5.1 STRING Sorts

- strings in Java
- key-indexed counting
- LSD radix sort
- MSD radix sort
- 3-way radix quicksort
- suffix arrays
5.1 String Sorts

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String processing

**String.** Sequence of characters.

**Important fundamental abstraction.**

- Programming systems (e.g., Java programs).
- Communication systems (e.g., email).
- Information processing.
- Genomic sequences.
- ...

“The digital information that underlies biochemistry, cell biology, and development can be represented by a simple string of G's, A's, T's and C's. This string is the root data structure of an organism's biology.” — M. V. Olson
The char data type

C char data type. Typically an 8-bit integer.
- Supports 7-bit ASCII.
- Can represent at most 256 characters.

Java char data type. A 16-bit unsigned integer.
- Supports original 16-bit Unicode.
- Supports 21-bit Unicode 3.0 (awkwardly).
I ♥ Unicode
The String data type

String data type in Java. Immutable sequence of characters.

Length. Number of characters.
Indexing. Get the $i^{th}$ character.
Concatenation. Concatenate one string to the end of another.
The String data type: immutability

Q. Why immutable?

A. All the usual reasons.
   • Can use as keys in symbol table.
   • Don't need to defensively copy.
   • Ensures consistent state.
   • Supports concurrency.
   • Improves security.

public class FileInputStream
{
    private String filename;
    public FileInputStream(String filename)
    {
        if (!allowedToReadFile(filename))
            throw new SecurityException();
        this.filename = filename;
    }
    ...
}

attacker could bypass security if string type were mutable
The String data type: representation

**Representation (Java 7).** Immutable `char[]` array + cache of hash.

<table>
<thead>
<tr>
<th>operation</th>
<th>Java</th>
<th>running time</th>
</tr>
</thead>
<tbody>
<tr>
<td>length</td>
<td><code>s.length()</code></td>
<td>1</td>
</tr>
<tr>
<td>indexing</td>
<td><code>s.charAt(i)</code></td>
<td>1</td>
</tr>
<tr>
<td>concatenation</td>
<td><code>s + t</code></td>
<td>$M + N$</td>
</tr>
<tr>
<td>:</td>
<td>:</td>
<td>:</td>
</tr>
</tbody>
</table>
String performance trap

Q. How to build a long string, one character at a time?

public static String reverse(String s) {
    String rev = "";
    for (int i = s.length() - 1; i >= 0; i--)
        rev += s.charAt(i);
    return rev;
}

A. Use StringBuilder data type (mutable char[] resizing array).

public static String reverse(String s) {
    StringBuilder rev = new StringBuilder();
    for (int i = s.length() - 1; i >= 0; i--)
        rev.append(s.charAt(i));
    return rev.toString();
}
Comparing two strings

Q. How many character compares to compare two strings, each of length $W$?

```
s.compareTo(t)
```

<table>
<thead>
<tr>
<th>s</th>
<th>p r e f e t c h</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>1 2 3 4 5 6 7</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>t</th>
<th>p r e f i x e s</th>
</tr>
</thead>
</table>

Running time. Proportional to length of longest common prefix.

- Proportional to $W$ in the worst case.
- But, often sublinear in $W$. 
Alphabets

**Digital key.** Sequence of digits over fixed alphabet.

**Radix.** Number of digits $R$ in alphabet.

<table>
<thead>
<tr>
<th>name</th>
<th>$R$</th>
<th>$\lg R$</th>
<th>characters</th>
</tr>
</thead>
<tbody>
<tr>
<td>BINARY</td>
<td>2</td>
<td>1</td>
<td>01</td>
</tr>
<tr>
<td>OCTAL</td>
<td>8</td>
<td>3</td>
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</tr>
<tr>
<td>DECIMAL</td>
<td>10</td>
<td>4</td>
<td>0123456789</td>
</tr>
<tr>
<td>HEXADECIMAL</td>
<td>16</td>
<td>4</td>
<td>0123456789ABCDEF</td>
</tr>
<tr>
<td>DNA</td>
<td>4</td>
<td>2</td>
<td>ACTG</td>
</tr>
<tr>
<td>LOWERCASE</td>
<td>26</td>
<td>5</td>
<td>abcdefghijklmnopqrstuvwxyz</td>
</tr>
<tr>
<td>UPPERCASE</td>
<td>26</td>
<td>5</td>
<td>ABCDEFGHIJKLMNOPQRSTUVWXYZ</td>
</tr>
<tr>
<td>PROTEIN</td>
<td>20</td>
<td>5</td>
<td>ACDEFGHIJKLMNOPQRSTUVWXYZ</td>
</tr>
<tr>
<td>BASE64</td>
<td>64</td>
<td>6</td>
<td>ABCDEFGHIJKLMNOPQRSTUVWXYZabcdefghijklmnopqrstuvwxzy0123456789+/</td>
</tr>
<tr>
<td>ASCII</td>
<td>128</td>
<td>7</td>
<td>ASCII characters</td>
</tr>
<tr>
<td>EXTENDED.ASCII</td>
<td>256</td>
<td>8</td>
<td>extended ASCII characters</td>
</tr>
<tr>
<td>UNICODE16</td>
<td>65536</td>
<td>16</td>
<td>Unicode characters</td>
</tr>
</tbody>
</table>
5.1 String Sorts

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- 3-way radix quicksort
- suffix arrays
Review: summary of the performance of sorting algorithms

Frequency of operations.

