5. **Divide and Conquer**

- *merge and count demo*
Merge and count demo

Given two sorted lists $A$ and $B$,
- Count number of inversions $(a, b)$ with $a \in A$ and $b \in B$.
- Merge $A$ and $B$ into sorted list $C$.

<table>
<thead>
<tr>
<th>sorted list A</th>
<th>3</th>
<th>7</th>
<th>10</th>
<th>14</th>
<th>18</th>
</tr>
</thead>
<tbody>
<tr>
<td>sorted list B</td>
<td>2</td>
<td>11</td>
<td>16</td>
<td>17</td>
<td>23</td>
</tr>
</tbody>
</table>
Given two sorted lists \( A \) and \( B \),

- Count number of inversions \( (a, b) \) with \( a \in A \) and \( b \in B \).
- Merge \( A \) and \( B \) into sorted list \( C \).

**Merge and count demo**

**sorted list A**

\[
\begin{array}{cccccc}
3 & 7 & 10 & 14 & 18 \\
\end{array}
\]

**sorted list B**

\[
\begin{array}{cccccc}
2 & 11 & 16 & 17 & 23 \\
\end{array}
\]

**compare minimum entry in each list**: copy 2 and add \( x \) to inversion count

**sorted list C**

\[
\begin{array}{cccccc}
\text{ } & \text{ } & \text{ } & \text{ } & \text{ } \\
\end{array}
\]

\( x = 5 \)

inversions = 0
Given two sorted lists $A$ and $B$,

- Count number of inversions $(a, b)$ with $a \in A$ and $b \in B$.
- Merge $A$ and $B$ into sorted list $C$.

### Merge and count demo

**Sorted List $A$**

| 3 | 7 | 10 | 14 | 18 |

**Sorted List $B$**

| 2 | 11 | 16 | 17 | 23 |

**Sorted List $C$**

| 2 |

**Inversions**

- $x = 5$
- Inversions = 5
Merge and count demo

Given two sorted lists $A$ and $B$,
- Count number of inversions $(a, b)$ with $a \in A$ and $b \in B$.
- Merge $A$ and $B$ into sorted list $C$.

**sorted list A**

| 3 | 7 | 10 | 14 | 18 |

**sorted list B**

| 2 | 11 | 16 | 17 | 23 |

compare minimum entry in each list: copy 7 and decrement $x$

**sorted list C**

| 2 | 3 |

$x = 4$

inversions = 5
**Merge and count demo**

Given two sorted lists \( A \) and \( B \),

- Count number of inversions \((a, b)\) with \( a \in A \) and \( b \in B \).
- Merge \( A \) and \( B \) into sorted list \( C \).

**Sorted list A**

\[
\begin{array}{cccc}
3 & 7 & 10 & 14 & 18 \\
\end{array}
\]

**Sorted list B**

\[
\begin{array}{cccc}
2 & 11 & 16 & 17 & 23 \\
\end{array}
\]

**Merge and count demo**

compare minimum entry in each list: copy 10 and decrement \( x \)

**Sorted list C**

\[
\begin{array}{cccc}
2 & 3 & 7 & \\
\end{array}
\]

\[
x = 3 \\
\text{inversions} = 5
\]
**Merge and count demo**

Given two sorted lists $A$ and $B$,
- Count number of inversions $(a, b)$ with $a \in A$ and $b \in B$.
- Merge $A$ and $B$ into sorted list $C$.

### sorted list $A$

| 3 | 7 | 10 | 14 | 18 |

### sorted list $B$

| 2 | 11 | 16 | 17 | 23 |

Compare minimum entry in each list: copy 11 and add $x$ to increment count.

### sorted list $C$

| 2 | 3 | 7 | 10 | 16 | 17 | 23 |

$x = 2$

inversions $= 5$
Merge and count demo

Given two sorted lists $A$ and $B$,
- Count number of inversions $(a, b)$ with $a \in A$ and $b \in B$.
- Merge $A$ and $B$ into sorted list $C$.

