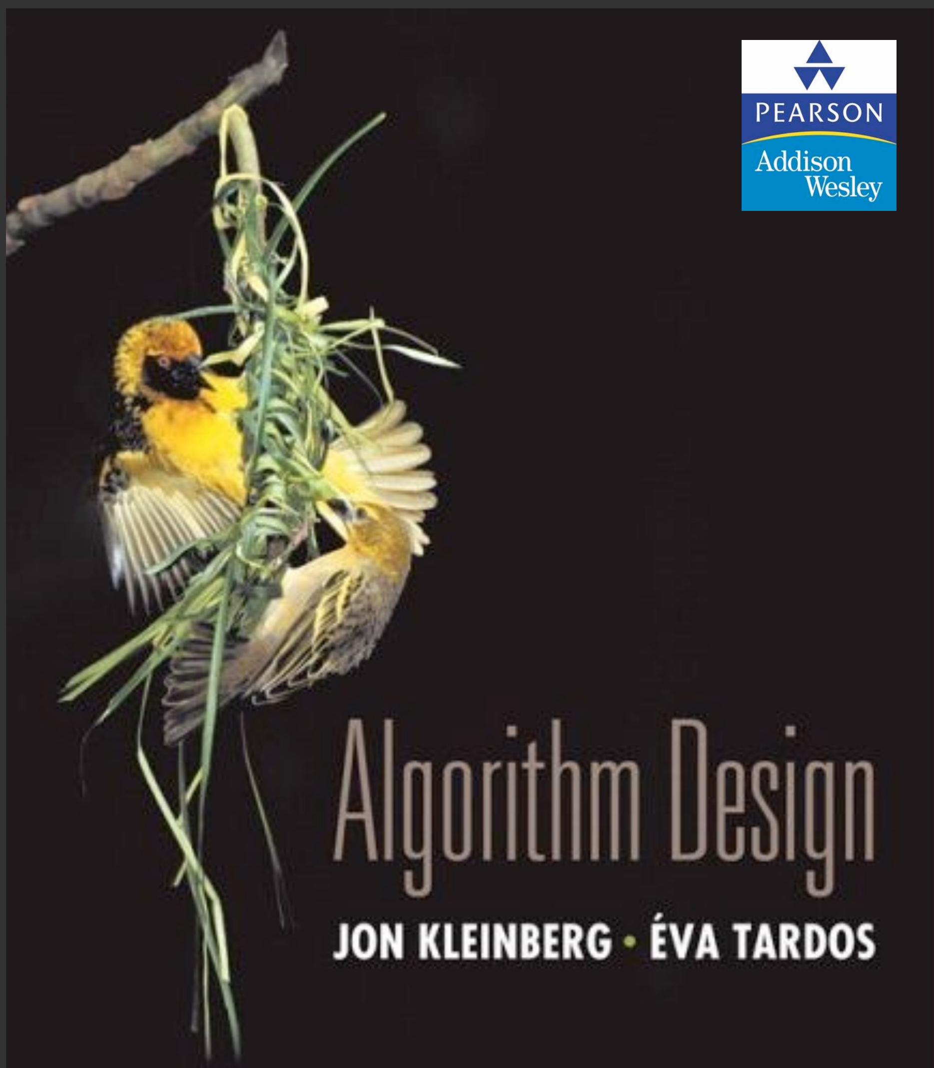


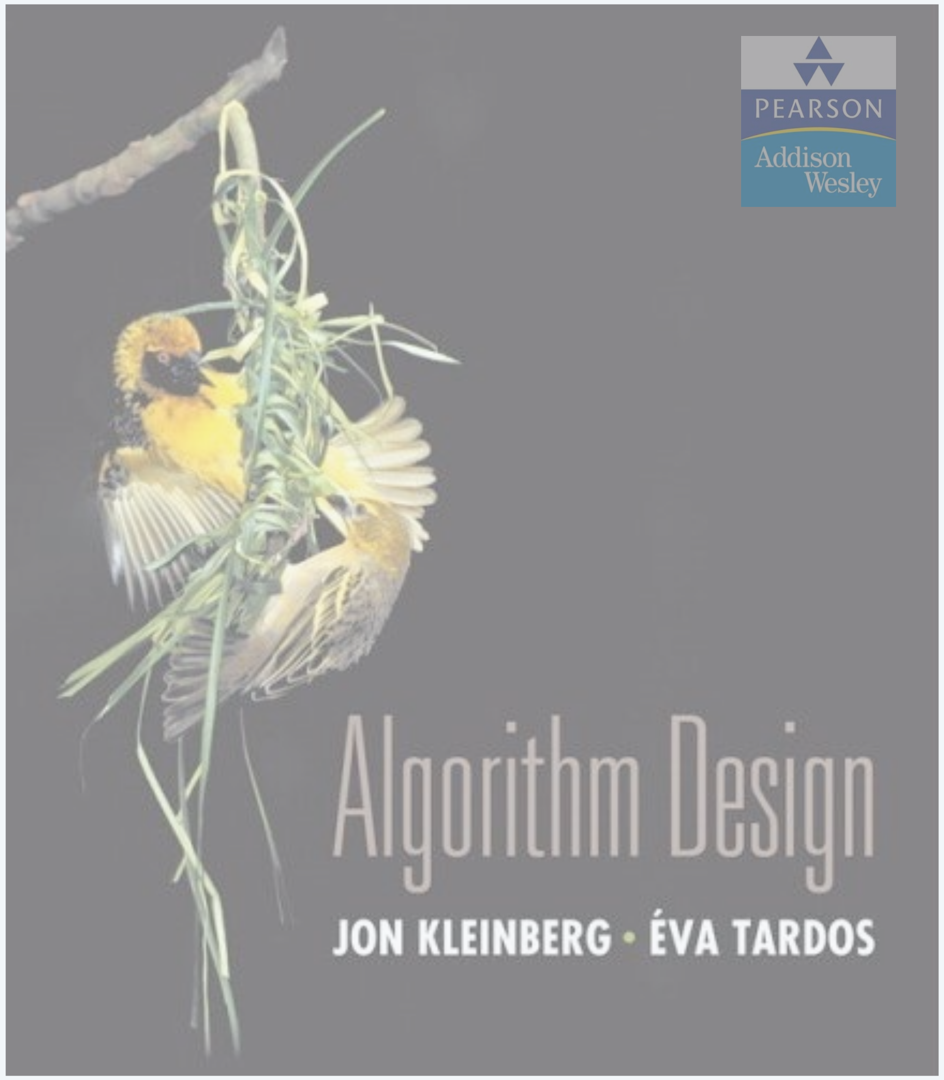
4. Greedy Algorithms II

- *Dijkstra's algorithm demo*
- *Dijkstra's algorithm demo (efficient implementation)*

Lecture slides by Kevin Wayne

http://www.cs.princeton.edu/~wayne/kleinberg-tardos

Last updated on 11/15/17 10:10 AM



4. Greedy Algorithms II

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Dijkstra's algorithm demo

- Initialize $S \leftarrow \{ s \}$ and $d[s] \leftarrow 0$.
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$$\pi(v) \;=\; \min_{e\,=\,(u,v)\;:\;u \in S} \boxed{d[u] + \ell_e}$$

the length of a shortest path from s to some node u in explored part S, followed by a single edge $e = (u, v)$

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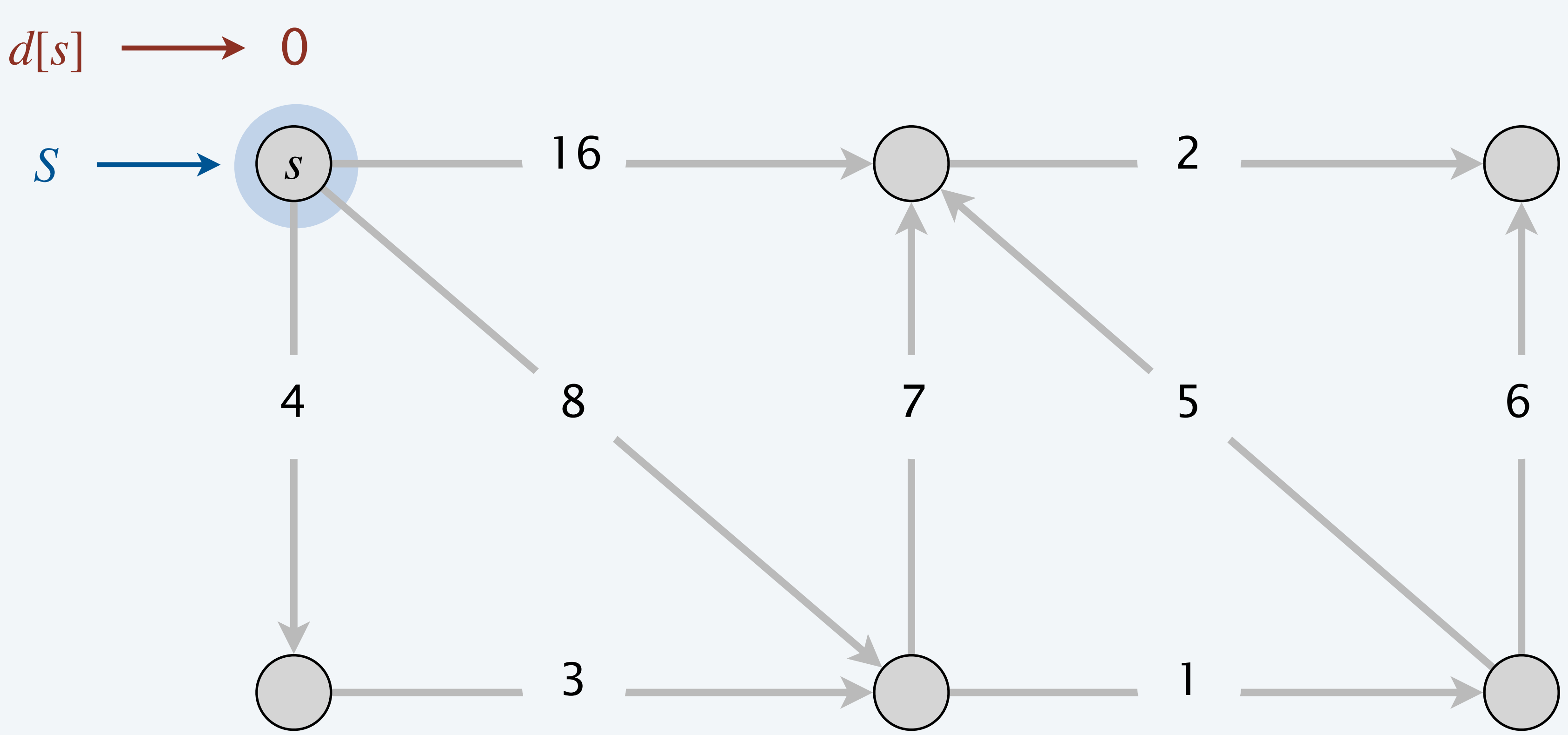

