

## 4. GREEDY ALGORITHMS II

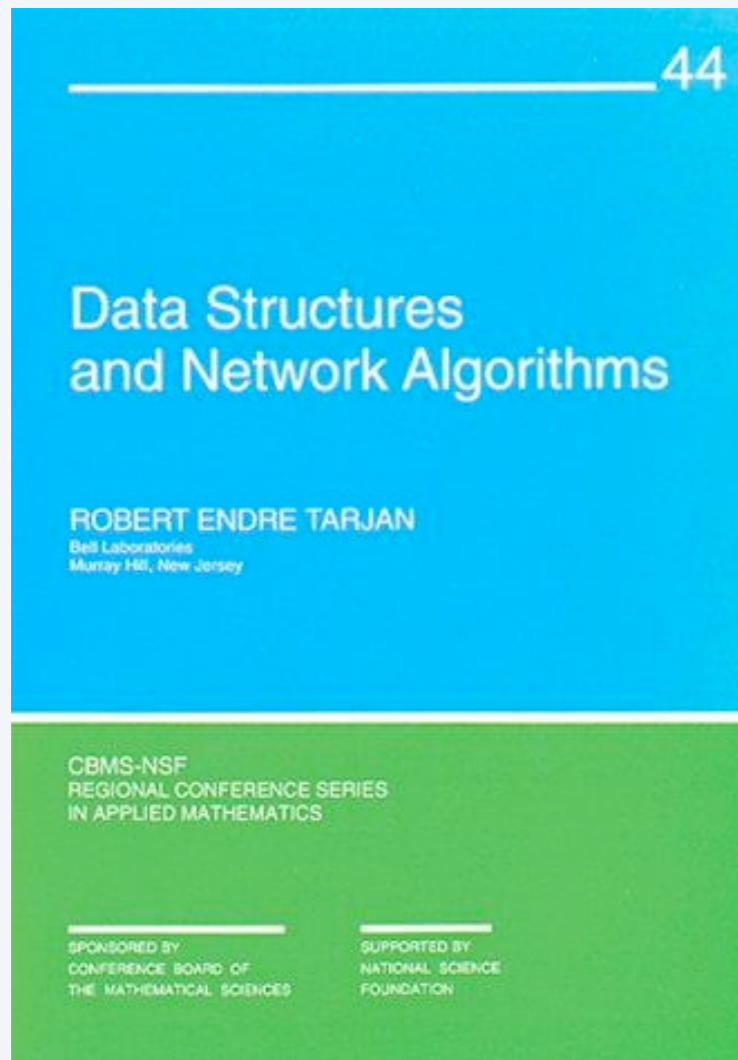
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- ▶ *red-rule blue-rule demo*
- ▶ *Prim's algorithm demo*
- ▶ *Kruskal's algorithm demo*
- ▶ *reverse-delete algorithm demo*
- ▶ *Boruvka's algorithm demo*

Lecture slides by Kevin Wayne

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<http://www.cs.princeton.edu/~wayne/kleinberg-tardos>



## SECTION 6.1

# 4. GREEDY ALGORITHMS II

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- ▶ *red-rule blue-rule demo*
- ▶ *Prim's algorithm demo*
- ▶ *Kruskal's algorithm demo*
- ▶ *reverse-delete algorithm demo*
- ▶ *Boruvka's algorithm demo*

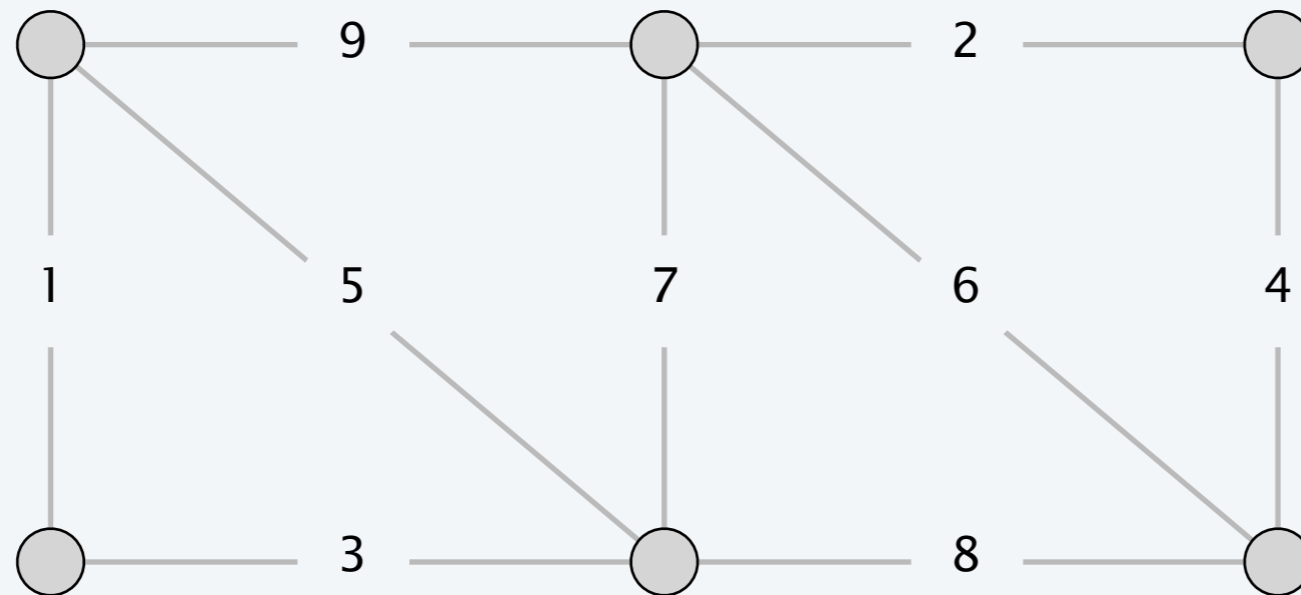
# Red-rule blue-rule demo

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**Red rule.** Let  $C$  be a cycle with no red edges. Select an uncolored edge of  $C$  of max weight and color it red.