<table>
<thead>
<tr>
<th>algorithm</th>
<th>guarantee</th>
<th>random</th>
<th>extra space</th>
<th>stable?</th>
<th>operations on keys</th>
</tr>
</thead>
<tbody>
<tr>
<td>insertion sort</td>
<td>( \frac{1}{2} N^2 )</td>
<td>( \frac{1}{4} N^2 )</td>
<td>1</td>
<td>✔</td>
<td>compareTo()</td>
</tr>
<tr>
<td>mergesort</td>
<td>( N \lg N )</td>
<td>( N \lg N )</td>
<td>( N )</td>
<td>✔</td>
<td>compareTo()</td>
</tr>
<tr>
<td>quicksort</td>
<td>( 1.39 , N \lg N ) *</td>
<td>( 1.39 , N \lg N )</td>
<td>( c , \lg N ) *</td>
<td>✔</td>
<td>compareTo()</td>
</tr>
<tr>
<td>heapsort</td>
<td>( 2 , N \lg N )</td>
<td>( 2 , N \lg N )</td>
<td>1</td>
<td></td>
<td>compareTo()</td>
</tr>
</tbody>
</table>

* probabilistic

Lower bound. \( \sim N \lg N \) compares required by any compare-based algorithm.

Q. Can we do better (despite the lower bound)?
A. Yes, if we don't depend on key compares.  

use array accesses to make R-way decisions (instead of binary decisions)
Key-indexed counting: assumptions about keys

Assumption. Keys are integers between 0 and $R - 1$.

Implication. Can use key as an array index.

Applications.

- Sort string by first letter.
- Sort class roster by section.
- Sort phone numbers by area code.
- Subroutine in a sorting algorithm. [stay tuned]

Remark. Keys may have associated data $\Rightarrow$
can't just count up number of keys of each value.
Key-indexed counting demo

**Goal.** Sort an array \( a[] \) of \( N \) integers between 0 and \( R - 1 \).

- Count frequencies of each letter using key as index.
- Compute frequency cumulates which specify destinations.
- Access cumulates using key as index to move items.
- Copy back into original array.

```java
int N = a.length;
int[] count = new int[R+1];
for (int i = 0; i < N; i++)
    count[a[i]+1]++;
for (int r = 0; r < R; r++)
    count[r+1] += count[r];
for (int i = 0; i < N; i++)
    aux[count[a[i]]++] = a[i];
for (int i = 0; i < N; i++)
    a[i] = aux[i];
```

**Example:**

<table>
<thead>
<tr>
<th>i</th>
<th>a[i]</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>d</td>
</tr>
<tr>
<td>1</td>
<td>a</td>
</tr>
<tr>
<td>2</td>
<td>c</td>
</tr>
<tr>
<td>3</td>
<td>f</td>
</tr>
<tr>
<td>4</td>
<td>f</td>
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<td>f</td>
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<tr>
<td>9</td>
<td>b</td>
</tr>
<tr>
<td>10</td>
<td>e</td>
</tr>
<tr>
<td>11</td>
<td>a</td>
</tr>
</tbody>
</table>

R = 6

Example output:

- Use `a` for 0
- Use `b` for 1
- Use `c` for 2
- Use `d` for 3
- Use `e` for 4
- Use `f` for 5
Key-indexed counting demo

**Goal.** Sort an array $a[]$ of $N$ integers between 0 and $R - 1$.
- Count frequencies of each letter using key as index.
- Compute frequency cumulates which specify destinations.
- Access cumulates using key as index to move items.
- Copy back into original array.

```java
int N = a.length;
int[] count = new int[R+1];

for (int i = 0; i < N; i++)
    count[a[i]+1]++;

for (int r = 0; r < R; r++)
    count[r+1] += count[r];

for (int i = 0; i < N; i++)
    aux[count[a[i]]++] = a[i];

for (int i = 0; i < N; i++)
    a[i] = aux[i];
```

<p>| | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>d</td>
</tr>
<tr>
<td>1</td>
<td>a</td>
</tr>
<tr>
<td>2</td>
<td>c</td>
</tr>
<tr>
<td>3</td>
<td>f</td>
</tr>
<tr>
<td>4</td>
<td>f</td>
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<td>b</td>
</tr>
<tr>
<td>10</td>
<td>e</td>
</tr>
<tr>
<td>11</td>
<td>a</td>
</tr>
</tbody>
</table>

offset by 1 [stay tuned]

a  0
b  2
c  3
d  1
e  2
f  1
-  3
**Key-indexed counting demo**

**Goal.** Sort an array `a[]` of `N` integers between 0 and `R - 1`.

- Count frequencies of each letter using key as index.
- Compute frequency cumulates which specify destinations.
- Access cumulates using key as index to move items.
- Copy back into original array.

```java
int N = a.length;
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for (int r = 0; r < R; r++)
    count[r+1] += count[r];

for (int i = 0; i < N; i++)
    aux[count[a[i]]++] = a[i];

for (int i = 0; i < N; i++)
    a[i] = aux[i];
```

<table>
<thead>
<tr>
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</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>d</td>
</tr>
<tr>
<td>1</td>
<td>a</td>
</tr>
<tr>
<td>2</td>
<td>c</td>
</tr>
<tr>
<td>3</td>
<td>f</td>
</tr>
<tr>
<td>4</td>
<td>f</td>
</tr>
<tr>
<td>5</td>
<td>b</td>
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<tr>
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<tr>
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<td>b</td>
</tr>
<tr>
<td>10</td>
<td>e</td>
</tr>
<tr>
<td>11</td>
<td>a</td>
</tr>
</tbody>
</table>

- 6 keys < d, 8 keys < e
  - so d's go in a[6] and a[7]
Goal. Sort an array \texttt{a[]} of \(N\) integers between 0 and \(R - 1\).
- Count frequencies of each letter using key as index.
- Compute frequency cumulates which specify destinations.
- Access cumulates using key as index to move items.
- Copy back into original array.

