
Problems (from lectures up to Nov. 13)

1. Show that no infinite subset of $MIN_{TM}$ is recognizable.

2. The axiomatic system given in class has a finite number of axioms and derivation rules. Argue briefly that Gödel’s Theorem also holds for any alternative system whose axioms and derivation rules are enumerable.

3. A 2CNF formula is an AND of clauses, where each clause is an OR of at most two literals. Let $2SAT = \{ \varphi : \varphi$ is a satisfiable 2CNF formula$\}$. Show that $2SAT \in P$.

4. Point out the fallacy in the following proof that the language $CLIQUE = \{ < G, k > :$ every clique in graph $G$ has size less than $k$ $\}$ is in NP.

   In class we saw that $CLIQUE$ is in NP. Just take that NDTM and swap its accept and reject states.

5. Suppose $P = NP$. Show that then we can do the following in polynomial time (be very careful, since the problems listed below are not decision problems!). (i) Given a boolean formula, find a satisfying assignment for it (if one exists). (ii) Given a graph and a number $k$, find a clique of size $k$ in the graph (if one exists). (iii) Given $(< T > 1^n)$, where $T$ is a number-theoretic statement, find a proof of $T$ in Peano Arithmetic of size $n$ (if one exists)$^1$.

6. For each of the following languages, state whether it is one or more of the following: in P, in NP, NP-complete, NP-hard. If you can’t classify some language in any of these four categories, mention if you think that the exact classification is an open problem. Justify your answer in each case.

   (i) $A_{TM} = \{ \langle M, w \rangle : M$ is a deterministic TM that accepts $w$$\}$.

   (ii) $\{ \langle M, w, k \rangle : M$ is an NDTM that accepts $w$ in time $k$$\}$.

   (iii) $\{ \langle M, w, 1^k \rangle : M$ is an NDTM that accepts $w$ in time $k$$\}$.

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$^1$In other words, if $P = NP$ mathematical proofs can be mechanically discovered in time that is polynomial in the number of symbols in the proof. Ergo, mathematicians had better hope that $P \neq NP$. 

HW4-1
7. \textit{(Just when you thought you would never see calculus ever again...)} Show that the following problem is NP-complete. Given integers $a_1, a_2, \ldots, a_n$, to decide if

$$
\int_0^{2\pi} \left(\prod_{i=1}^n \cos a_i \theta\right) d\theta \neq 0.
$$

(Hint: Generalize the formula $\cos A \cos B = (\cos(A + B) + \cos(A - B))/2$ and do a reduction from PARTITION.)

8. \textit{(Extra credit)} Describe an algorithm and some $c < 1$ such that the algorithm can decide satisfiability for 3CNF formulae on $n$ variables in $2^{cn}$ time.