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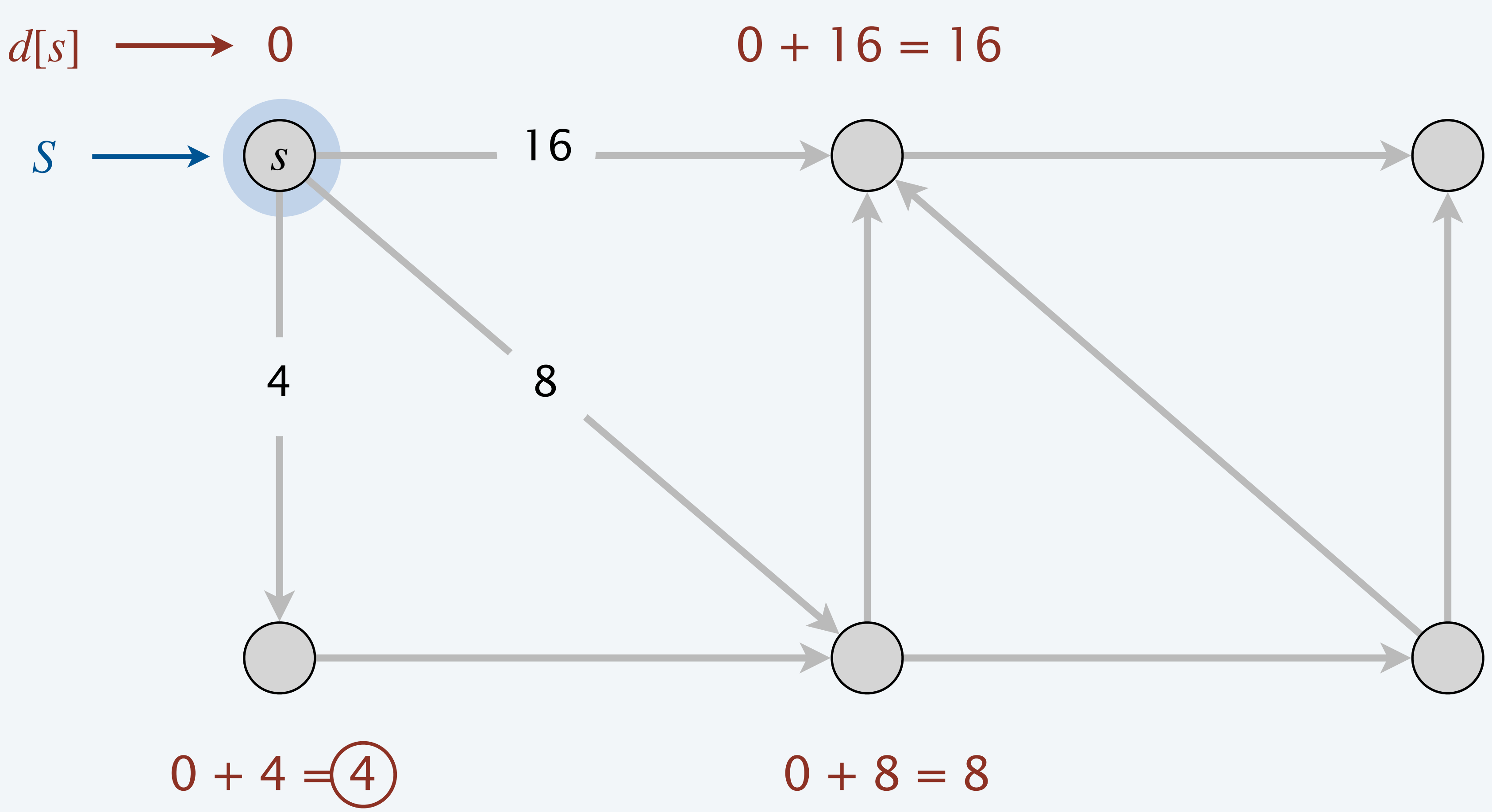

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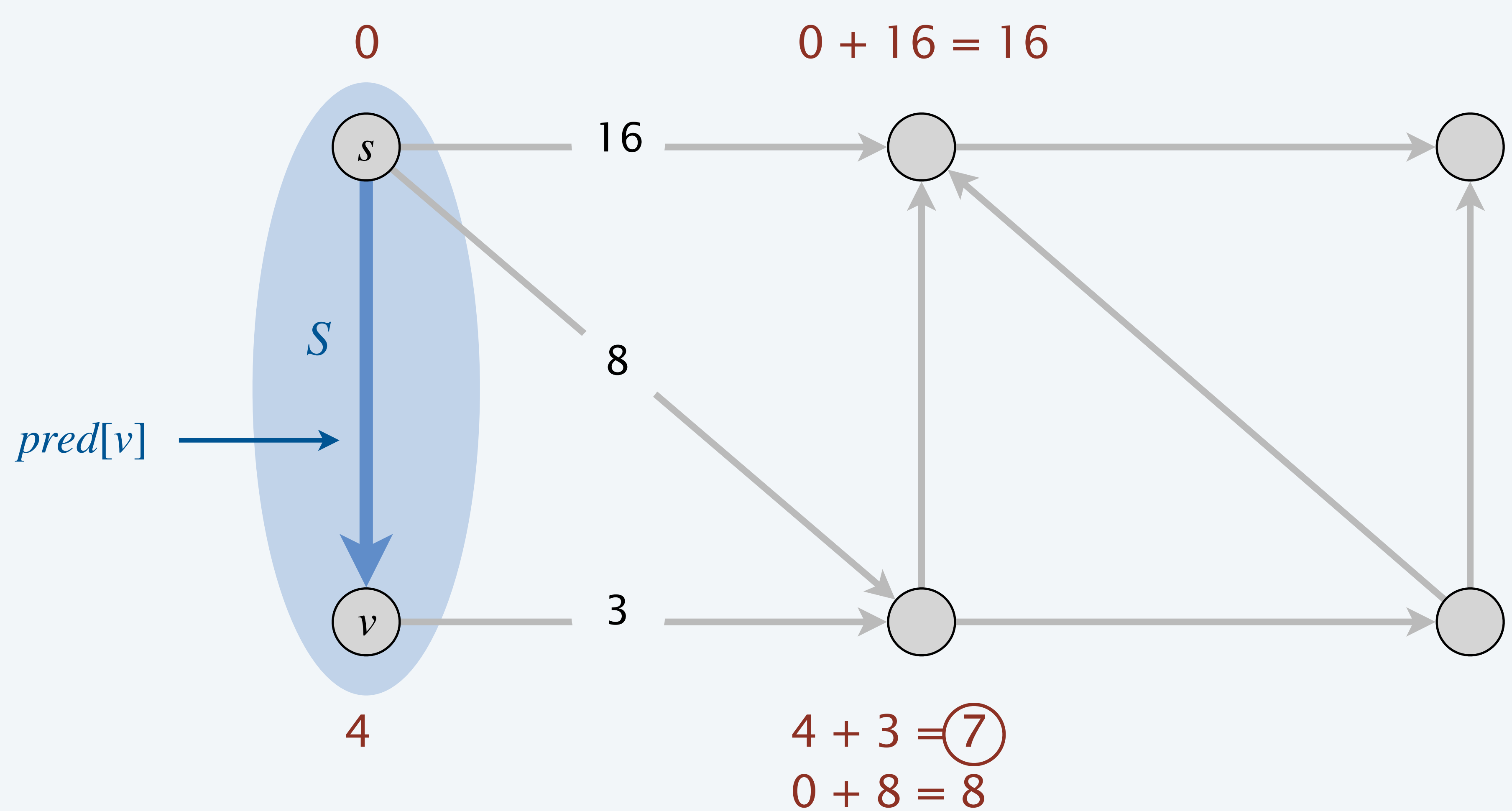

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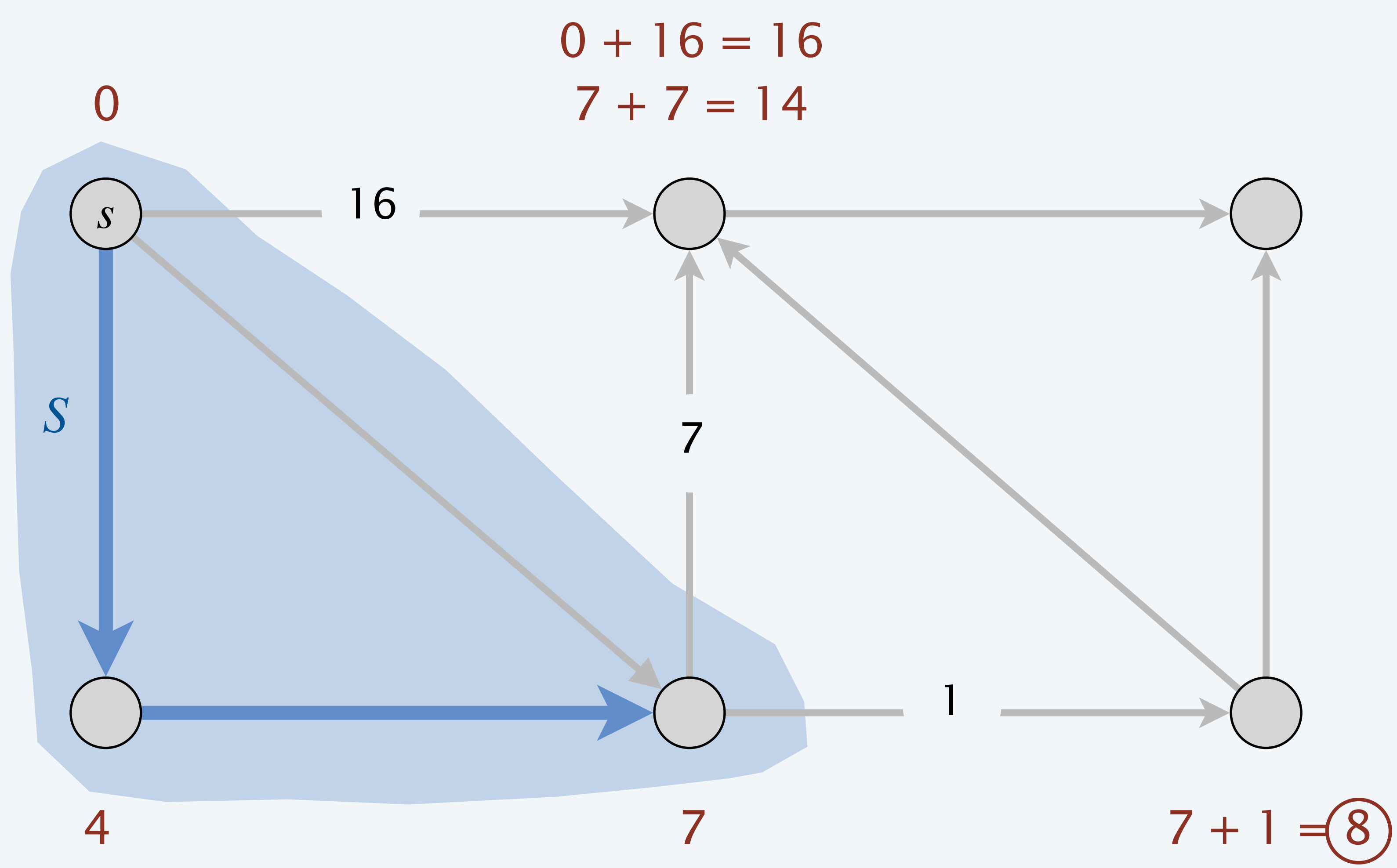