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int N = a.length;
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for (int i = 0; i < N; i++)
    count[a[i]+1]++;

for (int r = 0; r < R; r++)
    count[r+1] += count[r];

for (int i = 0; i < N; i++)
    aux[count[a[i]]++] = a[i];

for (int i = 0; i < N; i++)
    a[i] = aux[i];
```
Key-indexed counting demo

**Goal.** Sort an array \( a[] \) of \( N \) integers between 0 and \( R - 1 \).

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- Copy back into original array.

```java
int N = a.length;
int[] count = new int[R+1];

for (int i = 0; i < N; i++)
    count[a[i]+1]++;

for (int r = 0; r < R; r++)
    count[r+1] += count[r];

for (int i = 0; i < N; i++)
    aux[count[a[i]]++] = a[i];

for (int i = 0; i < N; i++)
    a[i] = aux[i];
```
Which of the following are properties of key-indexed counting?

A. Time proportional to $N + R$.
B. Extra space proportional to $N + R$.
C. Stable.
D. All of the above.
E. *I don't know.*

### Radix sorting: quiz 1

<p>| | | | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>a[0]</td>
<td>Anderson</td>
<td>2</td>
<td>Harris</td>
</tr>
<tr>
<td>a[1]</td>
<td>Brown</td>
<td>3</td>
<td>Martin</td>
</tr>
<tr>
<td>a[2]</td>
<td>Davis</td>
<td>3</td>
<td>Moore</td>
</tr>
<tr>
<td>a[4]</td>
<td>Harris</td>
<td>1</td>
<td>Martinez</td>
</tr>
<tr>
<td>a[5]</td>
<td>Jackson</td>
<td>3</td>
<td>Miller</td>
</tr>
<tr>
<td>a[6]</td>
<td>Johnson</td>
<td>4</td>
<td>Robinson</td>
</tr>
<tr>
<td>a[7]</td>
<td>Jones</td>
<td>3</td>
<td>White</td>
</tr>
<tr>
<td>a[8]</td>
<td>Martin</td>
<td>1</td>
<td>Brown</td>
</tr>
<tr>
<td>a[9]</td>
<td>Martinez</td>
<td>2</td>
<td>Davis</td>
</tr>
<tr>
<td>a[10]</td>
<td>Miller</td>
<td>2</td>
<td>Jackson</td>
</tr>
<tr>
<td>a[12]</td>
<td>Robinson</td>
<td>2</td>
<td>Taylor</td>
</tr>
<tr>
<td>a[13]</td>
<td>Smith</td>
<td>4</td>
<td>Williams</td>
</tr>
<tr>
<td>a[14]</td>
<td>Taylor</td>
<td>3</td>
<td>Garcia</td>
</tr>
<tr>
<td>a[15]</td>
<td>Thomas</td>
<td>4</td>
<td>Johnson</td>
</tr>
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<td>a[16]</td>
<td>Thompson</td>
<td>4</td>
<td>Smith</td>
</tr>
<tr>
<td>a[17]</td>
<td>White</td>
<td>2</td>
<td>Thomas</td>
</tr>
<tr>
<td>a[18]</td>
<td>Williams</td>
<td>3</td>
<td>Thompson</td>
</tr>
</tbody>
</table>
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**Least-significant-digit-first string sort**

**LSD string (radix) sort.**
- Consider characters from right to left.
- Stably sort using \(d^{th}\) character as the key (using key-indexed counting).

<table>
<thead>
<tr>
<th></th>
<th>0</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
<th>7</th>
<th>8</th>
<th>9</th>
<th>10</th>
<th>11</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>d</td>
<td>a</td>
<td>b</td>
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<td></td>
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<td></td>
<td></td>
</tr>
<tr>
<td>11</td>
<td>a</td>
<td>c</td>
<td>e</td>
<td></td>
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<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

- sort key (\(d = 2\))
- sort key (\(d = 1\))
- sort key (\(d = 0\))

Sort is stable (arrows do not cross)
LSD string sort: correctness proof

**Proposition.** LSD sorts fixed-length strings in ascending order.

**Pf.** [by induction on $i$]
After pass $i$, strings are sorted by last $i$ characters.
- If two strings differ on sort key, key-indexed sort puts them in proper relative order.
- If two strings agree on sort key, stability keeps them in proper relative order.

**Proposition.** LSD sort is stable.
**Pf.** Key-indexed counting is stable.
public class LSD
{
    public static void sort(String[] a, int W)
    {
        int R = 256;
        int N = a.length;
        String[] aux = new String[N];

        for (int d = W-1; d >= 0; d--)
        {
            int[] count = new int[R+1];
            for (int i = 0; i < N; i++)
                count[a[i].charAt(d) + 1]++;
            for (int r = 0; r < R; r++)
                count[r+1] += count[r];
            for (int i = 0; i < N; i++)
                aux[count[a[i].charAt(d)]++] = a[i];
            for (int i = 0; i < N; i++)
                a[i] = aux[i];
        }
    }
}
## Summary of the performance of sorting algorithms

### Frequency of operations.

<table>
<thead>
<tr>
<th>algorithm</th>
<th>guarantee</th>
<th>random</th>
<th>extra space</th>
<th>stable?</th>
<th>operations on keys</th>
</tr>
</thead>
<tbody>
<tr>
<td>insertion sort</td>
<td>$\frac{1}{2} N^2$</td>
<td>$\frac{1}{4} N^2$</td>
<td>1</td>
<td>✔</td>
<td>compareTo()</td>
</tr>
<tr>
<td>mergesort</td>
<td>$N \lg N$</td>
<td>$N \lg N$</td>
<td>$N$</td>
<td>✔</td>
<td>compareTo()</td>
</tr>
<tr>
<td>quicksort</td>
<td>$1.39 N \lg N^*$</td>
<td>$1.39 N \lg N$</td>
<td>$c \lg N$</td>
<td>✔</td>
<td>compareTo()</td>
</tr>
<tr>
<td>heapsort</td>
<td>$2 N \lg N$</td>
<td>$2 N \lg N$</td>
<td>1</td>
<td>✔</td>
<td>compareTo()</td>
</tr>
<tr>
<td>LSD sort †</td>
<td>$2 W (N + R)$</td>
<td>$2 W (N + R)$</td>
<td>$N + R$</td>
<td>✔</td>
<td>charAt()</td>
</tr>
</tbody>
</table>

* probabilistic  
† fixed-length W keys

**Q.** What if strings are not all of same length?
Which sorting method to use to sort 1 million 32-bit integers?

A. Insertion sort.
B. Mergesort.
C. Quicksort.
D. LSD radix sort.
E. I don't know.
Problem. Sort huge array of random 128-bit numbers.

Ex. Supercomputer sort, internet router.

Which sorting method to use?

- Insertion sort.
- Mergesort.
- Quicksort.
- Heapsort.
- LSD string sort.
How to take a census in 1900s?

1880 Census. Took 1500 people 7 years to manually process data.

Herman Hollerith. Developed a tabulating and sorting machine.
- Use punch cards to record data (e.g., sex, age).
- Machine sorts one column at a time (into one of 12 bins).
- Typical question: how many women of age 20 to 30?

Hollerith tabulating machine and sorter

1890 Census. Finished in 1 year (and under budget)!
How to get rich sorting in 1900s?

**Punch cards.** [1900s to 1950s]

- Also useful for accounting, inventory, and business processes.