**Blue rule.** Let  $D$  be a cutset with no blue edges. Select an uncolored edge in  $D$  of min weight and color it blue.

the input graph

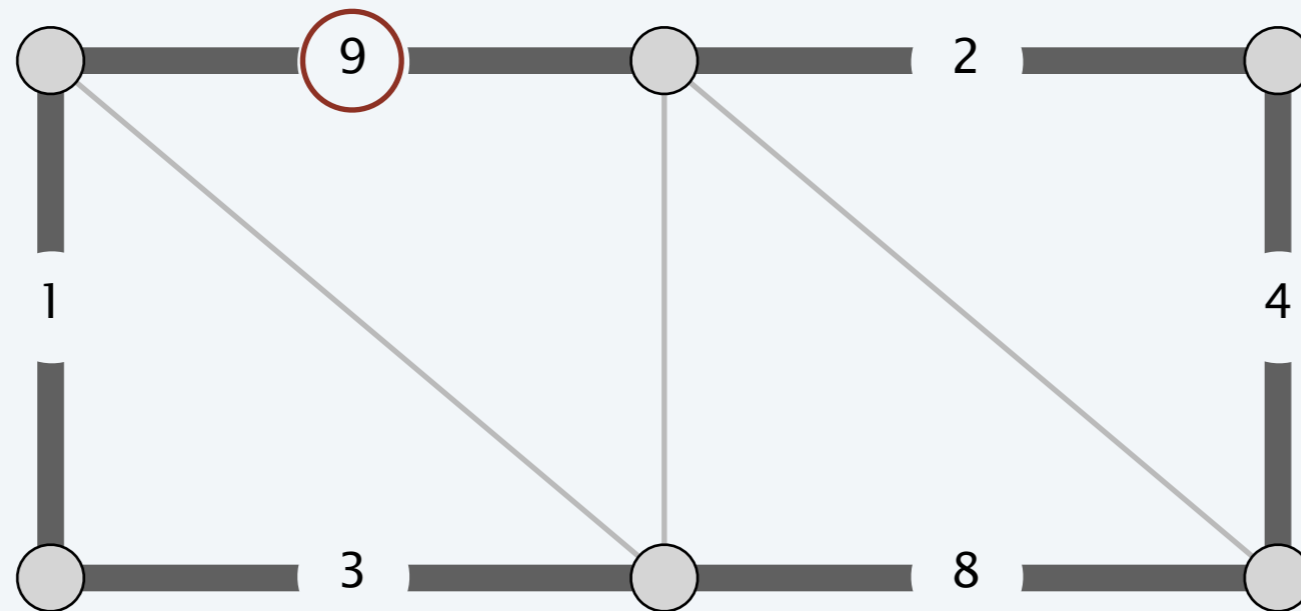


# Red-rule blue-rule demo

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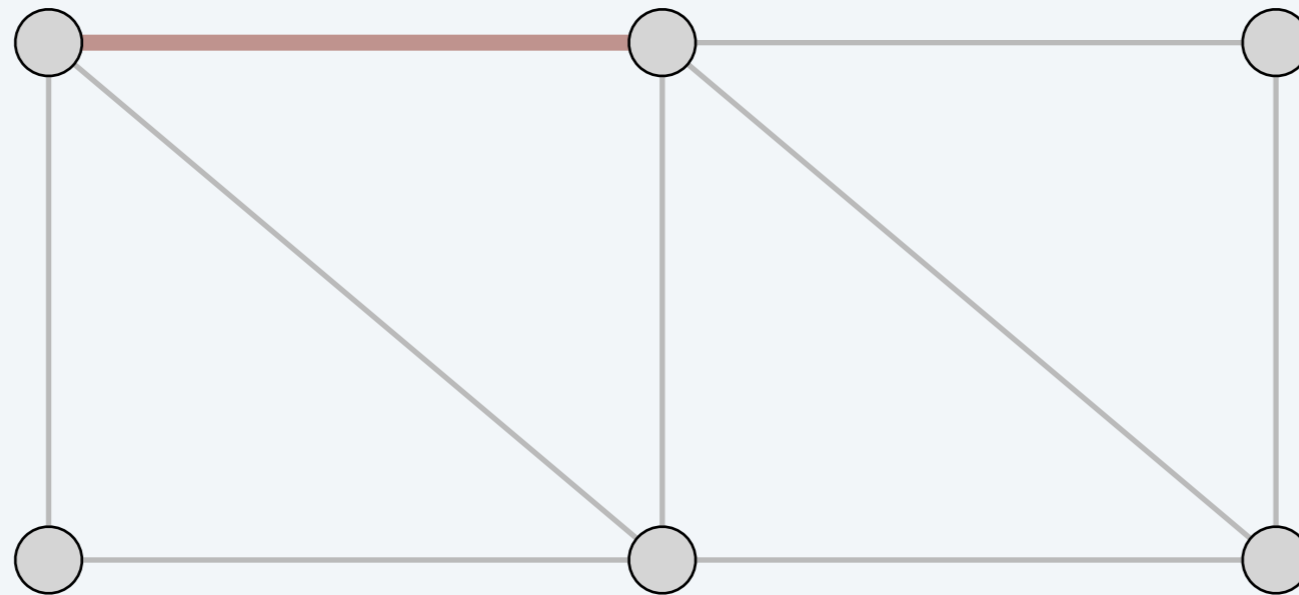
apply the red rule to the cycle