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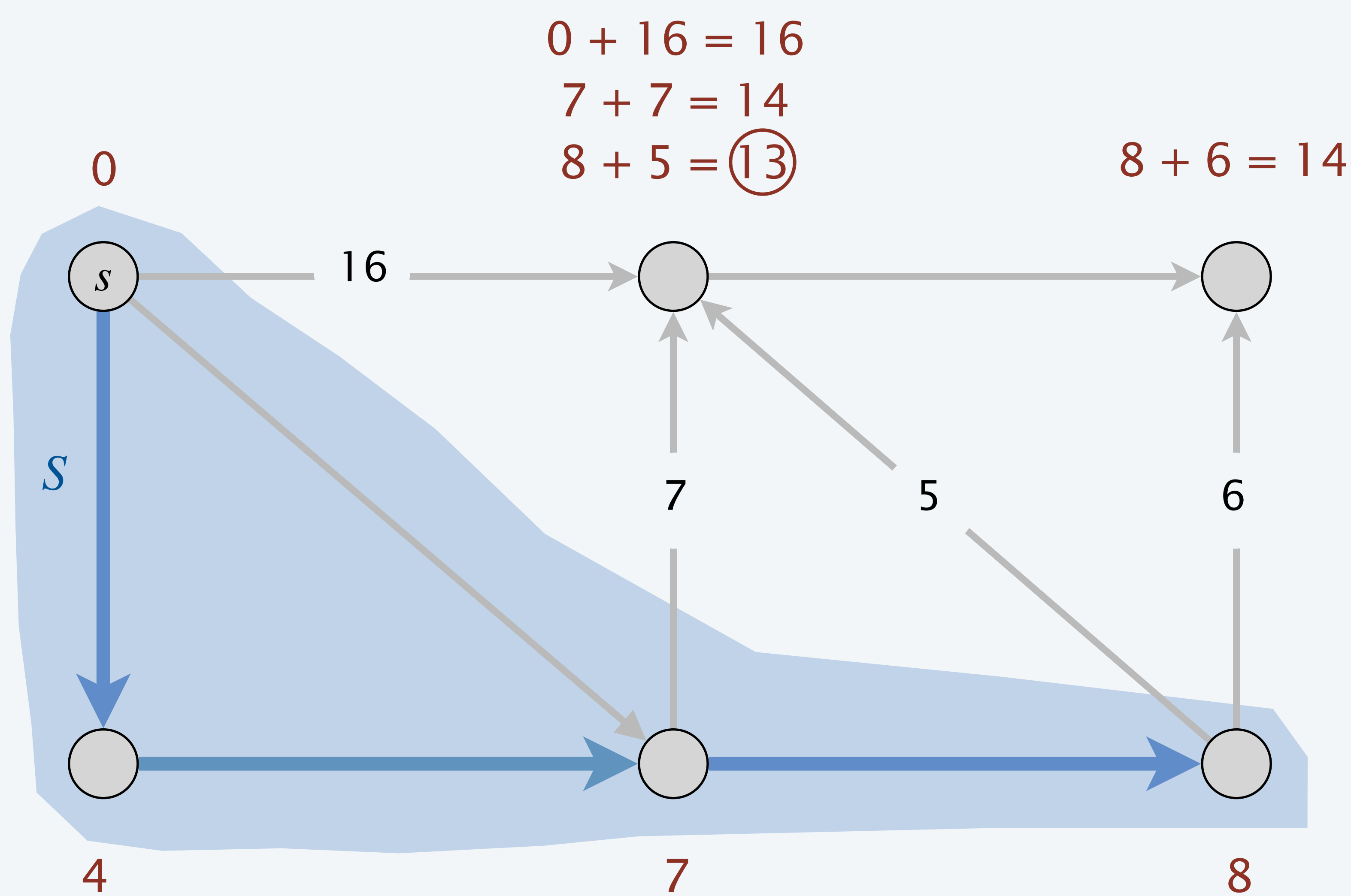

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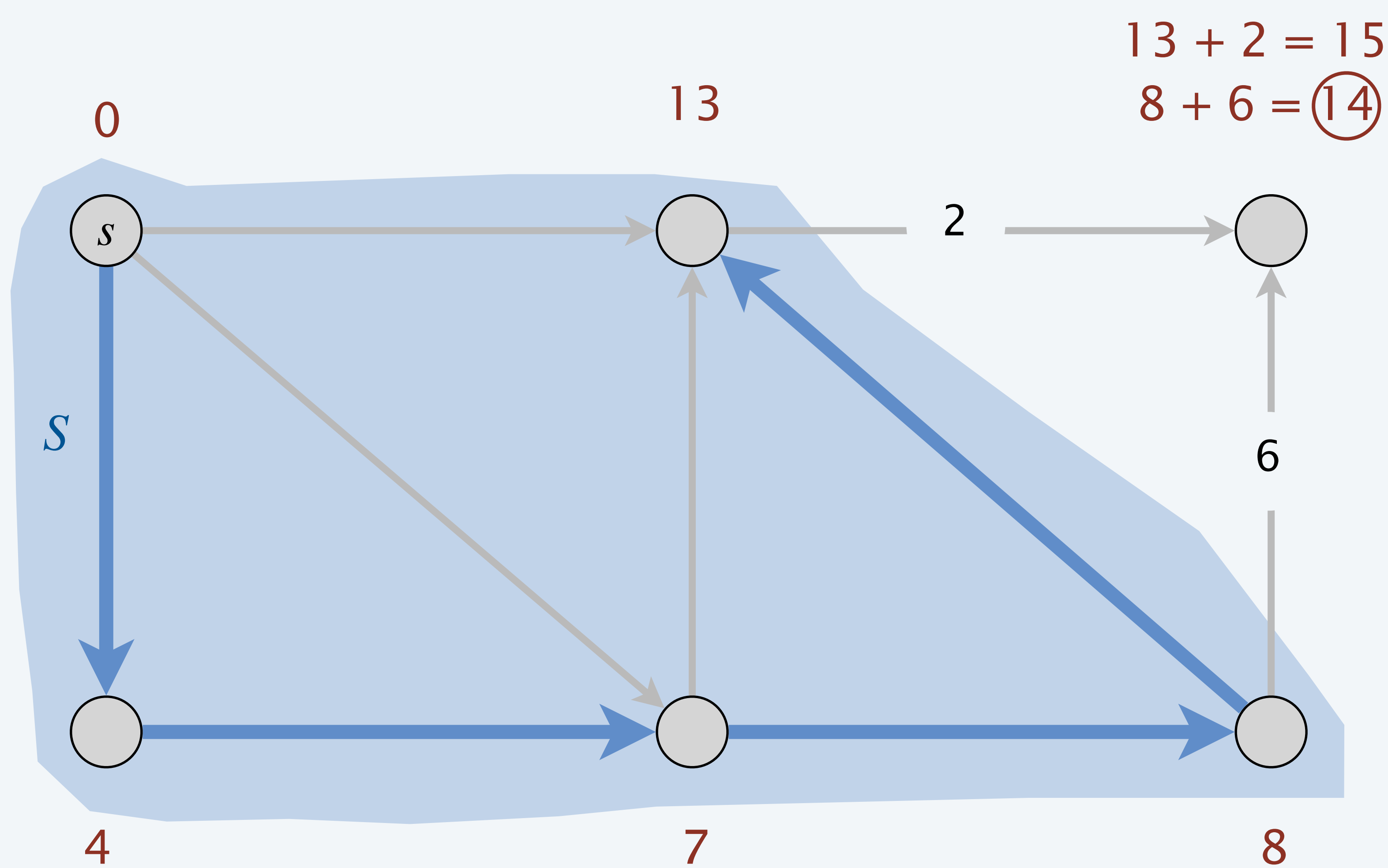

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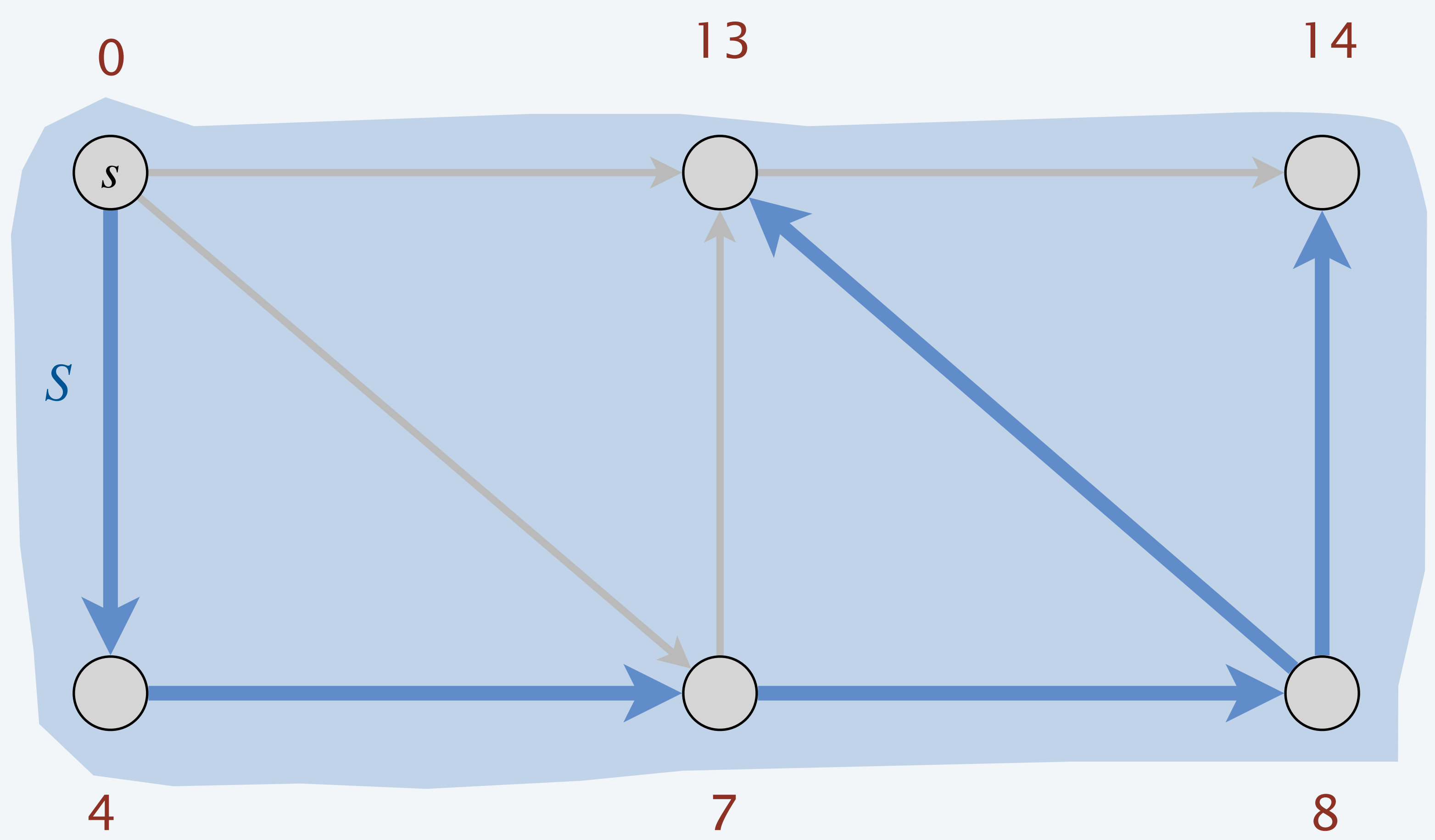

