Data Structures
Motivating Quotation

“Every program depends on algorithms and data structures, but few programs depend on the invention of brand new ones.”

-- Kernighan & Pike
“Programming in the Large” Steps

Design & Implement
• Program & programming style (done)
• Common data structures and algorithms
• Modularity
• Building techniques & tools (done)

Debug
• Debugging techniques & tools (done)

Test
• Testing techniques (done)

Maintain
• Performance improvement techniques & tools
Goals of this Lecture

Help you learn (or refresh your memory) about:
- Common data structures: linked lists and hash tables

Why? Deep motivation:
- Common data structures serve as “high level building blocks”
- A power programmer:
  - Rarely creates programs from scratch
  - Often creates programs using high level building blocks

Why? Shallow motivation:
- Provide background pertinent to Assignment 3
- … esp. for those who have not taken COS 226
Common Task

Maintain a collection of key/value pairs
  • Each key is a **string**; each value is an **int**
  • Unknown number of key-value pairs

Examples
  • (student name, grade)
    • (“john smith”, 84), (“jane doe”, 93), (“bill clinton”, 81)
  • (baseball player, number)
    • (“Ruth”, 3), (“Gehrig”, 4), (“Mantle”, 7)
  • (variable name, value)
    • (“maxLength”, 2000), (“i”, 7), (“j”, -10)
Agenda

Linked lists
Hash tables
Hash table issues
Linked List Data Structure

```c
struct Node
{   const char *key;
    int value;
    struct Node *next;
};

struct List
{   struct Node *first;
};
```

Your Assignment 3 data structures will be more elaborate

Really this is the address at which “Ruth” resides
Linked List Algorithms

Create

- Allocate List structure; set first to NULL
- Performance: O(1) => fast

Add (no check for duplicate key required)

- Insert new node containing key/value pair at front of list
- Performance: O(1) => fast

Add (check for duplicate key required)

- Traverse list to check for node with duplicate key
- Insert new node containing key/value pair into list
- Performance: O(n) => slow
Linked List Algorithms

Search
• Traverse the list, looking for given key
• Stop when key found, or reach end
• Performance: \(O(n)\) => slow

Free
• Free Node structures while traversing
• Free List structure
• Performance: \(O(n)\) => slow

Would it be better to keep the nodes sorted by key?
Agenda

Linked lists

Hash tables

Hash table issues
Hash Table Data Structure

Array of linked lists

```c
enum {BUCKET_COUNT = 1024};

struct Binding  
{   const char *key;  
    int value;  
    struct Binding *next; 
};

struct Table  
{   struct Binding *buckets[BUCKET_COUNT]; 
};
```

Your Assignment 3 data structures will be more elaborate

Really this is the address at which “Ruth” resides
Hash Table Data Structure

**Hash function** maps given key to an integer

Mod integer by $\text{BUCKET\_COUNT}$ to determine proper bucket
Example: \texttt{BUCKET\_COUNT} = 7

Add (if not already present) bindings with these keys:
- the, cat, in, the, hat
First key: “the”
  • hash(“the”) = 965156977; 965156977 % 7 = 1

Search `buckets[1]` for binding with key “the”; not found
Hash Table Example (cont.)

Add binding with key “the” and its value to buckets[1]
Second key: “cat”
  - hash(“cat”) = 3895848756; 3895848756 % 7 = 2

Search buckets[2] for binding with key “cat”; not found
Add binding with key “cat” and its value to buckets[2]
Third key: “in”
  - $\text{hash(“in”) = 6888005; 6888005}\mod 7 = 5$

Search $\text{buckets}[5]$ for binding with key “in”; not found

```
0 1 2 3 4 5 6
0
1
2
3
4
5
6
```

```
    the
```

```
cat
```
Add binding with key “in” and its value to \texttt{buckets[5]}

![Diagram of a hash table with keys and values]
Fourth word: “the”
  • hash(“the”) = 965156977; 965156977 % 7 = 1

Search buckets[1] for binding with key “the”; found it!
  • Don’t change hash table
Fifth key: “hat”
  • hash(“hat”) = 865559739; 865559739 % 7 = 2

Search buckets[2] for binding with key “hat”; not found
Add binding with key “hat” and its value to \textit{buckets}[2]

- At front or back? Doesn’t matter
- Inserting at the front is easier, so add at the front
Hash Table Algorithms

Create
- Allocate Table structure; set each bucket to NULL
- Performance: \( O(1) \) => fast

Add
- Hash the given key
- Mod by \( BUCKET\_COUNT \) to determine proper bucket
- Traverse proper bucket to make sure no duplicate key
- Insert new binding containing key/value pair into proper bucket
- Performance: \( O(1) \) => fast

Is the add performance always fast?
Hash Table Algorithms

Search
• Hash the given key
• Mod by BUCKET_COUNT to determine proper bucket
• Traverse proper bucket, looking for binding with given key
• Stop when key found, or reach end
• Performance: O(1) => fast

Free
• Traverse each bucket, freeing bindings
• Free Table structure
• Performance: O(n) => slow

Is the search performance always fast?
Agenda

Linked lists
Hash tables
Hash table issues
How Many Buckets?

