This test is 5 questions. Do all of your work on these pages (use the back for scratch space), giving
the answer in the space provided. This is a closed-book exam -- you may use one-page of notes
with writing on both sides during the exam. Please write out and sign the Honor Code pledge
before turning in the test.

``I pledge my honor that I have not violated the Honor Code during this examination.``

<table>
<thead>
<tr>
<th>Question</th>
<th>Score</th>
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<tr>
<td>1</td>
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Q1: Color Models (20 Points)

A) [5pts] Please circle True or False for each of the following statements:

   True  False :  A pixel is a little square.

   True  False :  The RGB color model is chosen for most computer monitors based on the response of cone cells in the human eye.

   True  False :  The gamut of a typical computer monitor includes all colors perceptible by the human eye.

   True  False :  Some colors that can be represented in the RGB color space cannot be represented in the CMY color space.

   True  False :  The CMY (or CMYK) color model is often used for printers.

C) [5pts] Please complete the table below where each row describes the same color in three different color models: RGB, CMY, and HSV. Note that hues for the HSV color model include Red=0°, Green=120°, and Blue = 240°.

<table>
<thead>
<tr>
<th>RGB</th>
<th>CMY</th>
<th>HSV</th>
</tr>
</thead>
<tbody>
<tr>
<td>1, 0, 0</td>
<td></td>
<td></td>
</tr>
<tr>
<td>1, 0, 0</td>
<td>1, 0, 0</td>
<td></td>
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<tr>
<td>1, 0, 0</td>
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<td>1, 0, 0</td>
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<tr>
<td>1, 1, 1</td>
<td></td>
<td></td>
</tr>
<tr>
<td>1, 1, 1</td>
<td></td>
<td>0.5, 0.5, 0.5</td>
</tr>
</tbody>
</table>
D) [5pts] If a color is represented by the spectral energy plot shown below, please explain whether/how you can determine its 1) hue, 2) saturation, 3) lightness, and 4) complementary color from properties of the plot. Please be specific (and concise) and mark the diagram to support your answer.

![Spectral Energy Plot](image)

E) [5pts] If a color is represented by the point marked X in the CIE chromaticity diagram below, please explain whether/how you can determine its 1) hue, 2) saturation, 3) lightness, and 4) complementary color from its position in the diagram. Please be specific (and concise) and mark the diagram to support your answer.

![CIE Chromaticity Diagram](image)
Q2: Image Processing (20 Points)

Below on the right is an image (of a set of concentric circles with thin lines that get closer as the circle radii get smaller) after it has been scaled down by MS Word to have 0.4 times the original resolution. In this image, it is possible to see artifact patterns (patterns that are not radially symmetric) in addition to the concentric circles; and if the image is scaled by a different factor, then different patterns appear. Please answer the following questions about this image with one phrase or sentence each.

A) [2pts] Why do these artifacts appear when the image is scaled down?

B) [2pts] Could they appear if the image is scaled down to have exactly 0.5 times the original resolution? Explain.

C) [2pts] Why are the artifacts more prominent closer to the center of the image?

D) [2pts] Please explain how these artifacts could be avoided, in theory.

E) [2pts] Please describe a practical process that could be implemented in the software to reduce these artifacts when scaling an image.

F) [2pts] When the process you described in part (E) is applied, there will probably still be visual artifacts when this image is scaled down. Why?
G) [4pts] Please write the result of blurring the following 1D image I by convolution with the filter f. Please provide one sentence to explain how you handled the boundary "pixels," but be sure not to brighten/darken the image.

\[ I = [4, 8, 12, 16, 8, 4, 4, 12, 8] \]

\[ f = [1/4, 1/2, 1/4] \]

\[ \text{Blur}(I, f) = [ \quad ] \]

H) [4pts] Please explain with a figure why there are 12 operators in the Poter-Duff image composition algebra. (hint: consider how they model combinations of partially covered pixels)
Q3: Parametric Curves and Surfaces (20 Points)

A) [5pts] Please circle true or false for each of the following statements:

True  False : It is impossible for a Bezier curve to intersect itself.

True  False : It is possible to define a family of spline curves (e.g., Bezier, BSpline, etc.) with piecewise cubic blending functions that has $G^2$ continuity everywhere, local control, and interpolates its control vertices.

True  False : The BSpline curve specified by four co-linear control points must lie on the line containing those control points.

True  False : A tensor product cubic Bezier surface patch interpolates all 16 of its control points except the four in the middle.

True  False : Parametric surfaces are often used for mechanical CAD applications because they are particularly efficient for collision detection.