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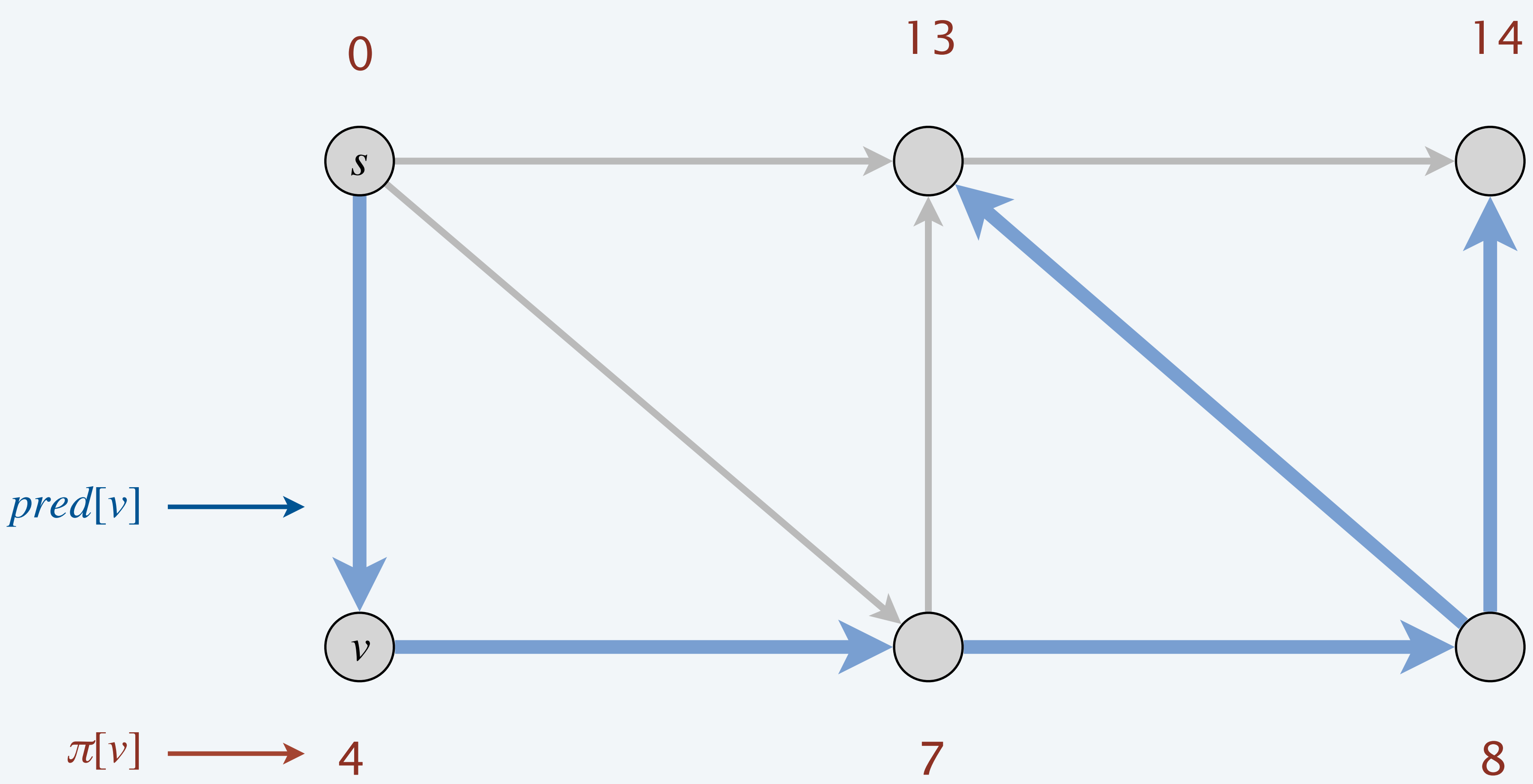
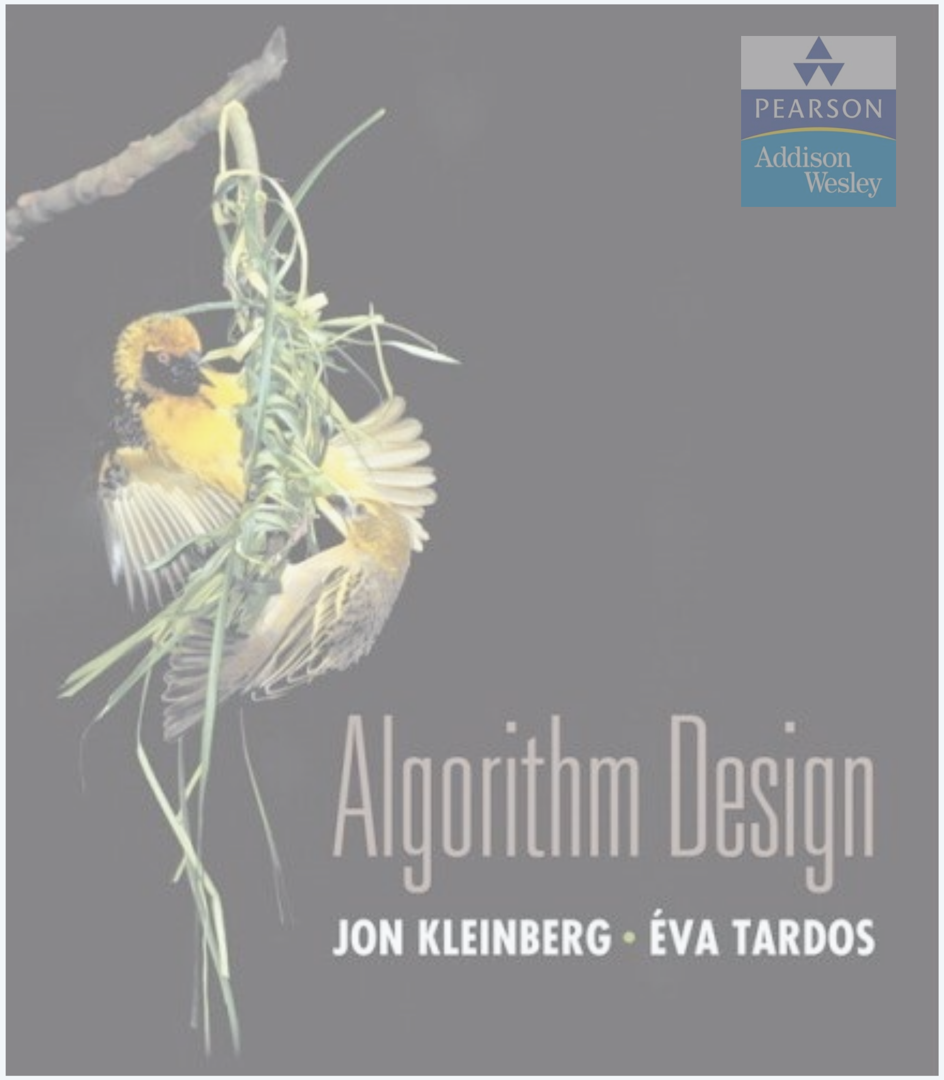




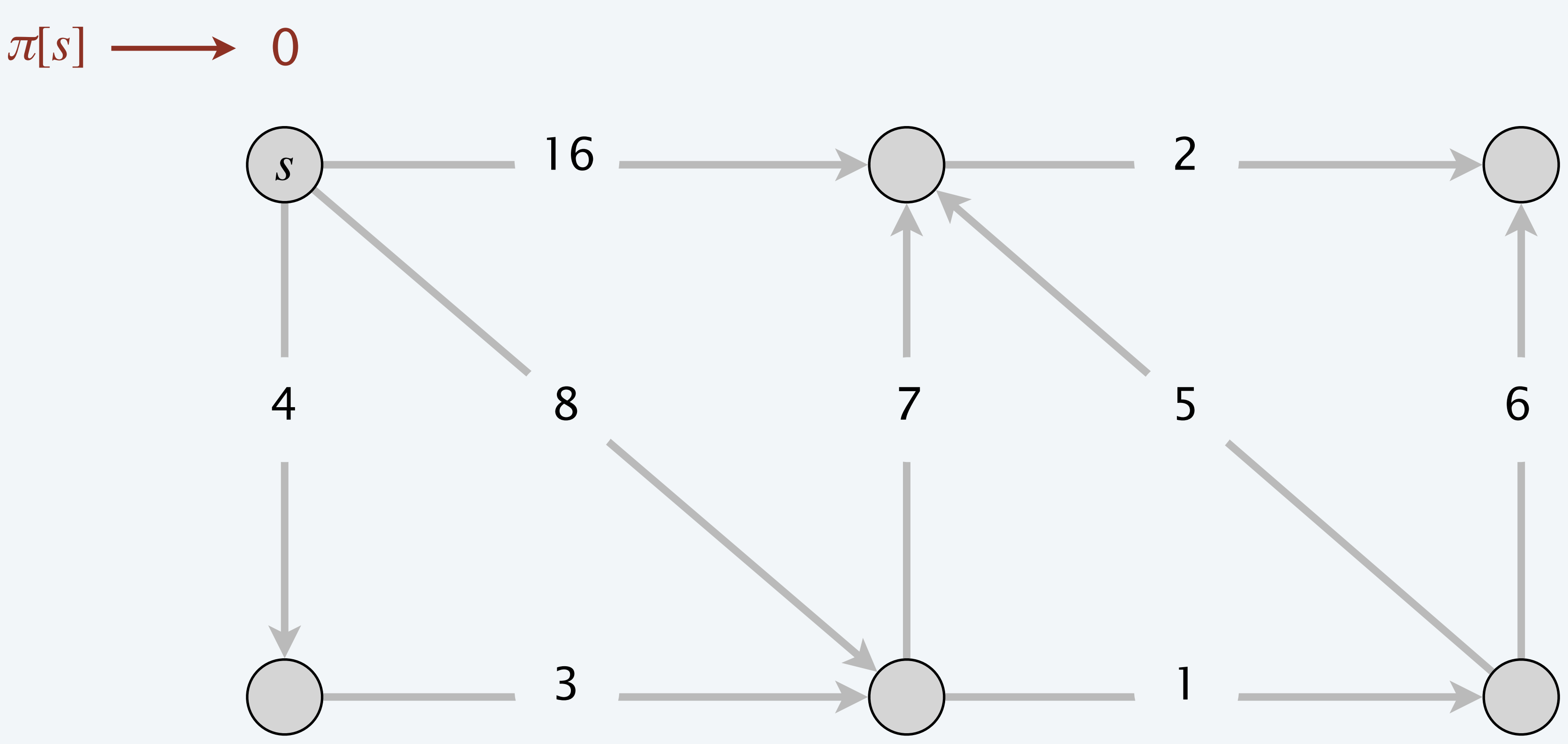
4. Greedy Algorithms II

- *Dijkstra's algorithm demo*
- *Dijkstra's algorithm demo (efficient implementation)*

Dijkstra's algorithm demo (efficient implementation)

Initialization.