- Primary medium for data entry, storage, and processing.

Hollerith's company later merged with 3 others to form Computing Tabulating Recording Corporation (CTRC); company renamed in 1924.

*IBM 80 Series Card Sorter (650 cards per minute)*
LSD string sort: a moment in history (1960s)

To sort a card deck
- start on right column
- put cards into hopper
- machine distributes into bins
- pick up cards (stable)
- move left one column
- continue until sorted

Lysergic Acid Diethylamide
(Lucy in the Sky with Diamonds)
5.1 STRING SORTS

- strings in Java
- key-indexed counting
- LSD radix sort
- MSD radix sort
- 3-way radix quicksort
- suffix arrays
Reverse LSD

- Consider characters from left to right.
- Stably sort using $d^{th}$ character as the key (using key-indexed counting).

<p>| | | | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
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</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>d</td>
<td>a</td>
<td>b</td>
</tr>
<tr>
<td>1</td>
<td>a</td>
<td>d</td>
<td>d</td>
</tr>
<tr>
<td>2</td>
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<td>a</td>
<td>b</td>
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<tr>
<td>3</td>
<td>f</td>
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<td>4</td>
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<td>b</td>
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<tr>
<td>10</td>
<td>e</td>
<td>b</td>
<td>b</td>
</tr>
<tr>
<td>11</td>
<td>a</td>
<td>c</td>
<td>e</td>
</tr>
</tbody>
</table>

- sort key (d = 0)

<p>| | | | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
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<td>d</td>
<td>d</td>
</tr>
<tr>
<td>1</td>
<td>a</td>
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<td>e</td>
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<td>a</td>
<td>d</td>
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<tr>
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<td>b</td>
<td>e</td>
<td>d</td>
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<tr>
<td>5</td>
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<td>a</td>
<td>b</td>
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<td>d</td>
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<td>e</td>
</tr>
<tr>
<td>11</td>
<td>f</td>
<td>e</td>
<td>d</td>
</tr>
</tbody>
</table>

- sort key (d = 1)

<p>| | | | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>b</td>
<td>a</td>
<td>d</td>
</tr>
<tr>
<td>1</td>
<td>c</td>
<td>a</td>
<td>b</td>
</tr>
<tr>
<td>2</td>
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<td>a</td>
<td>b</td>
</tr>
<tr>
<td>3</td>
<td>d</td>
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<td>d</td>
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<td>e</td>
</tr>
<tr>
<td>11</td>
<td>f</td>
<td>e</td>
<td>d</td>
</tr>
</tbody>
</table>

- sort key (d = 2)

<p>| | | | |</p>
<table>
<thead>
<tr>
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<th></th>
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<th></th>
</tr>
</thead>
<tbody>
<tr>
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<td>c</td>
<td>a</td>
<td>b</td>
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<td>d</td>
<td>a</td>
<td>b</td>
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<td>d</td>
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<td>b</td>
<td>e</td>
<td>e</td>
</tr>
<tr>
<td>11</td>
<td>f</td>
<td>e</td>
<td>e</td>
</tr>
</tbody>
</table>

not sorted!
Most-significant-digit-first string sort

MSD string (radix) sort.

- Partition array into $R$ pieces according to first character (use key-indexed counting).
- Recursively sort all strings that start with each character (key-indexed counts delineate subarrays to sort).
MSD string sort: example

Trace of recursive calls for MSD string sort (no cutoff for small subarrays, subarrays of size 0 and 1 omitted)
Variable-length strings

Treat strings as if they had an extra char at end (smaller than any char).

<table>
<thead>
<tr>
<th></th>
<th>s</th>
<th>e</th>
<th>a</th>
<th>-1</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>s</td>
<td>e</td>
<td>a</td>
<td>she l l s -1</td>
</tr>
<tr>
<td>2</td>
<td>s e l l s -1</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>3</td>
<td>s h e -1</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
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<td></td>
</tr>
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<td>s h e l l s -1</td>
<td></td>
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<td></td>
</tr>
<tr>
<td>6</td>
<td>s h o r e -1</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>7</td>
<td>s u r e l y -1</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

private static int charAt(String s, int d) {
    if (d < s.length()) return s.charAt(d);
    else return -1;
}

C strings. Have extra char '\0' at end ⇒ no extra work needed.
public static void sort(String[] a) {
    aux = new String[a.length];
    sort(a, aux, 0, a.length - 1, 0);
}

private static void sort(String[] a, String[] aux, int lo, int hi, int d) {
    if (hi <= lo) return;
    int[] count = new int[R+2];
    for (int i = lo; i <= hi; i++)
        count[charAt(a[i], d) + 2]++;
    for (int r = 0; r < R+1; r++)
        count[r+1] += count[r];
    for (int i = lo; i <= hi; i++)
        aux[count[charAt(a[i], d) + 1]++] = a[i];
    for (int i = lo; i <= hi; i++)
        a[i] = aux[i - lo];
    for (int r = 0; r < R; r++)
        sort(a, aux, lo + count[r], lo + count[r+1] - 1, d+1);
}
MSD string sort: potential for disastrous performance

Observation 1. Much too slow for small subarrays.
  • Each function call needs its own count[] array.
  • ASCII (256 counts): 100x slower than copy pass for $N = 2$.
  • Unicode (65,536 counts): 32,000x slower for $N = 2$.

Observation 2. Huge number of small subarrays because of recursion.
Cutoff to insertion sort

Solution. Cutoff to insertion sort for small subarrays.

- Insertion sort, but start at $d^{th}$ character.