B) [5pts] Please draw any example curve that has the following continuity properties (or indicate that no such curve is possible).

Not $G^0$:

$G^0$, but not $G^1$:

$G^1$, but not $G^2$:

$G^2$, but not $G^1$:

$G^2$: 
C) [5pts] Please draw the piecewise cubic **Bezier** spline curve through the following control vertices, \(V_0-V_{12}\). Pay particular attention to the positions, derivatives (slopes), and continuity of the spline at joints between curve segments -- i.e., mark the positions and derivatives of joints with dots and tangent lines, respectively, and write \(G^0, G^1, \) or \(G^2\) next to each joint to indicated its continuity.

D) [5pts] Same question as part (B), but for a piecewise cubic **B-Spline**.
Q4: Object Representations (20 Points)

Please answer the following questions in approximately one sentence each.

A) [2pts] Why would an implicit surface be a good representation for the blobs of wax in a lava lamp simulation?

B) [2pts] Please list two examples of industries for which $G^2$ continuity is particularly important in a surface design.

1) 
2) 

C) [2pts] Why would a quadtree be a better than a voxel grid for representing a 3D function indicating the temperature resulting from radiation of heat from the surface of an object?

D) [2pts] There is usually a trade-off between 1) the number and 2) the complexity of primitive elements (e.g., voxels, polygons, surface patches, etc.) in a 3D object representation. Please provide examples of two object representations at opposite extremes of the trade-off with a brief explanation.

E) [2pts] Please give both parametric and implicit equations for the following 3D surfaces. You may support your answer with a figure, if you wish.

<table>
<thead>
<tr>
<th>Implicit</th>
<th>Parametric</th>
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<tbody>
<tr>
<td>Plane</td>
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<tr>
<td>Sphere</td>
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F) [5pts] List the sequence of calls to R3SignedDistance(R3Plane plane, R3Point point) that would be executed to check whether the point marked X is within the interior (shaded region) of the Binary Space Partition (BSP) tree representation shown below.

![Binary Spatial Partition](image1)

G) [5pts] Please draw a constructive solid geometry representation for the CAD part shown below, assuming that holes through the center box are circular in cross-section (your answer may include cubes, spheres, cylinders, and cones as geometric primitives).

![CAD Part](image2)
Q5: Modeling Transformations (20 Points)

For each of the following scene graphs, please draw the scene it represents on the coordinate axes to its right, labeling each of the squares in the drawing with an A, B, C, or D placed in its top-right corner (an example for Square C is provided in part A). In these scene graphs, the symbol "M_i" represents a 3x3 homogeneous transformation matrix associated with a scene graph node, and the symbol "Square" represents a "unit" square -- i.e., a square that is axis-aligned, centered at (0,0), has sides of length 2, and has corners at (-1,-1) and (1,1) in its modeling coordinate system.

A)

\[
M_1 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}, \quad M_2 = \begin{bmatrix} 1 & 0 & -3 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}, \quad M_3 = \begin{bmatrix} 1 & 0 & 3 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}
\]

B)

\[
M_1 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}, \quad M_2 = \begin{bmatrix} 2 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 1 \end{bmatrix}, \quad M_3 = \begin{bmatrix} 3 & 0 & 0 \\ 0 & 3 & 0 \\ 0 & 0 & 1 \end{bmatrix}
\]

C)

\[
M_1 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}, \quad M_2 = \begin{bmatrix} 1 & 0 & 3 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}, \quad M_3 = \begin{bmatrix} 1 & 0 & -2 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}
\]
D)

\[ M_1 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \quad M_2 = \begin{bmatrix} \cos(45) & -\sin(45) & 0 \\ \sin(45) & \cos(45) & 0 \\ 0 & 0 & 1 \end{bmatrix} \]

\[ M_3 = \begin{bmatrix} 2 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 1 \end{bmatrix} \quad M_4 = \begin{bmatrix} \cos(45) & -\sin(45) & 0 \\ \sin(45) & \cos(45) & 0 \\ 0 & 0 & 1 \end{bmatrix} \]

E)

\[ M_1 = \begin{bmatrix} \cos(45) & -\sin(45) & 0 \\ \sin(45) & \cos(45) & 0 \\ 0 & 0 & 1 \end{bmatrix} \]

\[ M_2 = \begin{bmatrix} 1 & 0 & 3 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \quad M_3 = \begin{bmatrix} 2 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 1 \end{bmatrix} \]

\[ M_4 = \begin{bmatrix} 1 & 0 & -6 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \]