Inductive Data Flow Graphs

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Algorithmic verification

Goal

Given a (concurrent) program P and a specification $\varphi_{\mathbf{pre}}/\varphi_{\mathbf{post}}$, prove

$$\{\varphi_{\mathbf{pre}}\}P\{\varphi_{\mathbf{post}}\}$$

(or provide a counter-example)

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This talk presents **Inductive Data Flow Graphs** (iDFGs): a form of correctness proof for (concurrent) programs

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- Succinct
  - Present only the *essence* of a proof
  - Polynomial in the data complexity of a program
- Can be generated and checked automatically
  - Extend static analysis to concurrent control
  - Extend model checking to (unbounded) data

"Essence" of a proof

$\varphi_{\mathbf{pre}} : \mathbf{x} \geq 0 \wedge \mathbf{y} \geq 0$

Thread 1	Thread 2

`x ++`
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`z = x + y`
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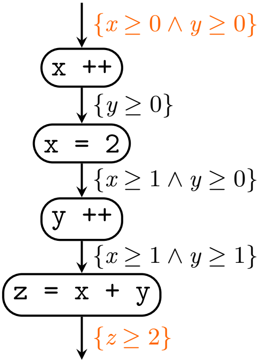
$\varphi_{\mathbf{post}} : \mathbf{z} \geq 2$



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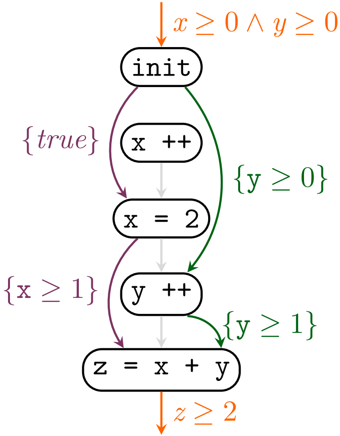

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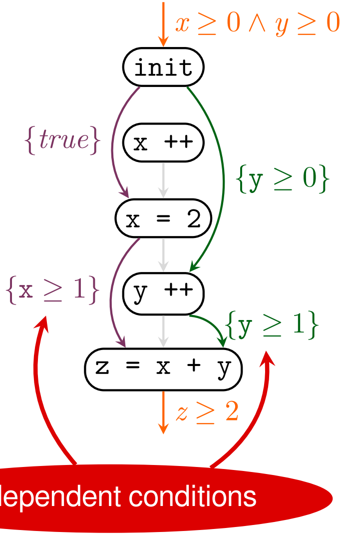

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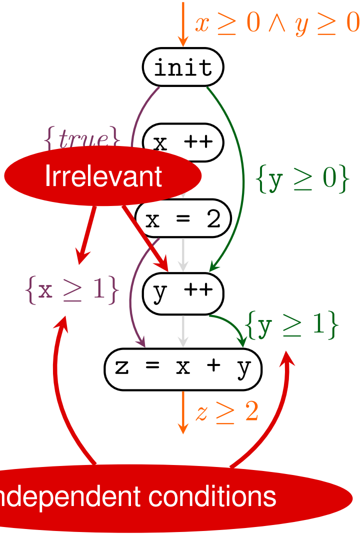

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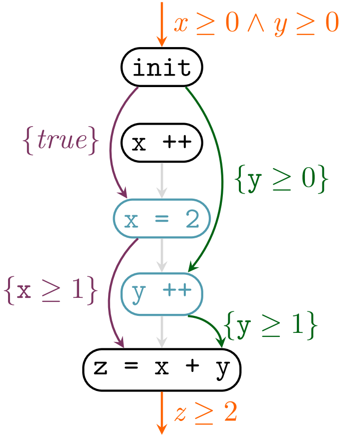

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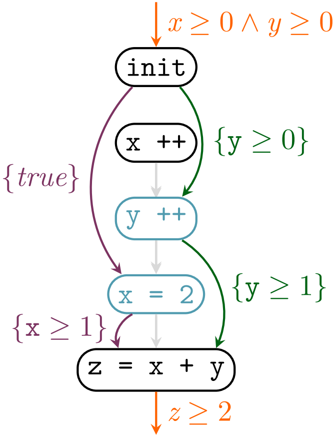
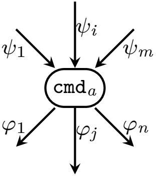
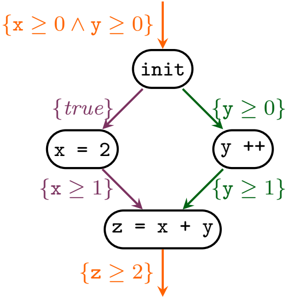