```java
private static void sort(String[] a, int lo, int hi, int d)
{
    for (int i = lo; i <= hi; i++)
        for (int j = i; j > lo && less(a[j], a[j-1], d); j--)
            exch(a, j, j-1);
}
```

- Implement `less()` so that it compares starting at $d^{th}$ character.

```java
private static boolean less(String v, String w, int d)
{
    for (int i = d; i < Math.min(v.length(), w.length()); i++)
    {
        if (v.charAt(i) < w.charAt(i)) return true;
        else if (v.charAt(i) > w.charAt(i)) return false;
    }
    return v.length() < w.length();
}
```
MSD string sort: performance

Number of characters examined.

- MSD examines just enough characters to sort the keys.
- Number of characters examined depends on keys.
- Can be sublinear in input size!

compareTo() based sorts can also be sublinear!

<table>
<thead>
<tr>
<th>Random (sublinear)</th>
<th>Non-random with duplicates (nearly linear)</th>
<th>Worst case (linear)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1EIO402</td>
<td>are</td>
<td>1DNB377</td>
</tr>
<tr>
<td>1HYL490</td>
<td>by</td>
<td>1DNB377</td>
</tr>
<tr>
<td>1ROZ572</td>
<td>sea</td>
<td>1DNB377</td>
</tr>
<tr>
<td>2HXE734</td>
<td>seashells</td>
<td>1DNB377</td>
</tr>
<tr>
<td>2IYE230</td>
<td>seashells</td>
<td>1DNB377</td>
</tr>
<tr>
<td>2XOR846</td>
<td>sells</td>
<td>1DNB377</td>
</tr>
<tr>
<td>3CDB573</td>
<td>sells</td>
<td>1DNB377</td>
</tr>
<tr>
<td>3CVP720</td>
<td>she</td>
<td>1DNB377</td>
</tr>
<tr>
<td>3IGJ319</td>
<td>she</td>
<td>1DNB377</td>
</tr>
<tr>
<td>3KNA382</td>
<td>shells</td>
<td>1DNB377</td>
</tr>
<tr>
<td>3TAV879</td>
<td>shore</td>
<td>1DNB377</td>
</tr>
<tr>
<td>4CQP781</td>
<td>surely</td>
<td>1DNB377</td>
</tr>
<tr>
<td>4QGI284</td>
<td>the</td>
<td>1DNB377</td>
</tr>
<tr>
<td>4YHV229</td>
<td>the</td>
<td>1DNB377</td>
</tr>
</tbody>
</table>

Characters examined by MSD string sort
Summary of the performance of sorting algorithms

Frequency of operations.

<table>
<thead>
<tr>
<th>algorithm</th>
<th>guarantee</th>
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<tr>
<td>insertion sort</td>
<td>( \frac{1}{2} N^2 )</td>
<td>( \frac{1}{4} N^2 )</td>
<td>1</td>
<td>✔</td>
<td>compareTo()</td>
</tr>
<tr>
<td>mergesort</td>
<td>( N \lg N )</td>
<td>( N \lg N )</td>
<td>( N )</td>
<td>✔</td>
<td>compareTo()</td>
</tr>
<tr>
<td>quicksort</td>
<td>( 1.39 N \lg N )</td>
<td>( 1.39 N \lg N )</td>
<td>( c \lg N )</td>
<td>✔</td>
<td>compareTo()</td>
</tr>
<tr>
<td>heapsort</td>
<td>( 2 N \lg N )</td>
<td>( 2 N \lg N )</td>
<td>1</td>
<td>✔</td>
<td>compareTo()</td>
</tr>
<tr>
<td>LSD sort †</td>
<td>( 2 W (N + R) )</td>
<td>( 2 W (N + R) )</td>
<td>( N + R )</td>
<td>✔</td>
<td>charAt()</td>
</tr>
<tr>
<td>MSD sort ‡</td>
<td>( 2 W (N + R) )</td>
<td>( N \log_R N )</td>
<td>( N + D R )</td>
<td>✔</td>
<td>charAt()</td>
</tr>
</tbody>
</table>

\( D \) = function-call stack depth (length of longest prefix match)

* probabilistic
† fixed-length W keys
‡ average-length W keys
MSD string sort vs. quicksort for strings

Disadvantages of MSD string sort.

- Extra space for aux[].
- Extra space for count[].
- Inner loop has a lot of instructions.
- Accesses memory "randomly" (cache inefficient).

Disadvantage of quicksort.

- Linearithmic number of string compares (not linear).
- Has to rescan many characters in keys with long prefix matches.

Goal. Combine advantages of MSD and quicksort.
Engineering a radix sort (American flag sort)

Optimization 0. Cutoff to insertion sort.

Optimization 1. Replace recursion with explicit stack.
- Push subarrays to be sorted onto stack.
- Now, one count[] array suffices.

Optimization 2. Do \(R\)-way partitioning in place.
- Eliminates aux[] array.
- Sacrifices stability.

American national flag problem

Dutch national flag problem

Engineering Radix Sort
Peter M. McIlroy and Keith Bostic
University of California at Berkeley;
and M. Douglas McIlroy
AT&T Bell Laboratories

ABSTRACT: Radix sorting methods have excellent asymptotic performance on string data, for which comparison is not a unit-time operation. Attractive for use in large byte-addressable memories, these methods have nevertheless long been eclipsed by more easily programmed algorithms. Three ways to sort strings by bytes left to right—a stable list sort, a stable two-array sort, and an in-place “American flag” sort—are illustrated with practical C programs. For heavy-duty sorting, all three perform comparably, usually running at least twice as fast as a good quicksort. We recommend American flag sort for general use.
5.1 STRING SORTS

- strings in Java
- key-indexed counting
- LSD radix sort
- MSD radix sort
- 3-way radix quicksort
- suffix arrays
3-way string quicksort (Bentley and Sedgewick, 1997)

Overview. Do 3-way partitioning on the $d^{th}$ character.

- Less overhead than $R$-way partitioning in MSD radix sort.
- Does not re-examine characters equal to the partitioning char.
  (but does re-examine characters not equal to the partitioning char)