# Red-rule blue-rule demo

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current set of red and blue edges

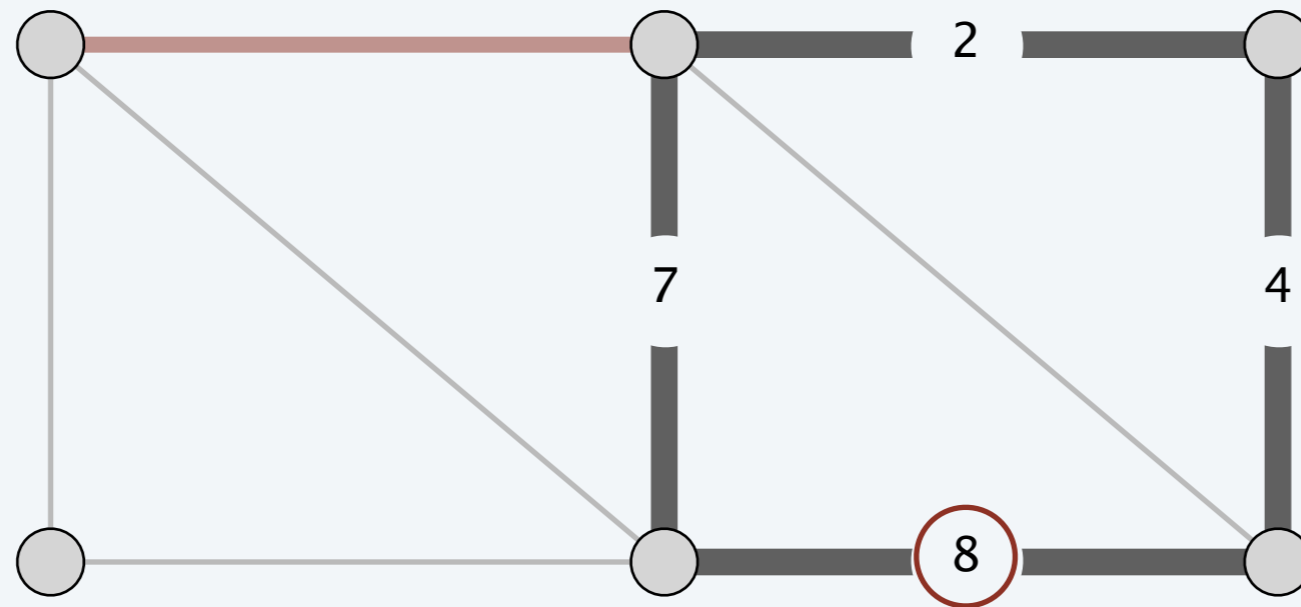


# Red-rule blue-rule demo

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apply the red rule to the cycle