Inductiveness condition:



for all j, $\{\psi_1 \wedge \cdots \wedge \psi_m\}\mathtt{cmd}_a\{\varphi_j\}$



Suppress irrelevant details of a partial correctness proof

- Irrelevant ordering constraints

  (`x = 2;y ++` vs `y ++;x = 2`)

- Irrelevant actions (`x ++`)

Inductive Data Flow Graphs (iDFGs)

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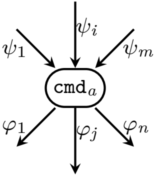

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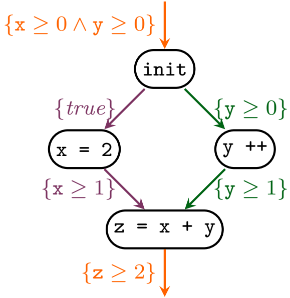

Parallelize a partial correctness proof

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Denotation of an iDFG

Data flow graph with inductive assertions (iDFG) proves correctness of traces that obey particular constraints

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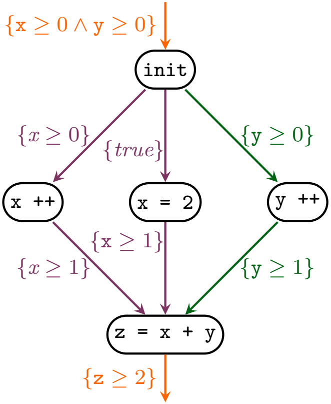

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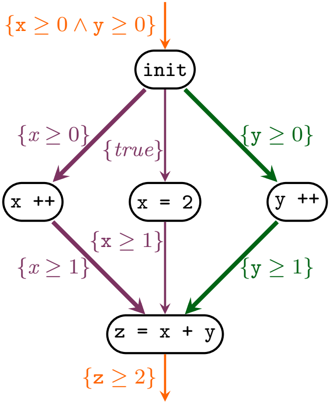

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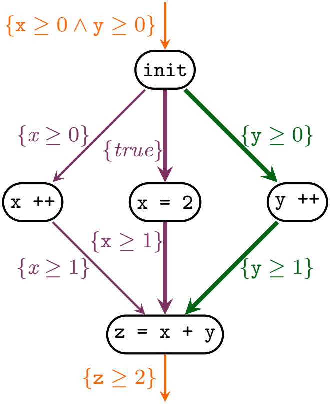

iDFGs as proof objects

Theorem

Let $G = \langle V, E, \varphi_{\mathbf{pre}}, \varphi_{\mathbf{post}}, v_o, V_{final} \rangle$ be an iDFG. For all $\tau \in [\![G]\!]$,

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Proof rule

Program P is correct w.r.t. $\varphi_{\mathbf{pre}}/\varphi_{\mathbf{post}}$ iff $\exists G. \mathcal{L}(P) \subseteq [\![G]\!]$

Data complexity

If there exists a small proof that P is correct (w.r.t. $\varphi_{\mathbf{pre}}/\varphi_{\mathbf{post}}$), then exists a small iDFG proof

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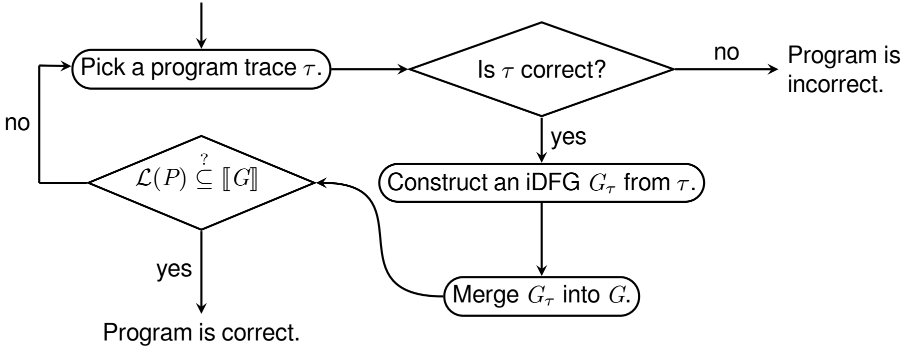
Data complexity measures how difficult a property is to prove.