<table>
<thead>
<tr>
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</thead>
<tbody>
<tr>
<td>3 7 10 14 18</td>
<td>2 11 16 17 23</td>
</tr>
</tbody>
</table>

compare minimum entry in each list: copy 14 and decrement $x$

<table>
<thead>
<tr>
<th>sorted list C</th>
</tr>
</thead>
<tbody>
<tr>
<td>2 3 7 10 11</td>
</tr>
</tbody>
</table>

$x = 2$

inversions = 7
Merge and count demo

Given two sorted lists $A$ and $B$,
- Count number of inversions $(a, b)$ with $a \in A$ and $b \in B$.
- Merge $A$ and $B$ into sorted list $C$.

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<td>11</td>
<td>16</td>
<td>17</td>
<td>23</td>
</tr>
</tbody>
</table>

compare minimum entry in each list: copy 16 and add $x$ to increment count

<table>
<thead>
<tr>
<th>sorted list C</th>
<th>2</th>
<th>3</th>
<th>7</th>
<th>10</th>
<th>11</th>
<th>14</th>
</tr>
</thead>
</table>

$x = 1$
Inversions = 7
Merge and count demo

Given two sorted lists $A$ and $B$,
- Count number of inversions $(a, b)$ with $a \in A$ and $b \in B$.
- Merge $A$ and $B$ into sorted list $C$.

<table>
<thead>
<tr>
<th>sorted list A</th>
<th>sorted list B</th>
</tr>
</thead>
<tbody>
<tr>
<td>3 7 10 14 18</td>
<td>2 11 16 17 23</td>
</tr>
<tr>
<td>5 2 1</td>
<td></td>
</tr>
</tbody>
</table>

compare minimum entry in each list: copy 17 and add $x$ to increment count

<table>
<thead>
<tr>
<th>sorted list C</th>
</tr>
</thead>
<tbody>
<tr>
<td>2 3 7 10 11 14 16</td>
</tr>
<tr>
<td>$x = 1$</td>
</tr>
<tr>
<td>inversions = 8</td>
</tr>
</tbody>
</table>
Merge and count demo

Given two sorted lists $A$ and $B$,
- Count number of inversions $(a, b)$ with $a \in A$ and $b \in B$.
- Merge $A$ and $B$ into sorted list $C$.

<table>
<thead>
<tr>
<th>sorted list A</th>
<th>sorted list B</th>
</tr>
</thead>
<tbody>
<tr>
<td>3  7  10  14  18</td>
<td>2  11  16  17  23</td>
</tr>
</tbody>
</table>

compare minimum entry in each list: copy 18 and decrement $x$

<table>
<thead>
<tr>
<th>sorted list C</th>
</tr>
</thead>
<tbody>
<tr>
<td>2  3  7  10  11  14  16  17</td>
</tr>
</tbody>
</table>

$x = 1$

inversions $= 9$
Merge and count demo

Given two sorted lists $A$ and $B$,
- Count number of inversions $(a, b)$ with $a \in A$ and $b \in B$.
- Merge $A$ and $B$ into sorted list $C$.

<table>
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<tbody>
<tr>
<td>3 7 10 14 18</td>
<td>2 11 16 17 23</td>
</tr>
<tr>
<td></td>
<td>5 2 1 1</td>
</tr>
</tbody>
</table>

list A exhausted: copy 23

<table>
<thead>
<tr>
<th>sorted list C</th>
</tr>
</thead>
<tbody>
<tr>
<td>2 3 7 10 11 14 16 17 18</td>
</tr>
</tbody>
</table>

$x = 0$

inversions = 9
Merge and count demo

Given two sorted lists $A$ and $B$,
- Count number of inversions $(a, b)$ with $a \in A$ and $b \in B$.
- Merge $A$ and $B$ into sorted list $C$.

### sorted list A

| 3 | 7 | 10 | 14 | 18 |

### sorted list B

| 2 | 11 | 16 | 17 | 23 |

### sorted list C

| 2 | 3 | 7 | 10 | 11 | 14 | 16 | 17 | 18 | 23 |

$x = 0$

inversions $= 9$

done: return 9 inversions