Assignment 1:
Introduction to SML and Inductive Definitions

COS441: Programming Languages
Fall 2004

Assigned: Wednesday, 15 September 2004
Due: in class on Wednesday, 22 September 2004

All problems should be turned in on paper at the beginning of class on the due date. Also submit problem 3(f) online.

Problem 1 [20 points] You should do all of the problems below, but only turn in those that are not starred. Solutions to the starred problems are available at http://www-db.stanford.edu/~ullman/emlpsols/sols.html.

(a) [2 points] Exercise 2.3.1 from Ullman (p33)
(b) [2 points] Exercise 2.4.1 from Ullman (p42)
(c) [2 points] Exercise 2.4.2 from Ullman (p43)
(d) [2 points] Exercise 2.4.3 from Ullman (p43)
(e) [3 points] Exercise 2.4.5 from Ullman (p43)
(f) [3 points] Exercise 3.1.3 from Ullman (p53)
(g) [2 points] Exercise 3.3.4 from Ullman (p75)
(h) [1 point] Exercise 3.3.5 from Ullman (p75)
(i) [3 points] Exercise 3.4.6 from Ullman (p83)

Problem 2 [30 points]

(a) [5 points] Exercise 5.3 (a) from Mitchell (p123)
(b) [5 points] Exercise 5.3 (b) from Mitchell (p123)
(c) [10 points] Exercise 5.3 (c) from Mitchell (p123)
(d) [10 points] Exercise 5.3 (d) from Mitchell (p123)

Problem 3 [50 points] Given the following definitons:

\[
\begin{align*}
  \text{zero} & : \text{nat} \\
  \text{succ} & : \text{nat} \rightarrow \text{nat} \\
  X & : \text{nat} \rightarrow \text{nat} \rightarrow \text{nat} \\
  A & : \text{nat} \rightarrow \text{nat} \\
  A-\text{S} & : \text{nat} \rightarrow \text{nat} \rightarrow \text{nat}
\end{align*}
\]
(a) [10 points] Inductively define a relation \( M(X, Y, Z) \) that defines multiplication as the repeated addition of natural numbers.

\[
\begin{align*}
M(\text{zero}, \text{succ}(\text{zero}), \text{zero}) \\
M(\text{succ}(\text{zero}), \text{succ}(\text{zero}), \text{succ}(\text{zero})) \\
M(\text{succ}(\text{succ}(\text{zero})), \text{succ}(\text{succ}(\text{zero})), \text{succ}(\text{succ}(\text{succ}(\text{succ}(\text{zero})))))
\end{align*}
\]

(b) [10 points] Show that the \( M(X, Y, Z) \) relation you define is also a function. You may assume: If \( X \) unique, \( Y \) unique and \( A(X, Y, Z) \) then \( Z \) unique.

(c) [5 points] Write a set of recursive equations based on the definition of \( M(X, Y, Z) \) that defines a function \( \text{mult}(X, Y) \).

(d) [5 points] Convert the following recursive equations into an inductively defined relation

\[
\begin{align*}
\text{fact}(\text{zero}) & \equiv \text{succ}(\text{zero}) \\
\text{fact}(\text{succ}(X)) & \equiv \text{mult}(\text{succ}(X), \text{fact}(X))
\end{align*}
\]

(e) [10 points] Show that relation you define for \( \text{fact} \) is also a function.

(f) [10 points] Translate the recursive equations defining \( \text{mult} \) and \( \text{fact} \) into SML code. Start with the definitions in \texttt{nat.sml} and modify them. Hand-in electronically copy.

(g) [10 points] (Extra Credit) Prove the following \( (A(X_1, Y, Z) \text{ and } A(X_2, X_3, Y')) \text{ iff } (A(Y', X_3, Z) \text{ and } A(X_1, X_2, Y')) \).