- For all $v \neq s$: $\pi[v] \leftarrow \infty$.
- For all $v \neq s$: $pred[v] \leftarrow null$.
- $S \leftarrow \varnothing$ and $\pi[s] \leftarrow 0$.



Dijkstra's algorithm demo (efficient implementation)

Basic step. Choose unexplored node $u \notin S$ with minimum $\pi[u]$.

- Add u to S.
- For each edge $e = (u, v)$ leaving u, if $\pi[v] > \pi[u] + \ell_e$ then:
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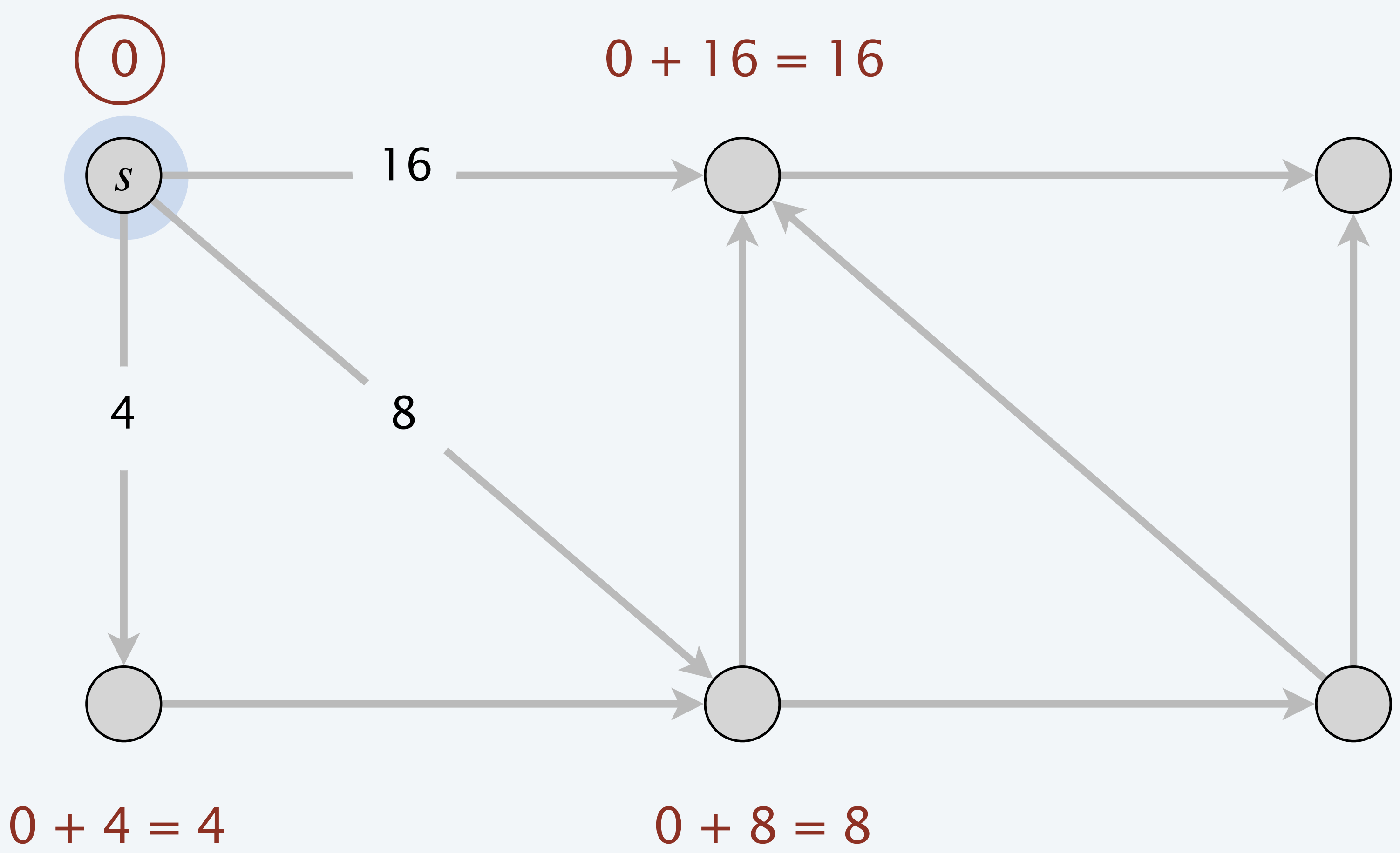

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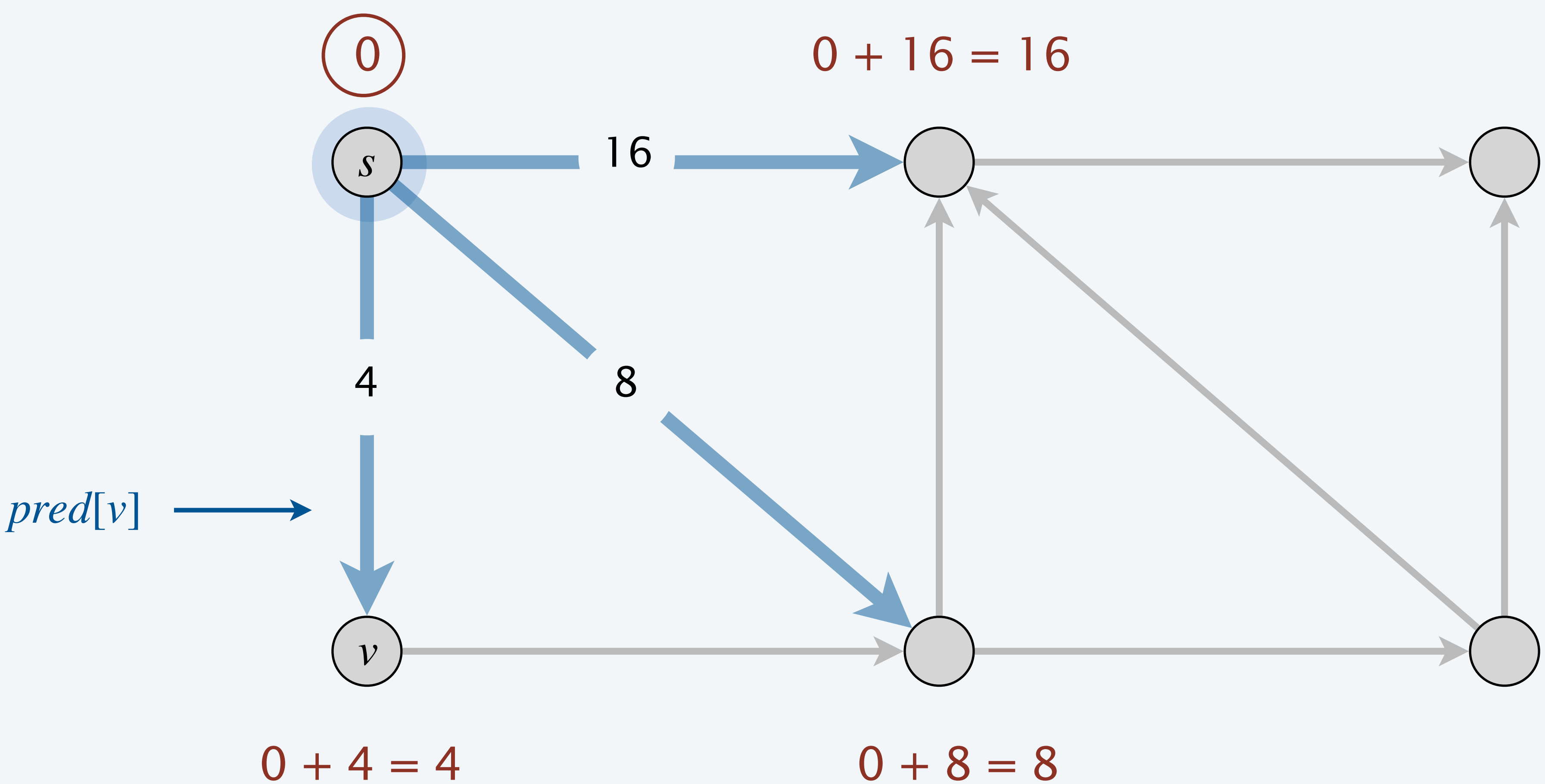