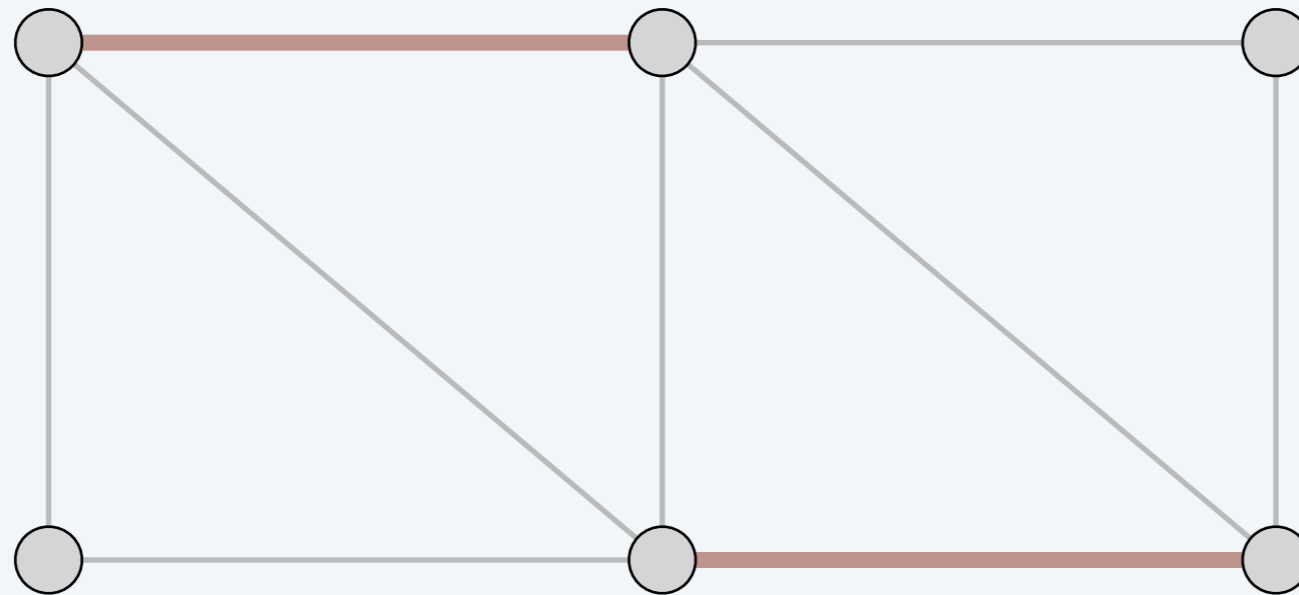


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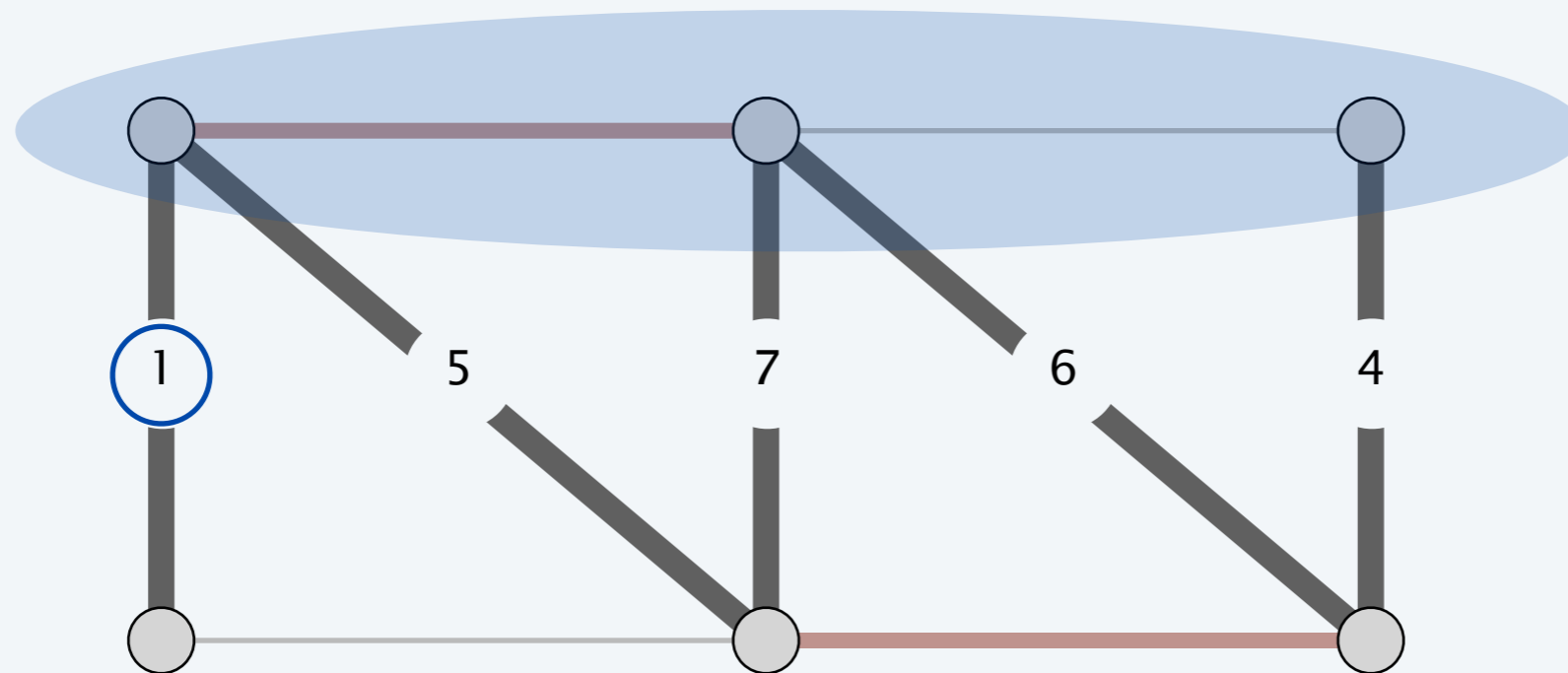


# Red-rule blue-rule demo

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**Blue rule.** Let  $D$  be a cutset with no blue edges. Select an uncolored edge in  $D$  of min weight and color it blue.