Many!
- Too few => large buckets => slow add, slow search

But not too many!
- Too many => memory is wasted

This is OK:

\[
\text{BUCKET\_COUNT}-1
\]
What Hash Function?

Should distribute bindings across the buckets well

- Distribute bindings over the range \(0, 1, \ldots, \text{BUCKET\_COUNT}-1\)
- Distribute bindings evenly to avoid very long buckets

This is not so good:

What would be the worst possible hash function?
How to Hash Strings?

Simple hash schemes don’t distribute the keys evenly enough
• Number of characters, mod $\text{BUCKET\_COUNT}$
• Sum the numeric codes of all characters, mod $\text{BUCKET\_COUNT}$
• ...

A reasonably good hash function:
• Weighted sum of characters $s_i$ in the string $s$
  • $(\Sigma \ a^i s_i) \ mod \ \text{BUCKET\_COUNT}$
  • Best if $a$ and $\text{BUCKET\_COUNT}$ are relatively prime
    • E.g., $a = 65599$, $\text{BUCKET\_COUNT} = 1024$
How to Hash Strings?

Potentially expensive to compute $\sum a^i s_i$

So let's do some algebra

- (by example, for string $s$ of length 5, $a=65599$):

\[
h = \sum 65599^i s_i
\]

\[
h = 65599^0 s_0 + 65599^1 s_1 + 65599^2 s_2 + 65599^3 s_3 + 65599^4 s_4
\]

Direction of traversal of $s$ doesn't matter, so...

\[
h = 65599^0 s_4 + 65599^1 s_3 + 65599^2 s_2 + 65599^3 s_1 + 65599^4 s_0
\]

\[
h = 65599^4 s_0 + 65599^3 s_1 + 65599^2 s_2 + 65599^1 s_3 + 65599^0 s_4
\]

\[
h = (((((s_0) * 65599 + s_1) * 65599 + s_2) * 65599 + s_3) * 65599) + s_4
\]
How to Hash Strings?

Yielding this function

```c
unsigned int hash(const char *s, int bucketCount) {
    int i;
    unsigned int h = 0U;
    for (i=0; s[i]!="\0"; i++)
        h = h * 65599U + (unsigned int)s[i];
    return h % bucketCount;
}
```
How to Protect Keys?

Suppose \texttt{Table\_add()} function contains this code:

```c
void Table_add(struct Table *t, const char *key, int value) {
    …
    struct Binding *p =
        (struct Binding*)malloc(sizeof(struct Binding));
    p->key = key;
    …
}
```
How to Protect Keys?

Problem: Consider this calling code:

```c
struct Table *t;
char k[100] = "Ruth";
...
Table_add(t, k, 3);
```
Problem: Consider this calling code:

```c
struct Table *t;
char k[100] = "Ruth";
...
Table_add(t, k, 3);
strcpy(k, "Gehrig");
```

What happens if the client searches \texttt{t} for "Ruth"? For Gehrig?
How to Protect Keys?

Solution: `Table_add()` saves a **defensive copy** of the given key

```c
void Table_add(struct Table *t, const char *key, int value)
{
    struct Binding *p =
        (struct Binding*)malloc(sizeof(struct Binding));
    p->key = (const char*)malloc(strlen(key) + 1);
    strcpy((char*)p->key, key);
    ...
}
```

Why add 1?
How to Protect Keys?

Now consider same calling code:

```c
struct Table *t;
char k[100] = "Ruth";
...
Table_add(t, k, 3);
```
How to Protect Keys?

Now consider same calling code:

```c
struct Table *t;
char k[100] = "Ruth";
...
Table_add(t, k, 3);
strcpy(k, "Gehrig");
```

Hash table is not corrupted
Who Owns the Keys?

Then the hash table **owns** its keys

- That is, the hash table owns the memory in which its keys reside
- `Hash_free()` function must free the memory in which the key resides
Common data structures and associated algorithms

- Linked list
  - (Maybe) fast add
  - Slow search
- Hash table
  - (Potentially) fast add
  - (Potentially) fast search
  - Very common

Hash table issues

- Hashing algorithms
- Defensive copies
- Key ownership