Homework Set 9

Reading Assignments  Read Chapter 6.

Written Assignments  Do Exercises 5, 19, and 30 in Chapter 13.6; do Exercise 2 in Chapter 6.6 (you should use Inclusion-Exclusion for this problem).

Remark  In your solution to Exercise 5 in Chapter 13.6, be sure to give a rigorous justification of your claim.

Special Problem 1  [Counted as 1 exercise]  For any \( n > 0 \), let \( G_n \) be the ladder graph with \( n \) rungs, which has \( 2n \) vertices and \( 3n - 2 \) edges. (The ladder graph \( G_6 \) is shown below.) Let \( t_n \) be the number of spanning trees of \( G_n \). Clearly, \( t_1 = 1, t_2 = 4 \). Let \( T(x) = \sum_{n \geq 1} t_n x^n \).

(a) Determine the value of \( t_3 \) and \( t_4 \).
(b) Derive a closed-form formula for \( T(x) \).

Hint:  You may want to set up recurrences involving two sequences, one of which being \( t_1, t_2, t_3, \ldots \).

Special Problem 2  [Counted as 2 exercises]  Let \( G = (V, E) \) where \( V = \{v_1, v_2, v_3, v_4, v_5, w_1, w_2, w_3\} \) and \( E \) consists of the edges \( \{v_i, v_j\}, 1 \leq i, j \leq 5 \) and \( \{v_i, w_j\}, 1 \leq i \leq 3, 1 \leq j \leq 3 \). Answer each of the following questions; justify your answers.

(a) What is \( \omega(G) \), the size of the largest clique?
(b) What is \( \alpha(G) \), the size of the largest independent set?
(c) What is \( \chi(G) \), the chromatic number?
(d) Does \( G \) have a (closed) Eulerian trail?
(e) Does \( G \) contain a Hamiltonian cycle?

Special Problem 3  (counted as 1 exercise)  An ancient DNA fragment of a dinosaur has just
been found. It is known that this critical fragment $\sigma$ contains some critical information. If the string $ACGAACT$ appears in $\bar{\sigma}$, then it can fly; if the string $CTCACG$ appears in $\bar{\sigma}$, then it is vegetarian; if the string $TGACCT$ appears in $\bar{\sigma}$, then it is a timid dinosaur.

The fragment has length 17, and you have subjected it to the hybridization procedure with $\ell = 4$. The spectrum $S$ you get consists of the strings $ACGA, AACT, ACTC, ACGT, GACT, CTC A, CGAA, CTGA, TGAC, GAAC, GACG, ACTG, CACG, TCAC$. What is $G_S$? Find all the $\sigma$ which has $S$ as its spectrum. What kind of attributes can you definitely infer about this dinosaur (ie, does it fly, is it a vegetarian, is it timid)?