- Minimum # of assertions in a *localized proof* that $\{\varphi_{\mathbf{pre}}\}P\{\varphi_{\mathbf{post}}\}$
  - Localized proofs: expose *"how compositional"* a Floyd proof is.

Automation


Pick a program trace τ.
Is τ correct?
no
Program is incorrect.
yes
Construct an iDFG G_τ from τ.
Merge G_τ into G.
$\mathcal{L}(P) \stackrel{?}{\subseteq} [\![G]\!]$
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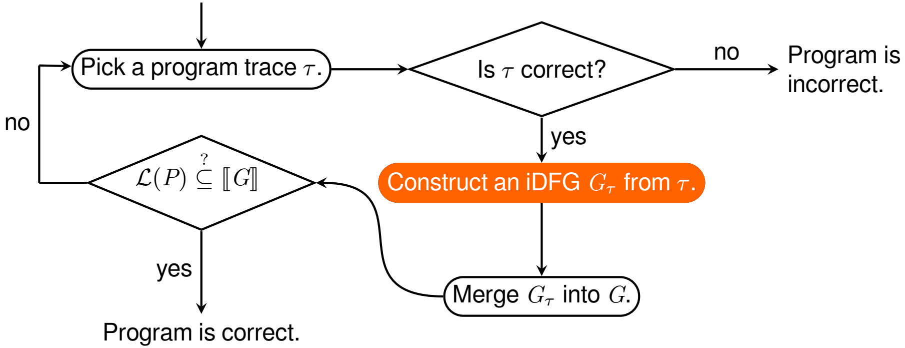

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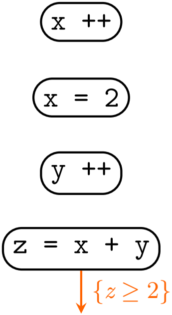

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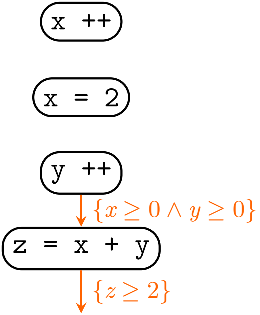

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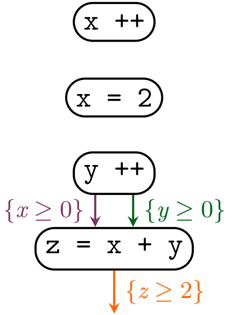

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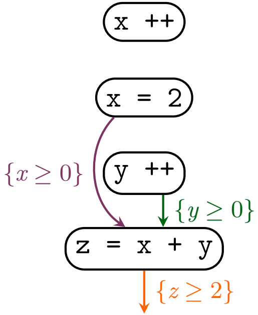

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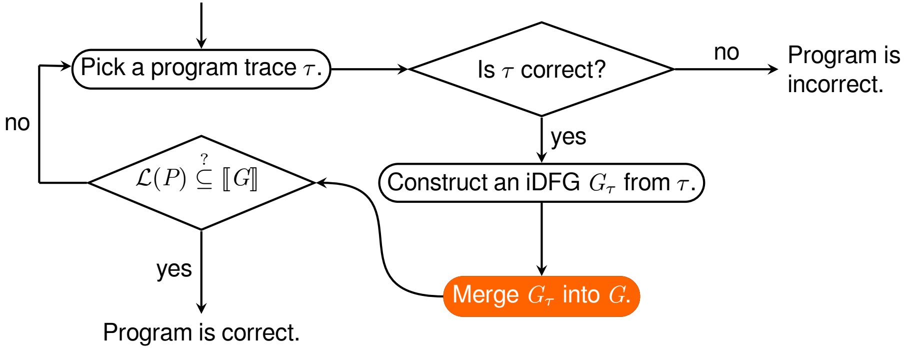

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Given iDFGs G_1, G_2, construct an iDFG $G_1 \curlywedge G_2$ such that