apply the blue rule to the cutset

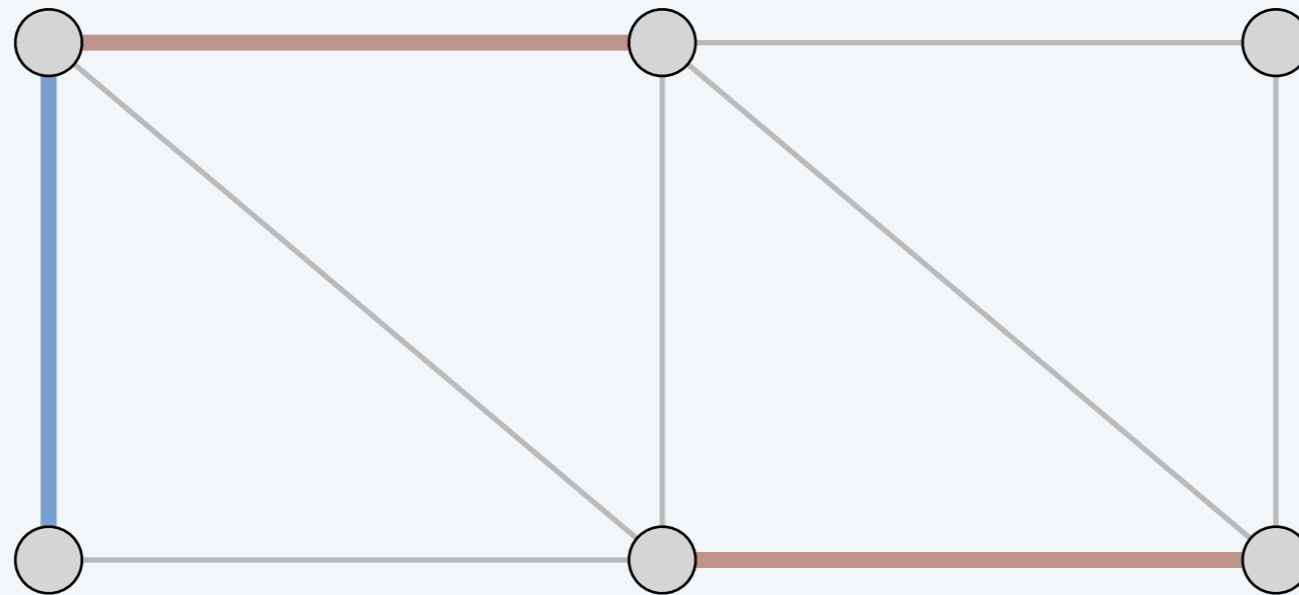




# Red-rule blue-rule demo

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current set of red and blue edges

