Unit T: Theory of CS

A Puzzle ("Post's Correspondence Problem")

Given a set of cards:

- N card types (can use as many of each type as possible).
- Each card has a top string and bottom string.

Example 1:

BAB	A	AB	BA
A	ABA	B	B
0	1	2	3

N = 4

Puzzle:

- Is it possible to arrange cards so that top and bottom strings are the same?

Solution 1.

A	BA	BAB	AB	A
ABA	B	A	B	ABA
1	3	0	2	1

A Puzzle ("Post's Correspondence Problem")

Given a set of cards:

- N card types (can use as many of each type as possible).
- Each card has a top string and bottom string.

Example 2:

A	ABA	B	A
BAB	B	A	B
0	1	2	3

N = 4

Puzzle:

- Is it possible to arrange cards so that top and bottom strings are the same?

Solution 2.

PCP Puzzle Contest

S[	X	X	11A	1	[A	]	[	B1	B]	[1A]E
S[11111X][	1X	A	A1	1	[B	]	[	1B	A]	E
0	1	2	3	4	5	6	7	8	9	10

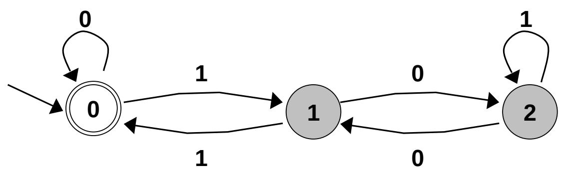
Contest:

- Additional restriction: string must start with 'S'.
- Be the first to solve this puzzle!
  (no credit, just fame and acclamation)
- Check solution by putting STRING ONLY (blanks and line breaks OK) in a file `solution.txt`, then type
  `/u/cs126/bin/pcp < solution.txt`

Extra credit for the bored:

- Write a program that reads a set of Post cards, and determines whether or not there is a solution.

Lecture T1: Pattern Matching



Introduction to Theoretical CS

Two fundamental questions.

- What can a computer do?
- What can a computer do with limited resources?

General approach.

- Don't talk about specific machines or problems.
- Consider minimal abstract machines.
- Consider general classes of problems.

Why Learn Theory

In theory . . .

- Deeper understanding of what is a computer and computing.
- Foundation of all modern computers.
- Pure science.
- Philosophical implications.

In practice . . .

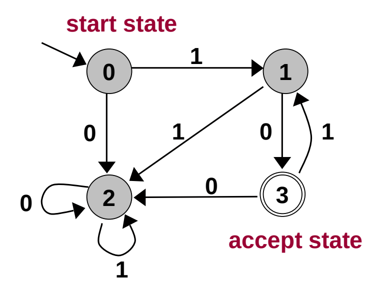
- Web search: theory of pattern matching.
- Sequential circuit: theory of finite state automata.
- Compilers: theory of context free grammar.
- Cryptography: theory of complexity.
- Data compression: theory of information.

Finite State Automata

Simple machine with N states.

- Start in state 0.
- Read an input bit.
- Move to new state
  - depends on input bit and current state
- Stop when last bit read.
  - 'yes' if end in accept state(s)
  - 'no' otherwise

'Yes' also called *accepted* or *recognized* inputs from a language.



C Code for FSA

fsa1.c

```
#include <stdio.h>
int main(void) {
  int c, state = 0;
  while ((c = getchar()) != EOF) {
     if (state == 0 && c == '0') state = 2;
     if (state == 0 && c == '1') state = 1;
     if (state == 1 && c == '0') state = 3;
     if (state == 1 && c == '1') state = 2;
     if (state == 2 && c == '0') state = 2;
     if (state == 2 && c == '1') state = 2;
     if (state == 3 && c == '0') state = 2;
     if (state == 3 && c == '1') state = 1;
  }

  if (state == 3)
     printf("Yes.\n");
  else
     printf("No.\n");
  return 0;
}
```

straightforward to convert FSA's into C program or to build with hardware

Better C Code for FSA

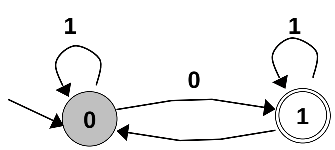
fsa2.c

```
#include <stdio.h>
#define STATES          4
#define ALPHABET_SIZE   2
#define START_STATE     0
#define ACCEPT_STATE    3

int main(void) {
  int c, state = START_STATE
  int transition[STATES][ALPHABET_SIZE] =
      { {2, 1}, {3, 2}, {2, 2}, {2, 1} };

  while ((c = getchar()) != EOF)
     if (c >= '0' && c < '0' + ALPHABET_SIZE)
        state = transition[state][c - '0'];

  if (state == ACCEPT_STATE) printf("Yes.\n");
  else printf("No.\n");
  return 0;
}
```

use 2D array, and don't hardwire constants

A Second Example

Consider the following two state FSA.