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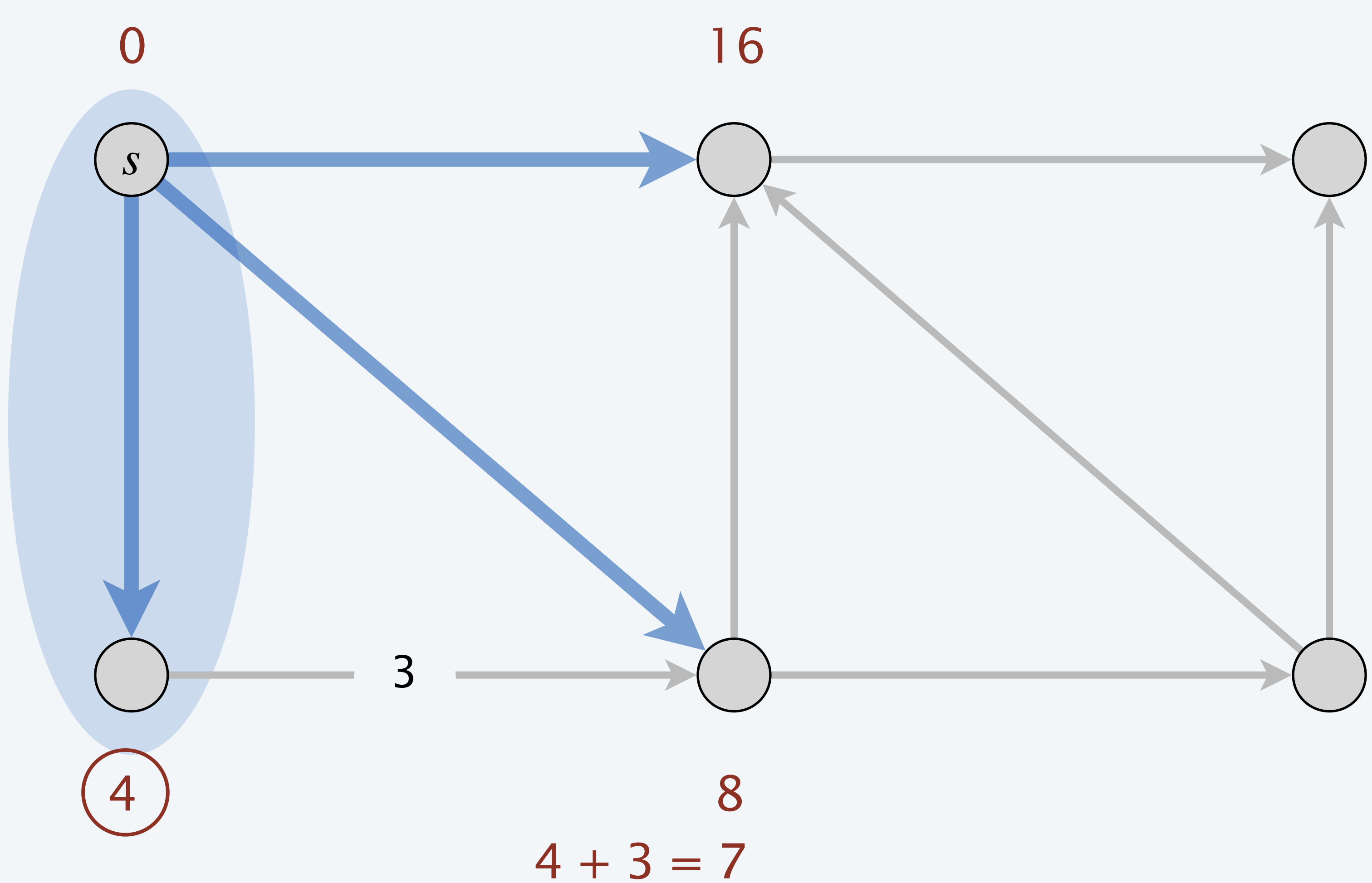

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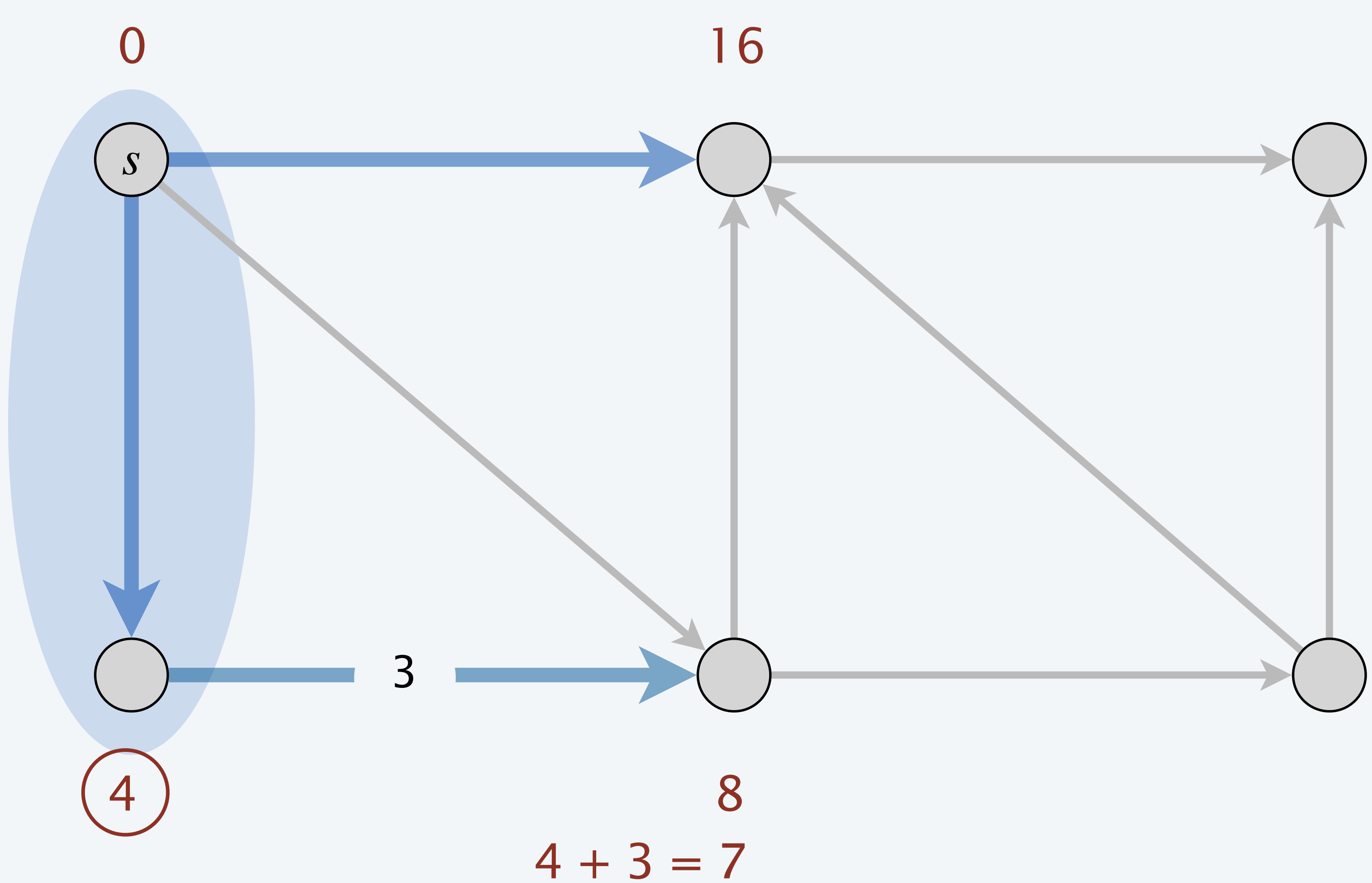

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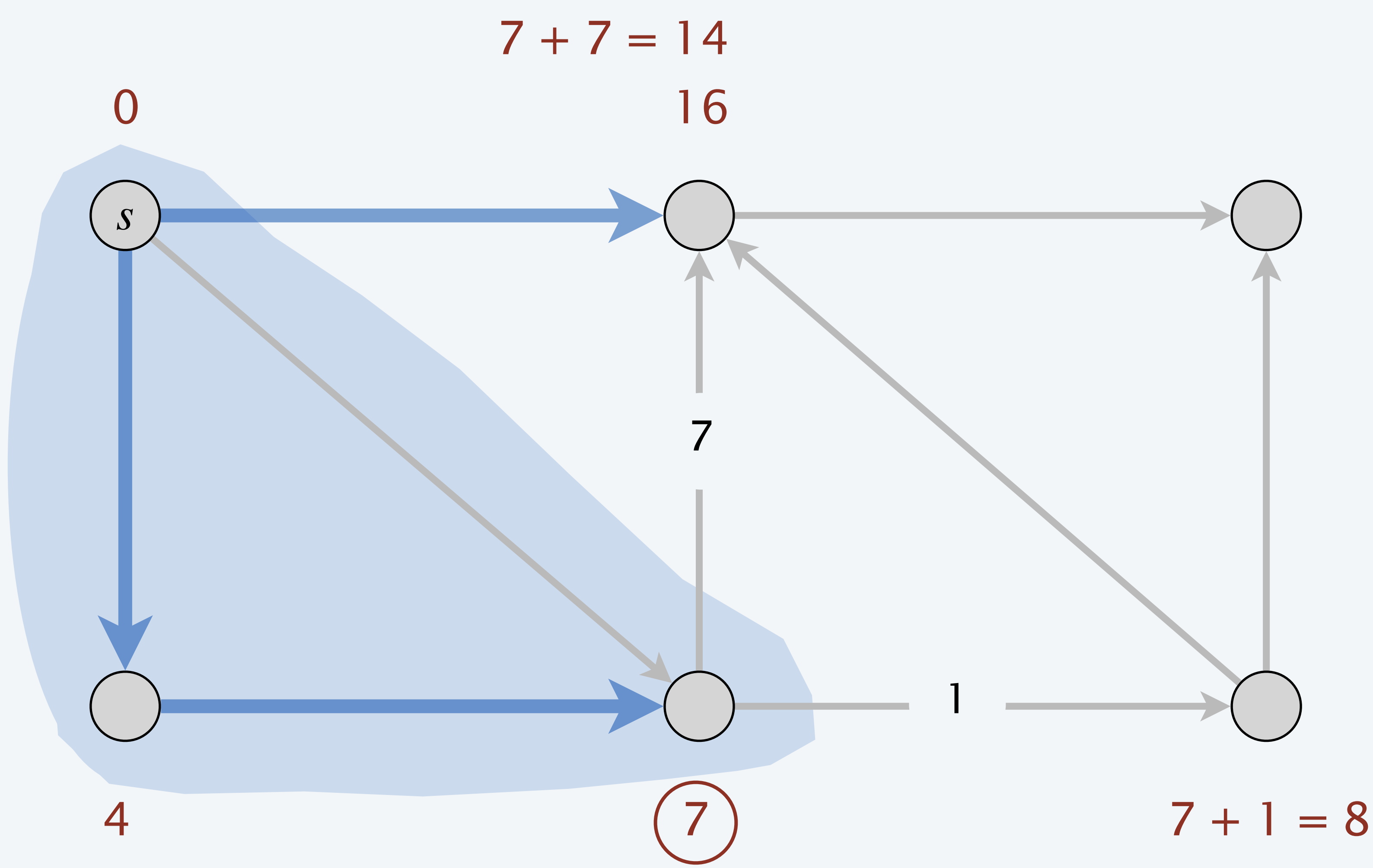