```plaintext
Overview. Do 3-way partitioning on the $d^{th}$ character.

- Less overhead than $R$-way partitioning in MSD radix sort.
- Does not re-examine characters equal to the partitioning char.
  (but does re-examine characters not equal to the partitioning char)
```
3-way string quicksort: trace of recursive calls

Trace of first few recursive calls for 3-way string quicksort (subarrays of size 1 not shown)
3-way string quicksort: Java implementation

```java
private static void sort(String[] a)
{
    sort(a, 0, a.length - 1, 0);
}

private static void sort(String[] a, int lo, int hi, int d)
{
    if (hi <= lo) return;
    int lt = lo, gt = hi;
    int v = charAt(a[lo], d);
    int i = lo + 1;
    while (i <= gt)
    {
        int t = charAt(a[i], d);
        if (t < v) exch(a, lt++, i);
        else if (t > v) exch(a, i, gt--);
        else i++;
    }
    sort(a, lo, lt-1, d);
    if (v >= 0) sort(a, lt, gt, d+1);
    sort(a, gt+1, hi, d);
}
```

3-way partitioning (using d\textsuperscript{th} character)

to handle variable-length strings

sort 3 subarrays recursively
3-way string quicksort vs. standard quicksort

Standard quicksort.

- Uses $\sim 2N \ln N$ string compares on average.
- Costly for keys with long common prefixes (and this is a common case!)

3-way string (radix) quicksort.

- Uses $\sim 2N \ln N$ character compares on average for random strings.
- Avoids re-comparing long common prefixes.

Abstract

We present theoretical algorithms for sorting and searching multikey data, and derive from them practical C implementations for applications in which keys are character strings. The sorting algorithm blends Quicksort and radix sort, it is competitive with the best known C sort codes. The searching algorithm blends tries and binary
3-way string quicksort vs. MSD string sort

MSD string sort.
- Is cache-inefficient.
- Too much memory storing `count[]`.
- Too much overhead reinitializing `count[]` and `aux[]`.

3-way string quicksort.
- Is in-place.
- Is cache-friendly.
- Has a short inner loop.
- But not stable.

Bottom line. 3-way string quicksort is method of choice for sorting strings.
### Summary of the performance of sorting algorithms

Frequency of operations.

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<tr>
<td>MSD sort ‡</td>
<td>$2 W (N + R)$</td>
<td>$N \log R$</td>
<td>$N + D R$</td>
<td>✔</td>
<td>charAt()</td>
</tr>
<tr>
<td>3-way string quicksort</td>
<td>$1.39 WN \lg R^*$</td>
<td>$1.39 N \lg N$</td>
<td>log $N + W^*$</td>
<td>✔</td>
<td>charAt()</td>
</tr>
</tbody>
</table>

* probabilistic
† fixed-length W keys
‡ average-length W keys
5.1 **String Sorts**

- strings in Java
- key-indexed counting
- LSD radix sort
- MSD radix sort
- 3-way radix quicksort
- suffix arrays
Keyword-in-context search

Given a text of $N$ characters, preprocess it to enable fast substring search (find all occurrences of query string context).

```plaintext
% more tale.txt
it was the best of times
it was the worst of times
it was the age of wisdom
it was the age of foolishness
it was the epoch of belief
it was the epoch of incredulity
it was the season of light
it was the season of darkness
it was the spring of hope
it was the winter of despair
```

Keyword-in-context search

Given a text of \( N \) characters, preprocess it to enable fast substring search (find all occurrences of query string context).

Applications. Linguistics, databases, web search, word processing, ....
### Suffix sort

**input string**

```
input string

it was best it was w
```

**form suffixes**

<table>
<thead>
<tr>
<th>Index</th>
<th>Suffix</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>it was best it was w</td>
</tr>
<tr>
<td>1</td>
<td>twas best it was w</td>
</tr>
<tr>
<td>2</td>
<td>was best it was w</td>
</tr>
<tr>
<td>3</td>
<td>as best it was w</td>
</tr>
<tr>
<td>4</td>
<td>sbest it was w</td>
</tr>
<tr>
<td>5</td>
<td>best it was w</td>
</tr>
<tr>
<td>6</td>
<td>est it was w</td>
</tr>
<tr>
<td>7</td>
<td>st it was w</td>
</tr>
<tr>
<td>8</td>
<td>t it was w</td>
</tr>
<tr>
<td>9</td>
<td>i t w as w</td>
</tr>
<tr>
<td>10</td>
<td>tw as w</td>
</tr>
<tr>
<td>11</td>
<td>w as w</td>
</tr>
<tr>
<td>12</td>
<td>a s w</td>
</tr>
<tr>
<td>13</td>
<td>s w</td>
</tr>
<tr>
<td>14</td>
<td>w</td>
</tr>
</tbody>
</table>

**sort suffixes to bring query strings together**