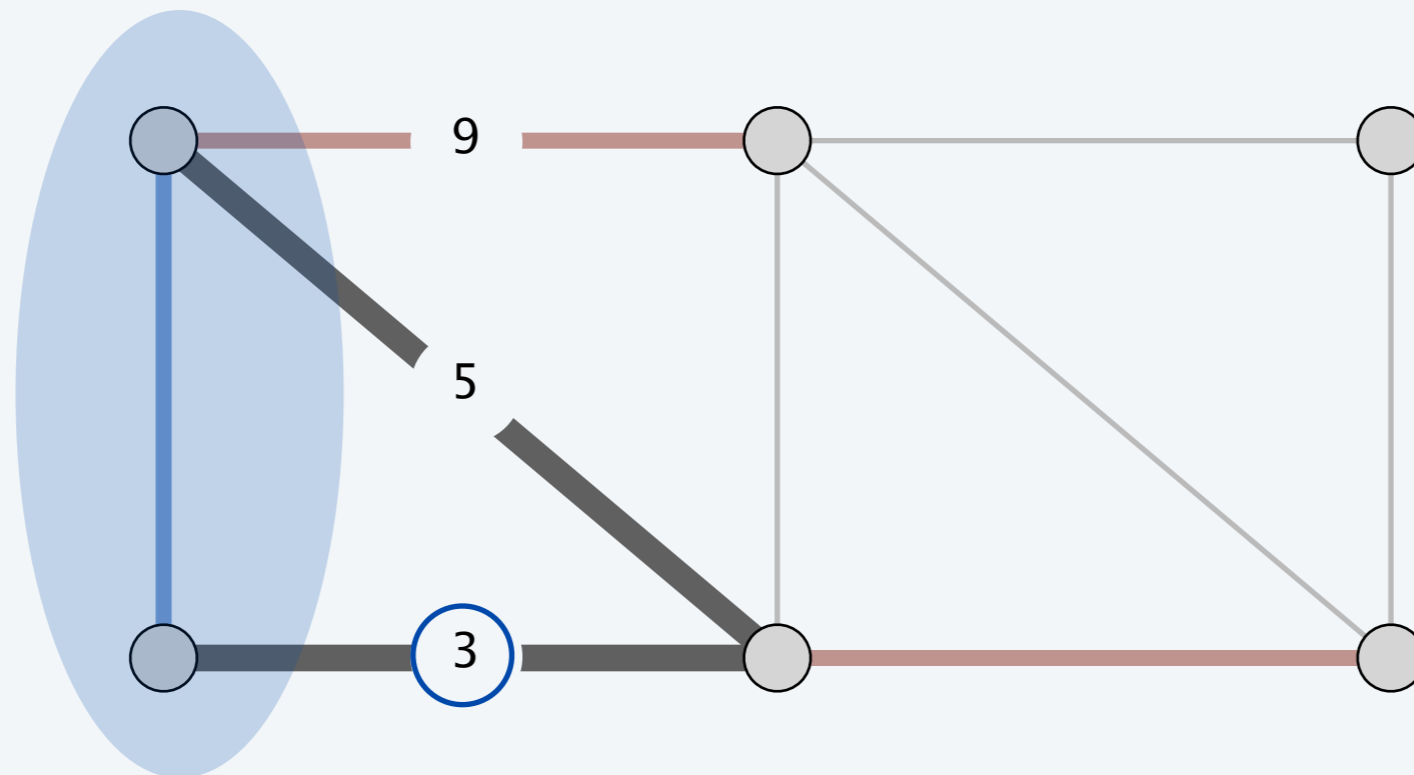


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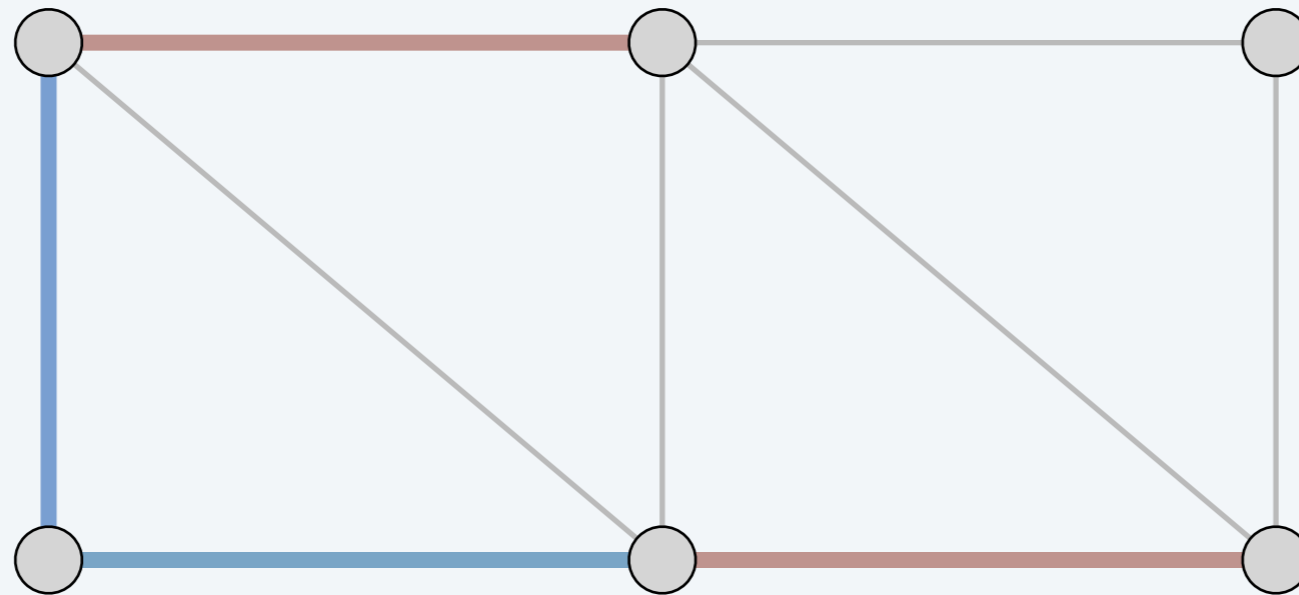
apply the blue rule to the cutset



# Red-rule blue-rule demo

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current set of red and blue edges

