5. **Divide and Conquer I**

- merge and count demo
Merge and count demo

Given two sorted lists $A$ and $B$,

- Count number of inversions $(a, b)$ with $a \in A$ and $b \in B$.
- Merge $A$ and $B$ into sorted list $C$.

<table>
<thead>
<tr>
<th>sorted list A</th>
<th>sorted list B</th>
</tr>
</thead>
<tbody>
<tr>
<td>3  7  10  14  18</td>
<td>2  11  16  17  23</td>
</tr>
</tbody>
</table>
Merge and count demo

Given two sorted lists $A$ and $B$,
- Count number of inversions $(a, b)$ with $a \in A$ and $b \in B$.
- Merge $A$ and $B$ into sorted list $C$.

| sorted list A | 3 | 7 | 10 | 14 | 18 |
| sorted list B | 2 | 11 | 16 | 17 | 23 |

compare minimum entry in each list: copy 2 and add $x$ to inversion count

sorted list C

$x = 5$
inversions = 0
Given two sorted lists $A$ and $B$,

- Count number of inversions $(a, b)$ with $a \in A$ and $b \in B$.
- Merge $A$ and $B$ into sorted list $C$.

**sorted list A**

<p>| | | | | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>3</td>
<td>7</td>
<td>10</td>
<td>14</td>
<td>18</td>
</tr>
</tbody>
</table>

**sorted list B**

<p>| | | | | | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>2</td>
<td>11</td>
<td>16</td>
<td>17</td>
<td>23</td>
<td></td>
</tr>
</tbody>
</table>

compare minimum entry in each list: copy 3 and decrement $x$

**sorted list C**

<p>| |</p>
<table>
<thead>
<tr>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>2</td>
</tr>
</tbody>
</table>

$x = 5$

inversions = 5
Merge and count demo

Given two sorted lists $A$ and $B$,
- Count number of inversions $(a, b)$ with $a \in A$ and $b \in B$.
- Merge $A$ and $B$ into sorted list $C$.

**sorted list A**

| 3 | 7 | 10 | 14 | 18 |

**sorted list B**

| 2 | 11 | 16 | 17 | 23 |

compare minimum entry in each list: copy 7 and decrement $x$

**sorted list C**

| 2 | 3 |

$x = 4$

inversions = 5
Given two sorted lists $A$ and $B$,

- Count number of inversions $(a, b)$ with $a \in A$ and $b \in B$.
- Merge $A$ and $B$ into sorted list $C$.

\[
\begin{array}{cccccc}
|   & 3 & 7 & 10 & 14 & 18 | \\
\hline
\end{array}
\quad
\begin{array}{cccccc}
|   & 2 & 11 & 16 & 17 & 23 | \\
\hline
\end{array}
\]

compare minimum entry in each list: copy 10 and decrement $x$

\[
\begin{array}{cccccc}
|   & 2 & 3 & 7 | \\
\hline
\end{array}
\]

$x = 3$

inversions $= 5$
Given two sorted lists $A$ and $B$,
- Count number of inversions $(a, b)$ with $a \in A$ and $b \in B$.
- Merge $A$ and $B$ into sorted list $C$.

**Merge and count demo**

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</tr>
</tbody>
</table>

compare minimum entry in each list: copy 11 and add $x$ to increment count

<table>
<thead>
<tr>
<th>sorted list C</th>
</tr>
</thead>
<tbody>
<tr>
<td>2  3  7  10</td>
</tr>
</tbody>
</table>

$x = 2$

inversions $= 5$
Merge and count demo

Given two sorted lists $A$ and $B$,

- Count number of inversions $(a, b)$ with $a \in A$ and $b \in B$.
- Merge $A$ and $B$ into sorted list $C$.

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</tr>
</tbody>
</table>

compare minimum entry in each list: copy 14 and decrement $x$

<table>
<thead>
<tr>
<th>sorted list C</th>
</tr>
</thead>
<tbody>
<tr>
<td>2  3  7  10  11</td>
</tr>
</tbody>
</table>

$x = 2$

inversions = 7
Given two sorted lists $A$ and $B$,
- Count number of inversions $(a, b)$ with $a \in A$ and $b \in B$.
- Merge $A$ and $B$ into sorted list $C$.

**Merge and count demo**

<table>
<thead>
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<th>sorted list A</th>
<th>3</th>
<th>7</th>
<th>10</th>
<th>14</th>
<th>18</th>
</tr>
</thead>
<tbody>
<tr>
<td>sorted list B</td>
<td>2</td>
<td>11</td>
<td>16</td>
<td>17</td>
<td>23</td>
</tr>
</tbody>
</table>

Compare minimum entry in each list: copy 16 and add $x$ to increment count

<table>
<thead>
<tr>
<th>sorted list C</th>
<th>2</th>
<th>3</th>
<th>7</th>
<th>10</th>
<th>11</th>
<th>14</th>
</tr>
</thead>
</table>

$x = 1$

inversions = 7
Merge and count demo

Given two sorted lists \( A \) and \( B \),
- Count number of inversions \((a, b)\) with \( a \in A \) and \( b \in B \).
- Merge \( A \) and \( B \) into sorted list \( C \).

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<tbody>
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</tr>
</tbody>
</table>

compare minimum entry in each list: copy 17 and add \( x \) to increment count

sorted list C

| 2  3  7  10  11  14  16 |

\( x = 1 \)

inversions = 8
Merge and count demo

Given two sorted lists $A$ and $B$,
- Count number of inversions $(a, b)$ with $a \in A$ and $b \in B$.
- Merge $A$ and $B$ into sorted list $C$.

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<td>2 11 16 17 23</td>
</tr>
</tbody>
</table>

compare minimum entry in each list: copy 18 and decrement $x$

<table>
<thead>
<tr>
<th>sorted list C</th>
</tr>
</thead>
<tbody>
<tr>
<td>2 3 7 10 11 14 16 17</td>
</tr>
</tbody>
</table>

$x = 1$

inversions = 9
# Merge and count demo

Given two sorted lists $A$ and $B$,

- Count number of inversions $(a, b)$ with $a \in A$ and $b \in B$.
- Merge $A$ and $B$ into sorted list $C$.

**sorted list A**

| 3 | 7 | 10 | 14 | 18 |

**sorted list B**

| 2 | 11 | 16 | 17 | 23 |

**list A exhausted: copy 23**

**sorted list C**

| 2 | 3 | 7 | 10 | 11 | 14 | 16 | 17 | 18 |

$x = 0$

inversions = 9
Merge and count demo

Given two sorted lists $A$ and $B$,
- Count number of inversions $(a, b)$ with $a \in A$ and $b \in B$.
- Merge $A$ and $B$ into sorted list $C$.

sorted list $A$

\[
\begin{array}{cccccc}
3 & 7 & 10 & 14 & 18 \\
\end{array}
\]

sorted list $B$

\[
\begin{array}{cccccc}
2 & 11 & 16 & 17 & 23 \\
\end{array}
\]

\[
\begin{array}{cccccc}
5 & 2 & 1 & 1 & 0 \\
\end{array}
\]

done: return 9 inversions

sorted list $C$

\[
\begin{array}{cccccccccc}
2 & 3 & 7 & 10 & 11 & 14 & 16 & 17 & 18 & 23 \\
\end{array}
\]

$x = 0$

inversions = 9