What bit strings does it accept?

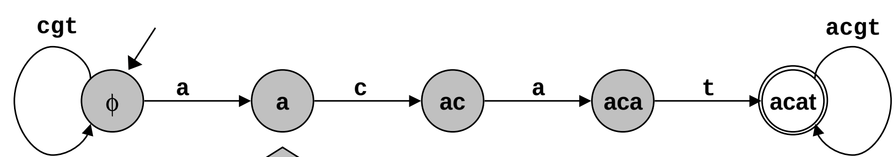
- Yes: `0, 11110, 00000, 100100111011`, all bit strings with an odd number of 0's.
- No: `1, 1111, 00, 1011100111011`, all bit strings with an even number of 0's.

A Third Example

Build an FSA that accepts all strings that contain 'acat' as a substring.

- tgacatg
- acacatg

Start building:



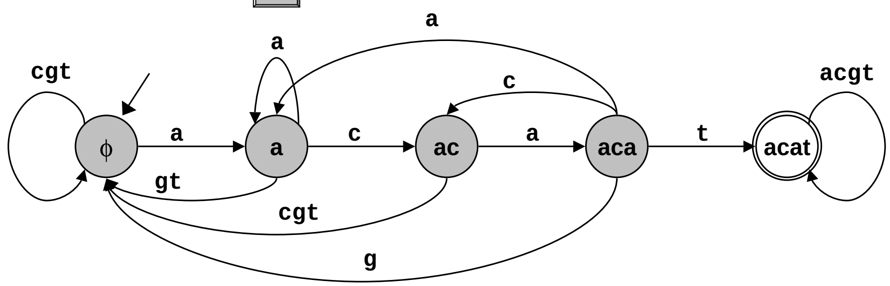
State name represents prefix of "acat" that is currently matches.

A Third Example

Build an FSA that accepts all strings that contain 'acat' as a substring.

- tgacatg
- acacatg

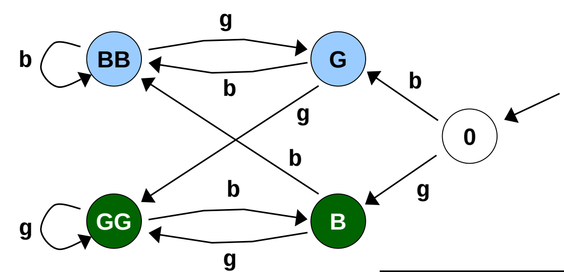
Finish building:



An Application: Bounce Filter

Bounce filter: remove isolated b's and g's in input.

- Input: b b g b b b g g b g g g g b b b b
- Output (one-bit delay): b b b b b b g g g g g g g b b b b



no accept state – instead output color of each state you visit

An Application: Bounce Filter

Bounce filter: remove isolated b's and g's in input.

- Input: b b g b b b g g b g g g g b b b b
- Output (one-bit delay): b b b b b b g g g g g g g b b b b

State interpretations.

- 0: start
- BB: at least two consecutive b's.
- G: sequence of b's followed by g.
- GG: at least two consecutive g's.
- B: sequence of g's followed by b.

egrep

General regular expressions pattern matching.

- Acts as filter.
- Sends lines from stdin to stdout that "match" argument string.

Elementary Examples

```
%egrep 'beth' classlist
03/Smythe/Elizabeth/6/esmythe
03/Bethke/Kristen/3/kbethke
```

Find all lines in file classlist with substring 'beth'

```
% egrep '/3/' classlist
03/Marin/Anthony/3/amarin
03/Arellano/Belen/3/arellano
. . .
03/Weiss/Jacob/3/weiss
```

List all people in precept 3.

```
%egrep 'zeuglodon' mobydick.txt
 rechristened the monster zeuglodon and in his

%egrep 'acat' human.data
gcaacgcacacaacatgcatttt
```

Crossword Puzzle or Scrabble Too Hard?