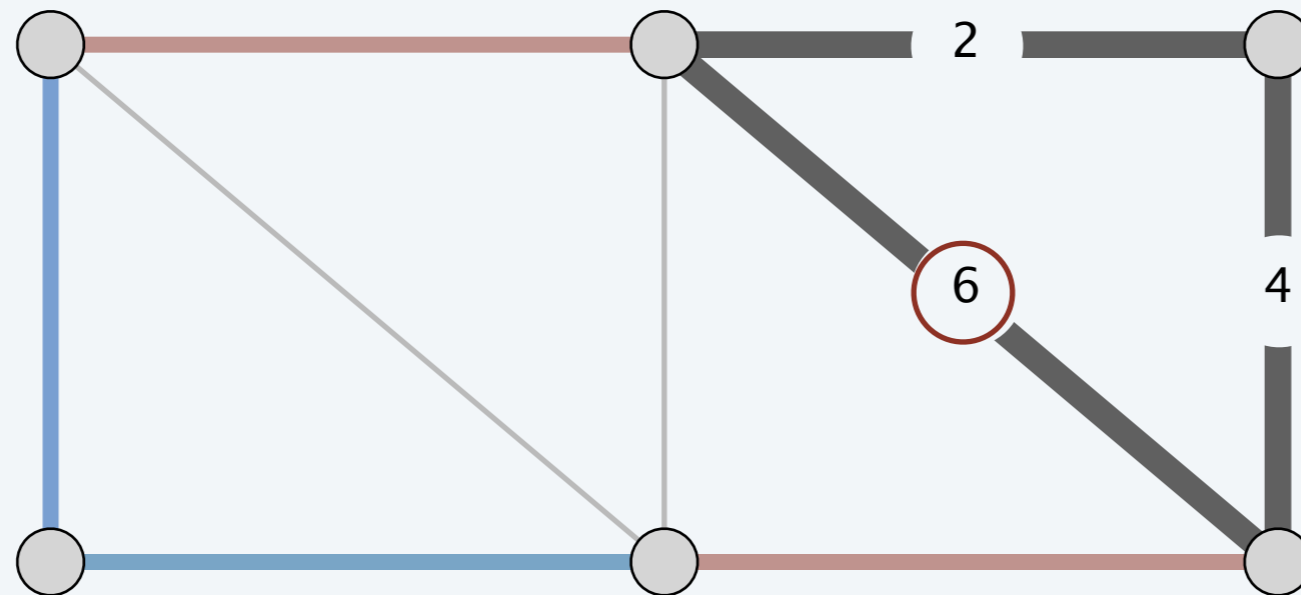


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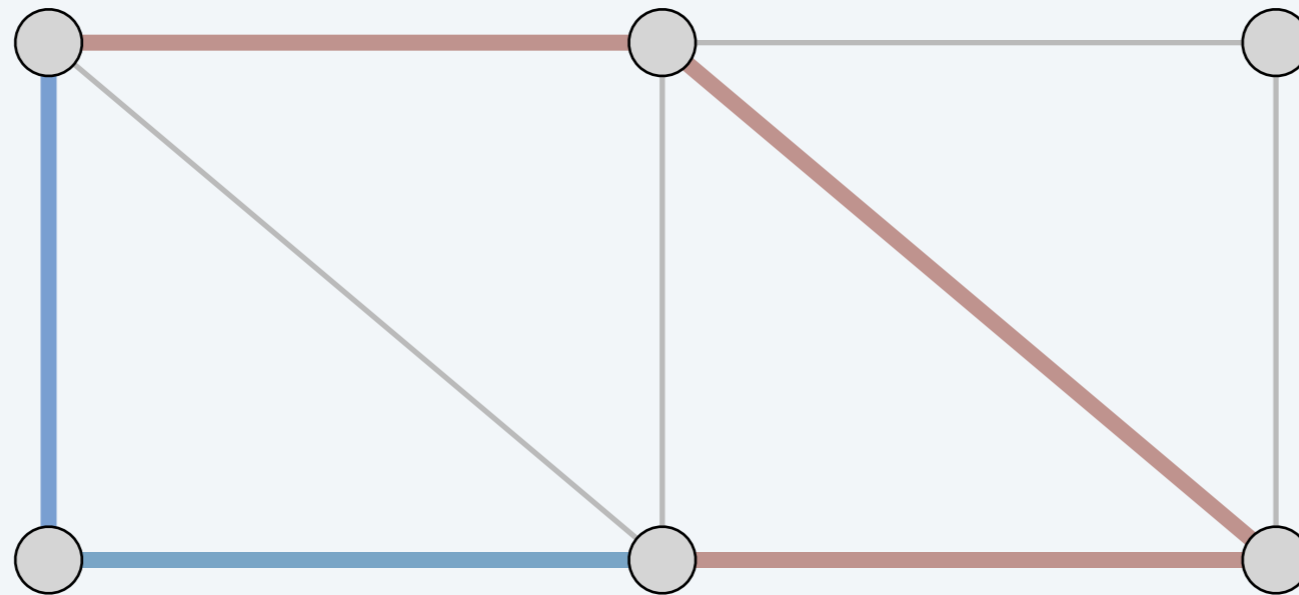
apply the red rule to the cycle



# Red-rule blue-rule demo

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current set of red and blue edges