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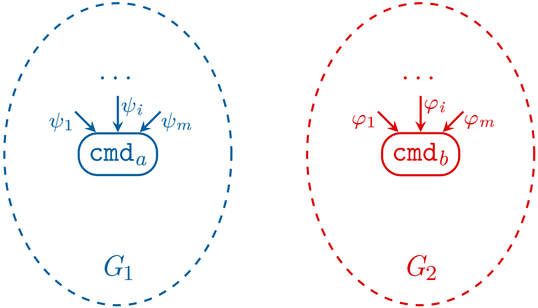

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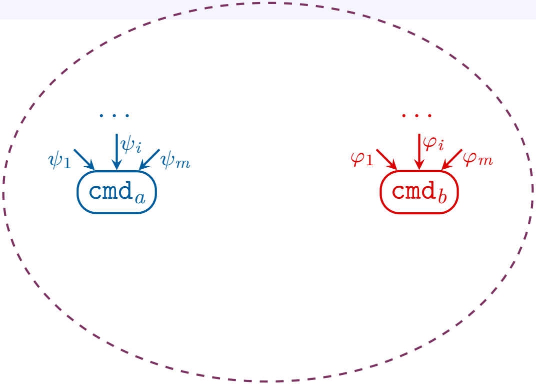

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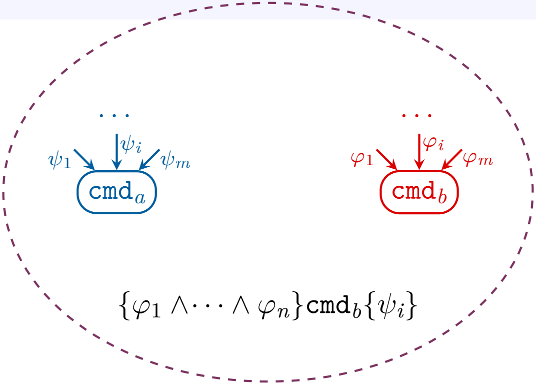

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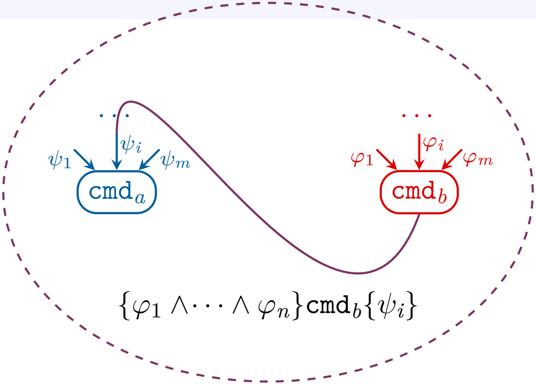

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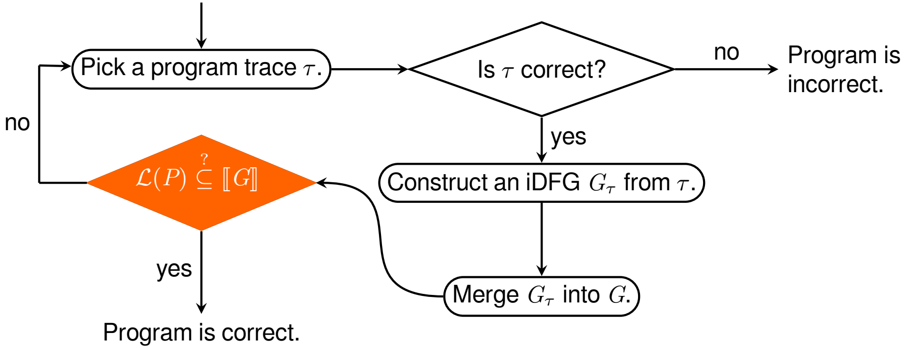

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Proof checking

For any iDFG G, we can efficiently (linear time, in the size of G) construct an *alternating finite automaton* A_G such that

$$\mathcal{L}(A_G) = [\![G]\!]^{rev}$$

Proof checking: $\mathcal{L}(P)^{rev} \overset{?}{\subseteq} \mathcal{L}(A_G)$

- Can be solved in PSPACE
- Combinatorial problem (non-reachability)
- Reuse techniques from (finite-state) model checking

Summary

- **Inductive Data Flow Graphs** are a proof method for partial correctness of (concurrent) programs
- (Provably) succinct
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Future work

- Can iDFGs be constructed more effectively?
- Efficient proof checking?
- Parameterized programs?
- Weak memory models?

Questions?

Thank you for your attention.