```
array of suffix indices in sorted order

3  as best it was w
12 as w
5 best it was w
6 est it was w
0 it was best it was w
9 it was w
4 sbest it was w
7 st it was w
13 s w
8 t it was w
1 twas best it was w
10 tw as w
14 w
2 w as best it was w
11 w as w
```
Keyword-in-context search: suffix-sorting solution

- Preprocess: **suffix sort** the text.
- Query: **binary search** for query; scan until mismatch.

### KWIC search for "search" in Tale of Two Cities

<table>
<thead>
<tr>
<th>Keyword Position</th>
<th>Context</th>
</tr>
</thead>
<tbody>
<tr>
<td>632698</td>
<td>sealed my letter and ...</td>
</tr>
<tr>
<td>713727</td>
<td>seamstress is lifted ...</td>
</tr>
<tr>
<td>660598</td>
<td>seamstress of twenty ...</td>
</tr>
<tr>
<td>67610</td>
<td>seamstress who was with ...</td>
</tr>
<tr>
<td><strong>4430</strong></td>
<td>search for contraband ...</td>
</tr>
<tr>
<td>42705</td>
<td>search for your father ...</td>
</tr>
<tr>
<td>499797</td>
<td>search of her husband ...</td>
</tr>
<tr>
<td>182045</td>
<td>search of impoverish ...</td>
</tr>
<tr>
<td>143399</td>
<td>search of other carries ...</td>
</tr>
<tr>
<td>411801</td>
<td>search the straw hold ...</td>
</tr>
<tr>
<td>158410</td>
<td>seaweed marking about ...</td>
</tr>
<tr>
<td>691536</td>
<td>sea and madame deaf ...</td>
</tr>
<tr>
<td>536569</td>
<td>sease an horrible pass ...</td>
</tr>
<tr>
<td>484763</td>
<td>sease that had brought ...</td>
</tr>
<tr>
<td></td>
<td>:</td>
</tr>
</tbody>
</table>
War story

**Q.** How to efficiently form (and sort) suffixes?

```java
String[] suffixes = new String[N];
for (int i = 0; i < N; i++)
    suffixes[i] = s.substring(i, N);
Arrays.sort(suffixes);
```

---

### input file | characters | Java 7u5 | Java 7u6
---|---|---|---
amendments.txt | 18 thousand | 0.25 sec | 2.0 sec
aesop.txt | 192 thousand | 1.0 sec | out of memory
mobydick.txt | 1.2 million | 7.6 sec | out of memory
chromosome11.txt | 7.1 million | 61 sec | out of memory

---

*3rd printing (2012)*
The String data type: Java 7u5 implementation

```java
public final class String implements Comparable<String> {
    private char[] value; // characters
    private int offset; // index of first char in array
    private int length; // length of string
    private int hash; // cache of hashCode()
    ...
}
```

String `s = "Hello, World"

- `value[]`: H E L L O ,  W O R L D
- `length = 12`
- `offset = 0`

String `t = s.substring(7, 12);

- `value[]`: H E L L O ,  W O R L D
- `length = 5`
- `offset = 7`
The String data type: Java 7u6 implementation

```java
public final class String implements Comparable<String> {
    private char[] value;  // characters
    private int hash;     // cache of hashCode()
    ...}
```

**String s = "Hello, World"

```
value[]   H   E   L   L   L   O   ,   W   O   R   L   D
0    1    2    3    4    5    6    7    8    9   10   11
```

**String t = s.substring(7, 12);

```
value[]   W   O   R   L   D
0    1    2    3    4
```
The String data type: performance

String data type (in Java). Sequence of characters (immutable).
Java 7u5. Immutable char[] array, offset, length, hash cache.
Java 7u6. Immutable char[] array, hash cache.

<table>
<thead>
<tr>
<th>operation</th>
<th>Java 7u5</th>
<th>Java 7u6</th>
</tr>
</thead>
<tbody>
<tr>
<td>length</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>indexing</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>substring extraction</td>
<td>1</td>
<td>N</td>
</tr>
<tr>
<td>concatenation</td>
<td>M + N</td>
<td>M + N</td>
</tr>
<tr>
<td>immutable?</td>
<td>✔️</td>
<td>✔️</td>
</tr>
<tr>
<td>memory</td>
<td>64 + 2N</td>
<td>56 + 2N</td>
</tr>
</tbody>
</table>
A Reddit exchange

I'm the author of the substring() change. As has been suggested in the analysis here there were two motivations for the change

• Reduce the size of String instances. Strings are typically 20-40% of common apps footprint.
• Avoid memory leakage caused by retained substrings holding the entire character array.

Changing this function, in a bugfix release no less, was totally irresponsible. It broke backwards compatibility for numerous applications with errors that didn't even produce a message, just freezing and timeouts... All pain, no gain. Your work was not just vain, it was thoroughly destructive, even beyond its immediate effect.

http://www.reddit.com/r/programming/comments/1qw73v/til_oracle_changed_the_internal_string
**Suffix sort**

Q. How to efficiently form (and sort) suffixes in Java 7u6?
A. Define Suffix class ala Java 7u5 String.