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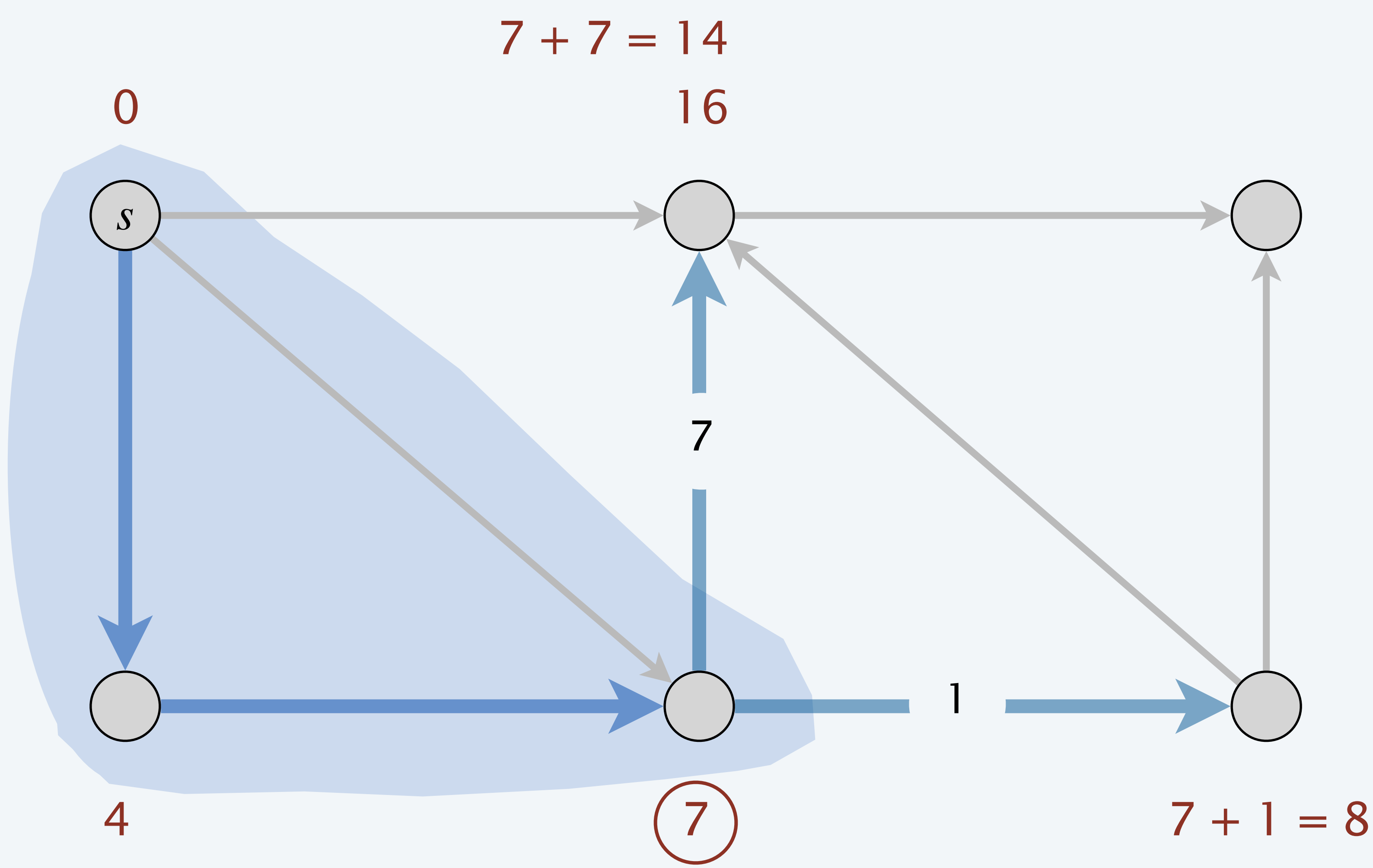

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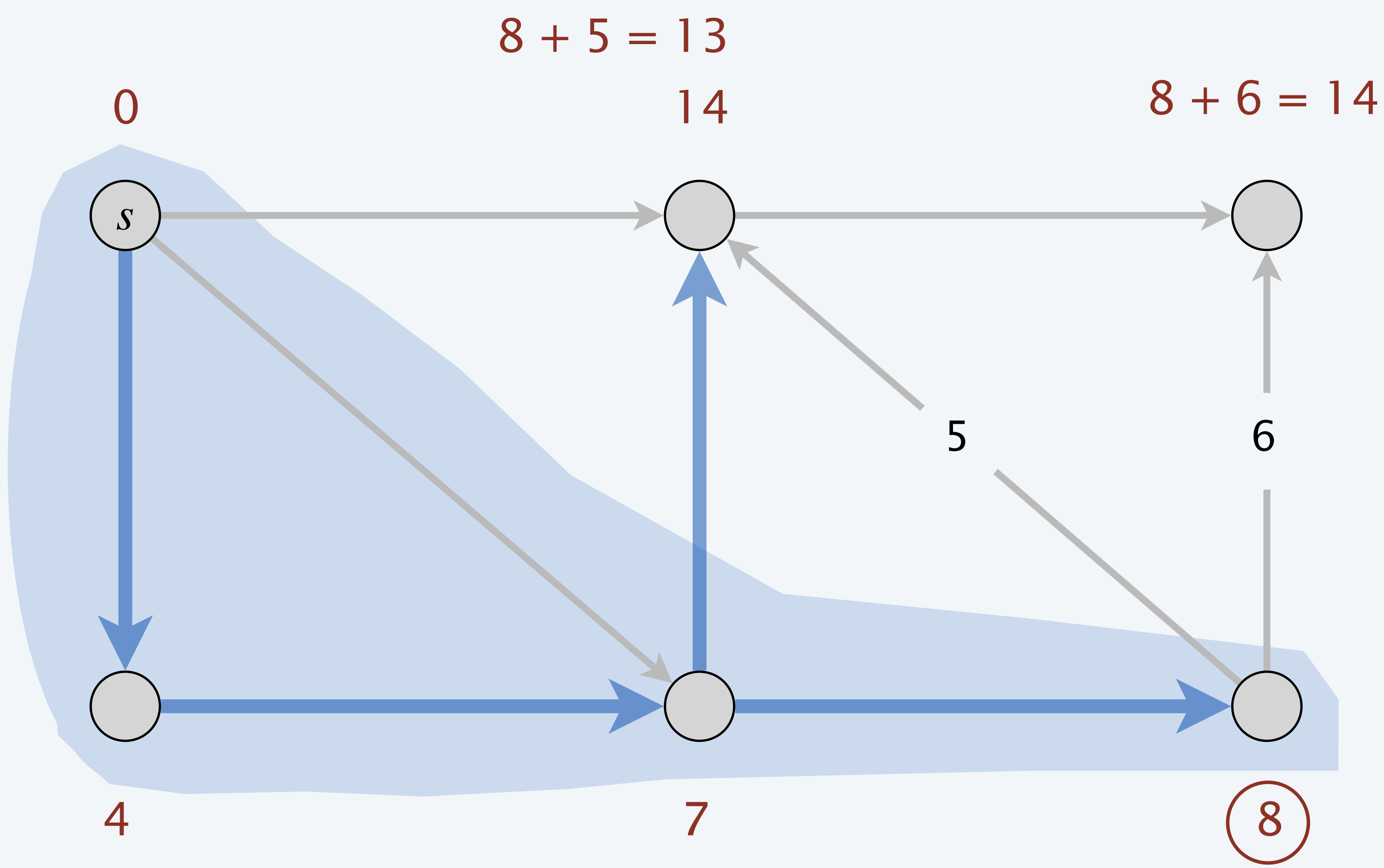

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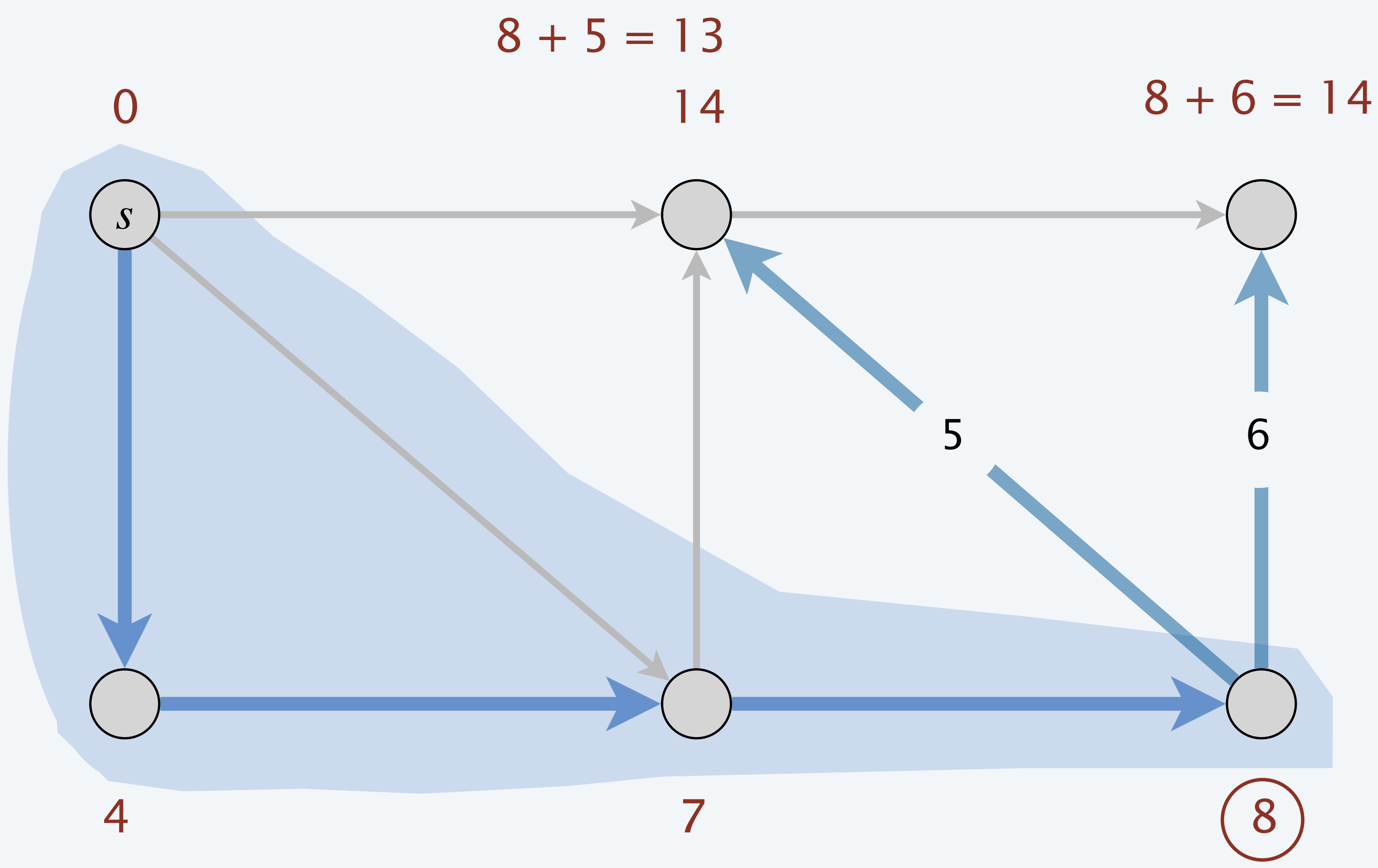