`/usr/dict/words` is a list of (25,143) words in dictionary.

More Examples

```
% egrep 'hh' /usr/dict/words
    beachhead
    highhanded
    withheld
    withhold
```

Two consecutive h's.

```
% egrep 'u.u.u' /usr/dict/words
    cumulus
```

A dot matches any single character

```
% egrep '..oo..oo' /usr/dict/words
    bloodroot
    schoolbook
    schoolroom
```

but not "cookbook"

Egrep Pattern Conventions

Conventions for egrep:

`c`	any non-special character matches itself
`.`	any single character
`r*`	zero or more occurrence of r
`(r)`	grouping
`r1|r2`	logical OR
`[aeiou]`	any vowel
`[^ aeiou]`	any non-vowel
`^`	beginning of line
`$`	end of line

Flags for egrep:

`egrep -v`	match all lines except those specified by pattern

Still More Examples

Unix

```
% egrep 'n(ie|ei)ther' /usr/dict/words
    neither
```

Do spell checking by specifying what you know.

```
% egrep 'actg(atac)*gcta' human.data
    ggtactggctaggac

% egrep 'actg(atac)*gcta' student.data
    tatactgatacatacatacgctattac

% egrep '^y.(..)*y$' /usr/dict/words
    yesterday
```

Starts and ends with y, odd number of characters.

```
% egrep -v '[aeiou]' /usr/dict/words |
  egrep '......'
    rhythm
    syzygy
```

Find all words with no vowels and 6 or more letters.

Fundamental Questions: Theoretical Minimum

Which aspects are essential?

- Unix `egrep` regular expressions are useful.
- But more complex than theoretical minimum.
- `egrep` theoretical minimum:

`c`	any non-special character matches itself
`r*`	zero or more occurrence of r
`(r)`	grouping
`r1|r2`	logical OR

`.`	any single character
`[aeiou]`	any vowel
`[^ aeiou]`	any non-vowel
`^`	beginning of line
`$`	end of line

not needed

Fundamental Questions: What Kinds of Patterns

What kinds of patterns can be specified?
(all but one of following)

All bit strings that:	Example
Begin with 0 and end with 1.	`00010110111`
Have more 1's than 0's.	`01111001100`
Have no consecutive 1's.	`01001010010`
Has and odd number of 0's.	`01001010010`
Has 011010 as a substring.	`00011010000`

Fundamental Questions: What Kinds of Patterns

What kinds of patterns can be specified?
(all but one of following)

All bit strings that:	Regular Expression
Begin with 0 and end with 1.	`0(0|1)*1`
Have more 1's than 0's.	`not possible`
Have no consecutive 1's.	`(0 | 10)*(1 | 0*)`
Has and odd number of 0's.	`(1*01*01*)*(1*01*)`
Has 011010 as a substring.	`(0|1)*011010(0|1)*`

Formal Languages

An alphabet is a finite set of symbols.

- Binary alphabet = `{0, 1}`
- Lower-case alphabet = `{a, b, c, d, ..., y, z}`
- Genetic alphabet = `{a, c, t, g}`

A string is a finite sequence of symbols in the alphabet.

- `'0111011011'` is a string in the binary alphabet.
- `'tigers'` is a string in the lower-case alphabet.
- `'acctgaacta'` is a string in the genetic alphabet.

A formal language is an (unordered) set of strings in an alphabet.

- Can have infinitely many strings.
- Examples:
  `{0, 010, 0110, 01110, 011110, 0111110, ...}`
  `{11, 1111, 111111, 11111111, 1111111111, ...}`

Formal Languages

Can cast any computation as a language recognition problem.

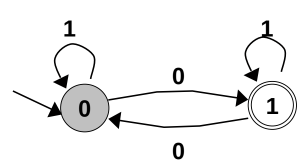
- Is x = 23,536,481,273 a prime number?

FSA.

- Machine determines whether a string is in language.

Regular expression.

- Shorthand method for specifying a language.



$$\underbrace{\texttt{(1*01*01*)*}}_{\text{even \# of 0's}}\underbrace{\texttt{(1*01*)}}_{\text{exactly one 0}}$$

Duality Between FSA's and RE's

Observation: for each FSA we create, we can find a regular expression that matches the same strings that the FSA accepts.

