COS426 Precept9

Rasterizer (Part 2)
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Rasterizer

• Render a lot of triangles in the image plane
  • Projection – orthogonal (naïve) or perspective
  • Which triangles are in the front? (z buffering)
  • How does the triangle react to the light? (reflection model)
  • Meshes are coarse. How to cheat our eyes? (interpolation)
  • How does the material affect the color? (texture mapping)
  • How to add fine details at low cost? (normal mapping)
Rigid Transformation

1. all the triangle vertices (3D) are represented in the world coordinate system ($z_{cam}$ isn’t directly accessible).
2. projection is done in the camera coordinate system.
3. rendering is again done in the world coordinate system.

source: http://www.robots.ox.ac.uk/~vgg/hzbook/all_figs/gif/fig5.3.gif
Rigid Transformation

\[
\begin{pmatrix}
R_{11} & R_{12} & R_{13} & T_x \\
R_{21} & R_{22} & R_{23} & T_y \\
R_{31} & R_{32} & R_{33} & T_z \\
0 & 0 & 0 & 1
\end{pmatrix}
\]
Transformation

\[
\begin{pmatrix} x_{cam} \\ y_{cam} \\ z_{cam} \\ 1 \end{pmatrix} = \begin{pmatrix} R_{11} & R_{12} & R_{13} & T_x \\ R_{21} & R_{22} & R_{23} & T_y \\ R_{31} & R_{32} & R_{33} & T_z \\ 0 & 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} x \\ y \\ z \\ 1 \end{pmatrix}
\]

homogeneous representation.
Why? it’s easier to concatenate square matrices.
Transformation

- Transformation can be very complicated. Don’t get confused.

Each joint is a coordinate system.
Now we are in the camera coordinate system!!

Perspective Projection

objects must be on the negative z axis, otherwise cannot be seen.
Which triangles should we render? – near and far planes

1. \( n \) and \( f \) are usually positive values. But near plane locates at \(-n\) and far plane locates at \(-f\).
2. if you \( z_{\text{cam}} \) is out side \([-f, -n]\), skip that triangle.
3. project triangle vertices using the projection matrix.
Graphics Projection

- Map x-component of a point to (-1, 1)
- Map y-component of a point to (-1, 1)
- Map z-component of a point from (near, far) to (-1, 1)
- Believe it or not, this matrix does the transformation:

\[
\begin{pmatrix}
\frac{2n}{r-l} & 0 & \frac{r+l}{r-l} & 0 \\
0 & \frac{2n}{t-b} & \frac{r-l}{t+b} & 0 \\
0 & 0 & \frac{f+n}{f-n} & -\frac{2fn}{f-n} \\
0 & 0 & -1 & 0 \\
\end{pmatrix}
\]
Mixing Projection and Transformation

\[
\begin{pmatrix}
    x' \\
y' \\
z' \\
w
\end{pmatrix} =
\begin{pmatrix}
    \frac{2n}{r-l} & 0 & \frac{r+l}{t-b} & 0 \\
    0 & \frac{2n}{t-b} & \frac{t+b}{t-b} & 0 \\
    0 & 0 & \frac{f+n}{f-n} & -1 \\
    0 & 0 & \frac{f-n}{f-n} & 0
\end{pmatrix}
\begin{pmatrix}
    R_{11} & R_{12} & R_{13} & T_x \\
    R_{21} & R_{22} & R_{23} & T_y \\
    R_{31} & R_{32} & R_{33} & T_z \\
    0 & 0 & 0 & 1
\end{pmatrix}
\begin{pmatrix}
    x \\
y \\
z \\
1
\end{pmatrix}
\]

\[
\begin{pmatrix}
    x_{\text{cam}} \\
y_{\text{cam}} \\
z_{\text{cam}} \\
1
\end{pmatrix}
\]

\[w = -z_{\text{cam}}!\]
Mixing Projection and Transformation

viewMat = \[
\begin{pmatrix}
\frac{2n}{r-l} & 0 & \frac{r+l}{r-l} & 0 \\
0 & \frac{2n}{t-b} & \frac{t+b}{t-b} & 0 \\
0 & 0 & \frac{f+n}{f-n} & -1 \\
0 & 0 & \frac{2fn}{f-n} & 0 \\
\end{pmatrix}
\]
\[
\times
\begin{pmatrix}
R_{11} & R_{12} & R_{13} & T_x \\
R_{21} & R_{22} & R_{23} & T_y \\
R_{31} & R_{32} & R_{33} & T_z \\
0 & 0 & 0 & 1 \\
\end{pmatrix}
\]

\[w = -z_{cam}!\]

We can simply check whether w is within [n, f]
Pipeline of Rendering a Triangle

In the world coordinate system: verts[], normals[], uvs[] (optional), material (optional).

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In the camera coordinate system: projectedVerts[].
Pipeline of Rendering a Triangle (Flat Shader)

For a pixel \((x, y)\) in the bounding box:

1. determine whether it’s inside the triangle (barycentric coordinates). If not, go to the next pixel.
2. use barycentric coordinates to interpolate \(z'/w\) for the pixel.
3. If \(z'/w\) is not smaller(closer) than \(zBuffer[x][y]\), go to the next pixel.
4. If the pixel survives, render the pixel!
Render a Pixel

• To render a pixel, we need the following ingredients.
  • normal of the pixel in the world coordinate system (interpolate using the three vertex normals and barycentric coordinates).
  • position of the pixel in the world coordinate system (interpolate using the three vertex positions and barycentric coordinates).
  • view position (where your camera/eye is, in the world coordinate system).
  • light position(s) (where the light source is, in the world coordinate system).
  • material of the pixel:
    • case 1: material is uniform (k_a, k_d, k_s, shininess).
    • case 2: texture maps. (we need uv coordinates to look up k_a, k_d, k_s, shininess of the pixel).
      • uv coordinates: (interpolate using the three uv coordinates and barycentric coordinates).
  • All ingredients are essential for Phong reflection model.
UV coordinates

• Can be computed automatically (a lot of papers). None of them is perfect.
• Done by artists.
• Specify where a triangle vertex should map to in the texture map.
• Not always available! Make sure to check whether uvs[] is defined or not.
Barycentric Coordinates

- Any point in the triangle can be represented as a linear combination of the three vertices
  - Q is a linear combination of A2 and A3
  - P is a linear combination of Q and A1
Barycentric Coordinates

- \( P = \alpha A_1 + \beta A_2 + \gamma A_3 \)
- \( \alpha + \beta + \gamma = 1 \)
- If any of \( \alpha, \beta, \gamma < 0 \), \( P \) is not in the triangle.
- Barycentric coordinate of \( A_1 \) is computed using \( A_2 \) and \( A_3 \)

See this article for detailed computation:
https://fgiesen.wordpress.com/2013/02/06/the-barycentric-conspirac/
Q&A