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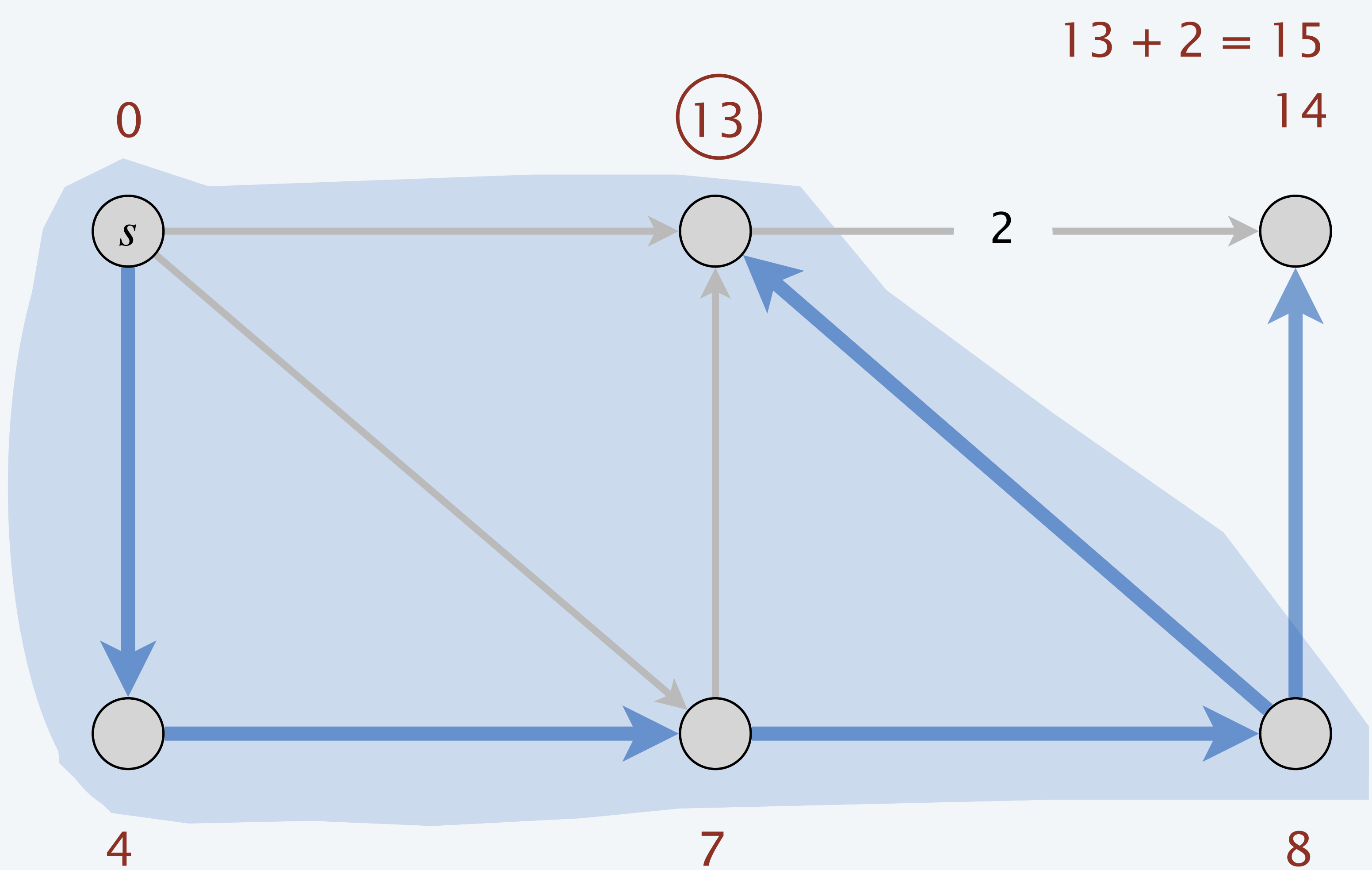

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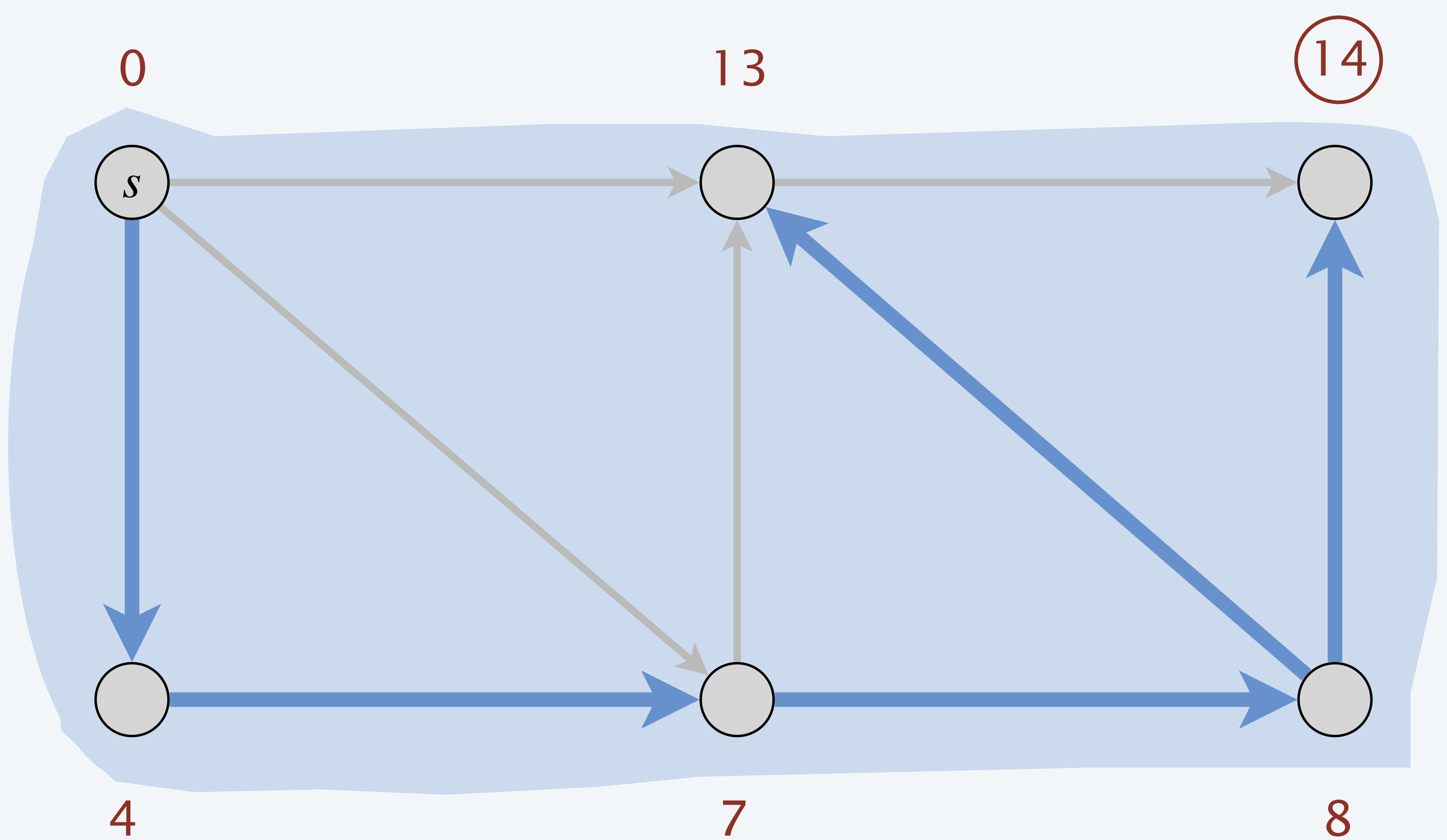

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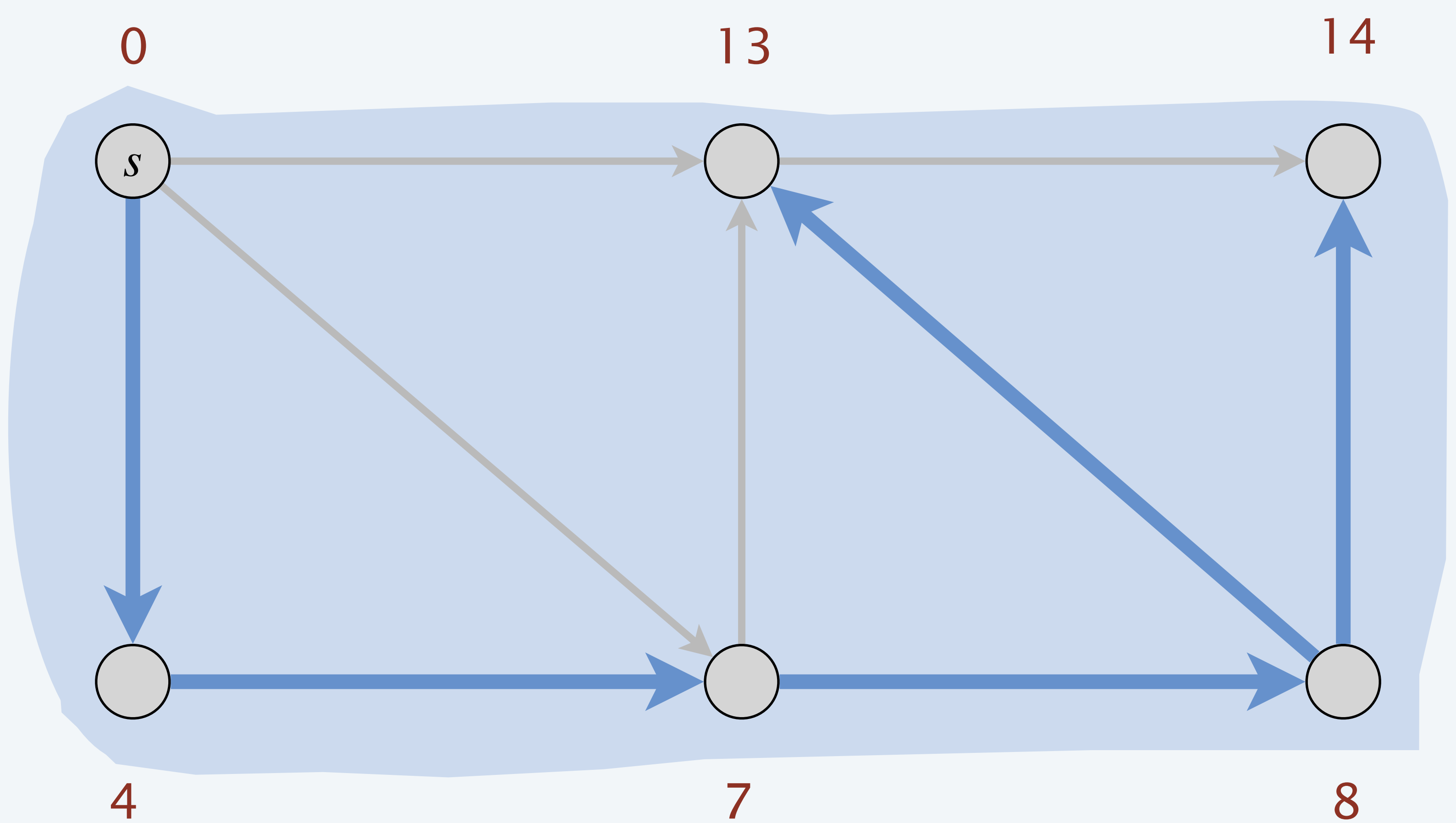

Dijkstra's algorithm demo (efficient implementation)

Termination.

- $\pi[v]$ = length of a shortest $s \rightarrow v$ path.
- $pred[v]$ = last edge on a shortest $s \rightarrow v$ path.