```java
public class Suffix implements Comparable<Suffix>
{
    private final String text;
    private final int offset;
    public Suffix(String text, int offset)
    {
        this.text = text;
        this.offset = offset;
    }
    public int length() { return text.length() - offset; }
    public char charAt(int i) { return text.charAt(offset + i); }
    public int compareTo(Suffix that) { /* see textbook */ }
}
```

<table>
<thead>
<tr>
<th>text[]</th>
<th>HELLO, WORLD</th>
<th>offset</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>1</td>
<td>4</td>
</tr>
<tr>
<td>2</td>
<td>3</td>
<td>5</td>
</tr>
<tr>
<td>4</td>
<td>5</td>
<td>6</td>
</tr>
<tr>
<td>6</td>
<td>7</td>
<td>8</td>
</tr>
<tr>
<td>8</td>
<td>9</td>
<td>10</td>
</tr>
<tr>
<td>10</td>
<td>11</td>
<td></td>
</tr>
</tbody>
</table>
Suffix sort

Q. How to efficiently form (and sort) suffixes in Java 7u6?
A. Define Suffix class ala Java 7u5 String.

```java
String[] suffixes = new String[N];
for (int i = 0; i < N; i++)
    suffixes[i] = new Suffix(s, i);
Arrays.sort(suffixes);
```
Lessons learned

Lesson 1. Put performance guarantees in API.
Lesson 2. If API has no performance guarantees, don't rely upon any!

Corollary. May want to avoid String data type for huge strings.
- Are you sure charAt() and length() take constant time?
- If lots of calls to charAt(), overhead for function calls is large.
- If lots of small strings, memory overhead of String is large.

Ex. Our optimized algorithm for suffix arrays is 5x faster and uses 32x less memory than our original solution in Java 7u5!
Radix sorting: quiz 3

What is worst-case running time of our suffix array algorithm?

A. Quadratic.
B. Linearithmic.
C. Linear.
D. None of the above.
E. I don't know.

```
 0  a a a a a a a a a a
 1  a a a a a a a a a a
 2  a a a a a a a a a a
 3  a a a a a a a a a a
 4  a a a a a a a a a a
 5  a a a a a a a a a a
 6  a a a a a a a a a a
 7  a a a a a a a a a a
 8  a a a a a a a a a a
 9  a a a a a a a a a a
```

suffixes
Radix sorting: quiz 4

What is the worst-case complexity of the suffix array problem?

A. Quadratic.
B. Linearithmic.
C. Linear.
D. None of the above.
E. I don't know.

“has no practical virtue... but a historic monument in the area of string processing.”

---

**Linear Pattern Matching Algorithms**

Peter Weiner

The Rand Corporation, Santa Monica, California

Abstract

In 1970, Knuth, Pratt, and Morris [1] showed how to do basic pattern matching in linear time. Related problems, such as those discussed in [4], have previously been solved by efficient but sub-optimal algorithms. In this paper, we introduce an interesting data structure called a bi-tree. A linear time algorithm for obtaining a compacted version of a bi-tree associated with a given string is presented. With this construction as the basic tool, we indicate how to solve several pattern matching problems, including some from [4], in linear time.

---

**Suffix arrays: A new method for on-line string searches**

Udi Manber

Gene Myers

Department of Computer Science

University of Arizona

Tucson, AZ 85721

May 1989

Revised August 1991

Abstract

A new and conceptually simple data structure, called a suffix array, for on-line string searches is introduced in this paper. Constructing and querying suffix arrays is reduced to a sort and search paradigm that employs novel algorithms. The main advantage of suffix arrays over suffix trees is that, in practice, they use three to five times less space. From a complexity standpoint, suffix arrays permit on-line string searches of the type, “Is W a substring of A?” to be answered in time O(P + log N), where P is the length of W and N is the length of A, which is competitive with (and in some cases slightly better than) suffix trees. The only drawback is that in those instances where the underlying alphabet is finite and small, suffix trees can be constructed in O(N) time in the worst case, versus O(N log N) time for suffix arrays.

However, we give an augmented algorithm that, regardless of the alphabet size, constructs suffix arrays in O(N) expected time, albeit with lesser space efficiency. We believe that suffix arrays will prove to be better in practice than suffix trees for many applications.
Suffix arrays: practice

Applications. Bioinformatics, information retrieval, data compression, ...

Many ingenious algorithms.
- Memory footprint very important.
- State-of-the-art still changing.

<table>
<thead>
<tr>
<th>year</th>
<th>algorithm</th>
<th>worst case</th>
<th>memory</th>
</tr>
</thead>
<tbody>
<tr>
<td>1990</td>
<td>Manber–Myers</td>
<td>$N \log N$</td>
<td>$8N$</td>
</tr>
<tr>
<td>1999</td>
<td>Larsson–Sadakane</td>
<td>$N \log N$</td>
<td>$8N$</td>
</tr>
<tr>
<td>2003</td>
<td>Kärkkäinen–Sanders</td>
<td>$N$</td>
<td>$13N$</td>
</tr>
<tr>
<td>2003</td>
<td>Ko–Aluru</td>
<td>$N$</td>
<td>$10N$</td>
</tr>
<tr>
<td>2008</td>
<td>divsufsort2</td>
<td>$N \log N$</td>
<td>$5N$</td>
</tr>
<tr>
<td>2010</td>
<td>sais</td>
<td>$N$</td>
<td>$6N$</td>
</tr>
</tbody>
</table>
String sorting summary

We can develop linear-time sorts.
- Key compares not necessary for string keys.
- Use characters as index in an array.

We can develop sublinear-time sorts.
- Input size is amount of data in keys (not number of keys).
- Not all of the data has to be examined.

3-way string quicksort is asymptotically optimal.
- $1.39 N \log N$ chars for random data.

Long strings are rarely random in practice.
- Goal is often to learn the structure!
- May need specialized algorithms.