Is this always the case?

What about the OTHER way around?

Stay tuned: see Lecture T2.

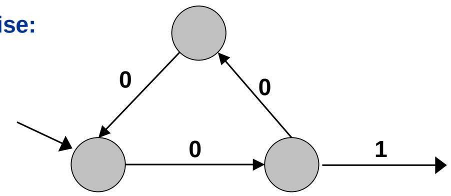
Limitations of FSA

FSA are simple machines.

- N states $\Rightarrow$ can't remember more than N things.
- Some languages require remembering more than N things.

No FSA can recognize the language of all bit strings with an equal number of 0's and 1's.

A warmup exercise:

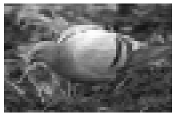


If `01xyz` accepted then so is `00001xyz`

Limitations of FSA

No FSA can recognize the language of all bit strings with an equal number of 0's and 1's.

- Suppose an N-state FSA can recognize this language.
- Consider following input: $\underbrace{\texttt{000000000}}_{\text{N+1 0's}}\underbrace{\texttt{11111111}}_{\text{N+1 1's}}$
- FSA must accept this string.
- Some state x is revisited during first N+1 0's since only N states.



```
0000000011111111
  x    x
```

- Machine would accept same string without intervening 0's.

```
000011111111
```

- This string doesn't have an equal number of 0's and 1's.

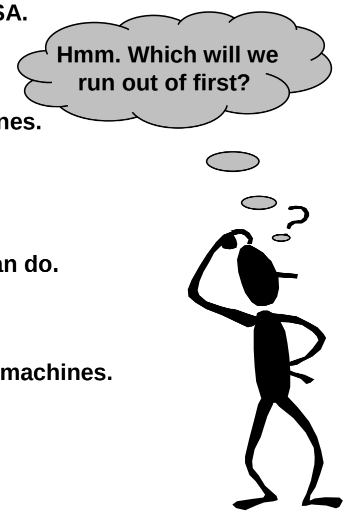
Looking Ahead

Today.

- Defined a simple abstract machine = FSA.
- Capable of pattern matching.
- Incapable of "counting."
- Need to consider more powerful machines.

Future lectures.

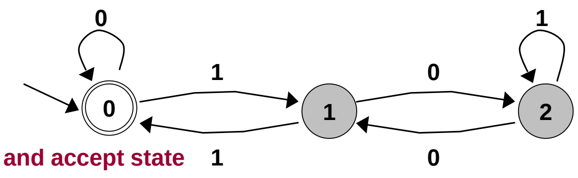
- Define an abstract machine.
- Understand how it works and what it can do.
- Find things it can't do.
- Define a more powerful machine.
- Repeat until we run out of problems or machines.



Lecture T1: Supplemental Notes

A Fourth Example

FSA to decide if integer (represented in binary) is divisible by 3?



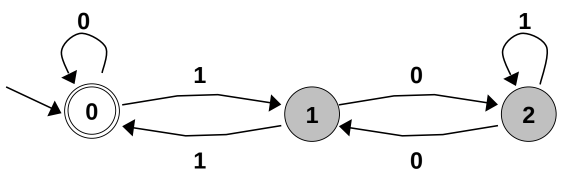
0 is start and accept state

What bit strings does it accept?

- **Yes: 11 (3_{10}), 110 (6_{10}), 1001 (9_{10}), 1100 (12_{10}), 1111 (15_{10}), 10011 (18_{10}), integers divisible by 3.**
- **No: 1 (1_{10}), 10 (2_{10}), 100 (4_{10}), 101 (5_{10}), 111 (7_{10}), integers not divisible by 3.**

A Fourth Example

FSA to decide if input (convert binary to decimal) is divisible by 3?



How does it work?

- **State 0: input so far is divisible by 3.**
- **State 1: input has remainder 1 upon division by 3.**
- **State 2: input has remainder 2 upon division by 3.**
- **Transition example.**
  - **Input 1100 (12_{10}) ends in state 0.**
  - **If next bit is 0 then stay in state 0: 11000 (24_{10}).**
  - **Adding 0 to last bit is same as multiplying number by 2. Remains divisible by 3.**