Lecture 8: Applications of solving CNF
Outline

• Brief review on search techniques
  – Blind search, heuristic search, and game search

• Brief review on logical inference
  – Propositional logic, model checking, and theorem proving

• Applications of solving CNF
  – Many problems can be reduced to SAT problems
5 components of search problems

- Initial state
- Actions
- Transitional model
- Goal test
- Path cost
Blind search

- Breadth-First Search (BFS)
- Depth-First Search (DFS)
- Depth-Limited Search (DLS)
  - The depth of the root node is 0.
- Iterative-Deepening Search (IDS)
  - Start at $l = 0$.
- Bidirectional Search
Heuristic Search

• Admissible and consistent heuristics

• Greedy-First Search
  – \( f(n) = h(n) \)

• A* Search
  – \( f(n) = g(n) + h(n) \)
  – A* graph search is optimal when using consistent heuristics
  – A* tree search is optimal when using admissible heuristics
Search in Games

• Games
  – 2 player
  – Zero-sum

• The Minimax algorithm
  – Complete and optimal

• Alpha-beta pruning
  – Significantly reduce the number of nodes searched while maintaining the optimality of the Minimax algorithm.
Logical inference

• Problem: Can we infer a new fact given a set of known facts (KB |= α ?)

• Propositional logic
  – Propositional symbols, Syntax and semantics

• Model checking
  – DPLL
  – WALKSAT

• Theorem proving
  – Resolution algorithm (is KB ∧ ¬α unsatisfiable? )
  – Forward/backward chaining (KB: Horn clauses, α : single positive symbol )
DPLL and WALKSAT

• **DPLL**
  
  • Complete and sound
  
  • Determine KB |= α
  
  • Check satisfiability of a cnf + find a model if it is satisfiable

• **WALKSAT**
  
  • Sound, but not complete
  
  • Mostly used for finding a model when a cnf is satisfiable
Applications of solving CNF

• SAT is used in problems other than logical inference
  – N-queen problem
  – 3-coloring graph
  – Hamiltonian path
  – Planning
Reduce 3-coloring graph to SAT

- Define Symbols:
  - \( P_{ij} \): node \( i \) is colored in color \( j \)
  - \( i = 1, 2, 3 \) or 4
  - \( j = r, g \) or \( b \)

- Express facts/rules in clauses
  1. Each node gets one color
  2. Two nodes sharing a common edge can’t be colored the same

Diagram:

- Nodes: 1, 2, 3, 4
- Edges: 1-2, 1-3, 2-3, 4-1, 4-2, 4-3
Reduce 3-coloring graph to SAT

1. Each node gets one color

   (1) Each node gets at least one color

   \[ P_{1r} \lor P_{1g} \lor P_{1b} \]
   \[ P_{2r} \lor P_{2g} \lor P_{2b} \]
   \[ P_{3r} \lor P_{3g} \lor P_{3b} \]
   \[ P_{4r} \lor P_{4g} \lor P_{4b} \]

   (2) Each node gets only one color

   \[ (\neg P_{1r} \lor \neg P_{1g}) \land (\neg P_{1r} \lor \neg P_{1b}) \land (\neg P_{1g} \lor \neg P_{1b}) \]
   \[ (\neg P_{2r} \lor \neg P_{2g}) \land (\neg P_{2r} \lor \neg P_{2b}) \land (\neg P_{2g} \lor \neg P_{2b}) \]
   \[ (\neg P_{3r} \lor \neg P_{3g}) \land (\neg P_{3r} \lor \neg P_{3b}) \land (\neg P_{3g} \lor \neg P_{3b}) \]
   \[ (\neg P_{4r} \lor \neg P_{4g}) \land (\neg P_{4r} \lor \neg P_{4b}) \land (\neg P_{4g} \lor \neg P_{4b}) \]
2. Two nodes sharing a common edge can’t be colored the same

• For edge 1-4
  \[(\neg P_{1r} \lor \neg P_{4r}) \land (\neg P_{1g} \lor \neg P_{4g}) \land (\neg P_{1b} \lor \neg P_{4b})\]

• For edge 2-4
  \[- (\neg P_{2r} \lor \neg P_{4r}) \land (\neg P_{2g} \lor \neg P_{4g}) \land (\neg P_{2b} \lor \neg P_{4b})\]

• For edge 1-2
  \[- (\neg P_{1r} \lor \neg P_{2r}) \land (\neg P_{1g} \lor \neg P_{2g}) \land (\neg P_{1b} \lor \neg P_{2b})\]

• For edge 2-3
  \[- (\neg P_{2r} \lor \neg P_{3r}) \land (\neg P_{2g} \lor \neg P_{3g}) \land (\neg P_{2b} \lor \neg P_{3b})\]

--- Put all clauses in a cnf and pass to a sat-solver.

--- A model for the constructed cnf is a solution to the original problem.

--- Legal coloring is guaranteed by the rules in 1 and 2.
Announcement & Reminder

• P1 is due today

--- due by midnight, upload your files to CS dropbox.

--- remember to press the “check all submitted files” button. No credit will be given the code that does not compile.

• P2 has been released and is due on Tuesday Oct. 27th

--- due by midnight, upload your files to CS dropbox.