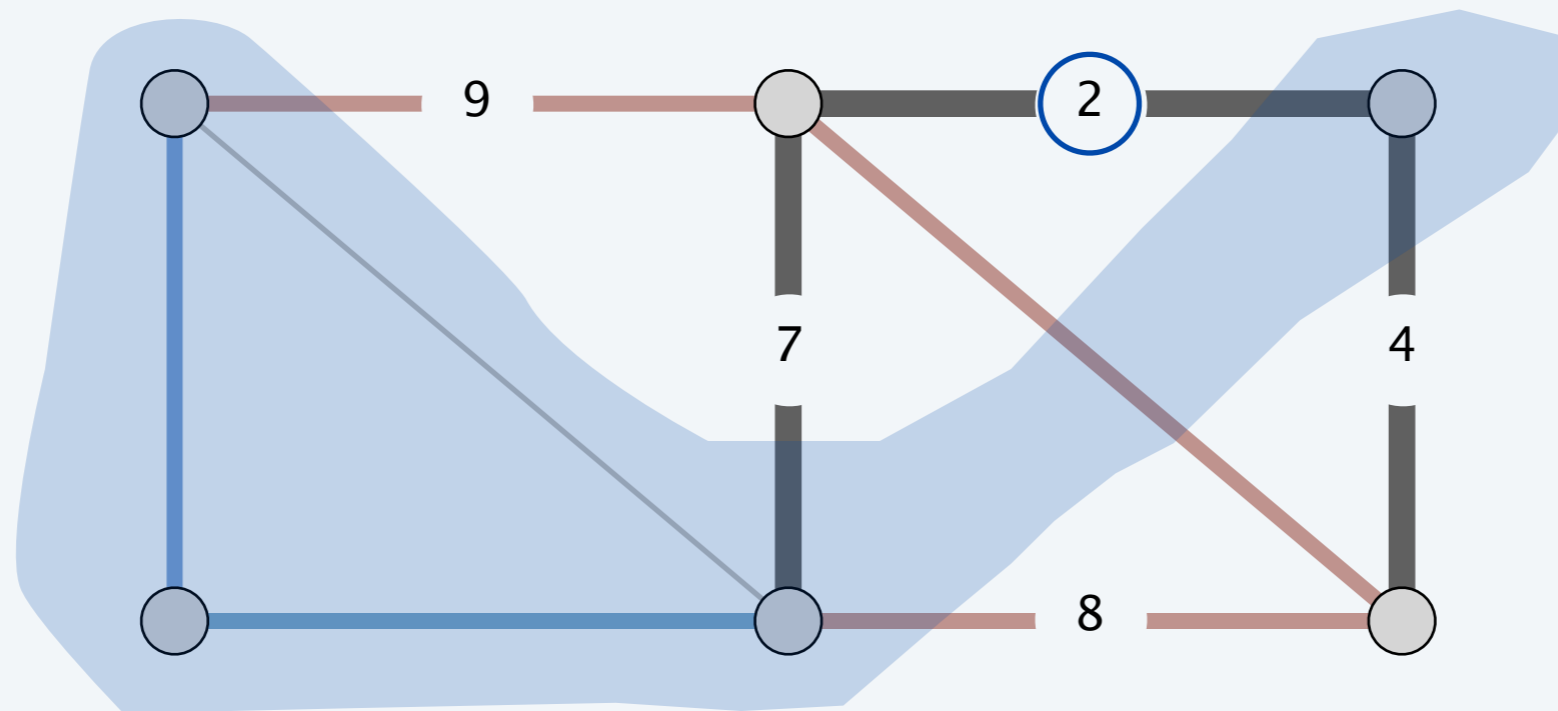


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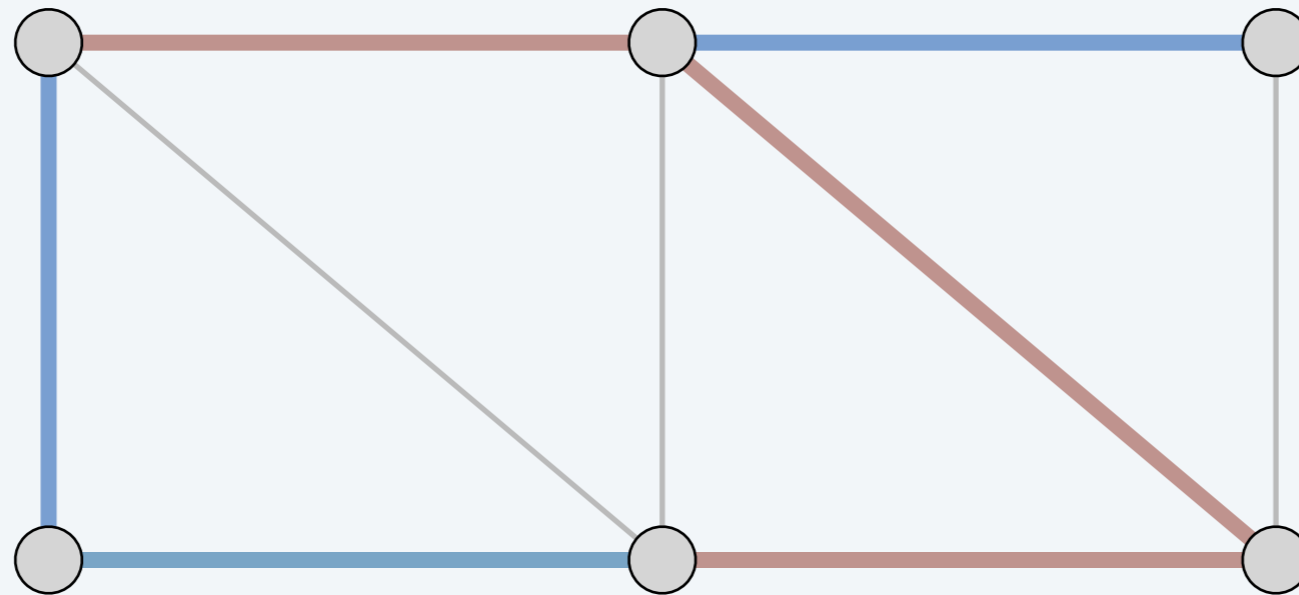
apply the blue rule to the cutset



# Red-rule blue-rule demo

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current set of red and blue edges

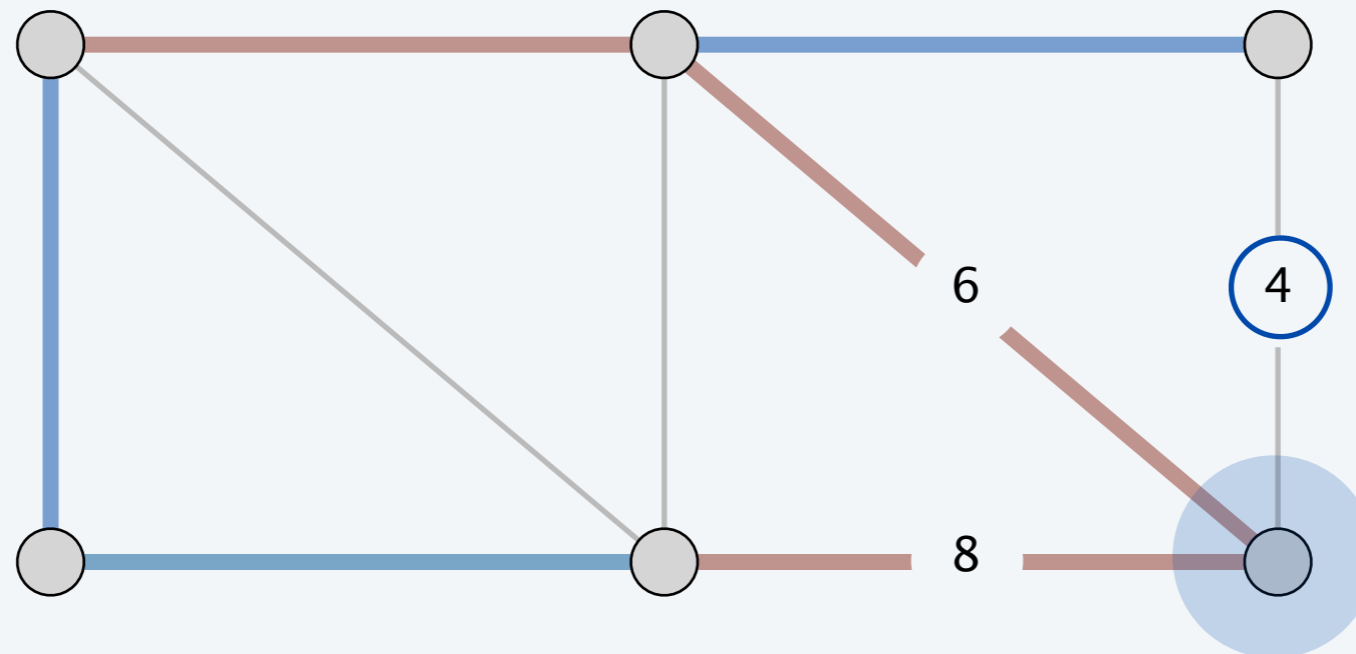


# Red-rule blue-rule demo

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**Blue rule.** Let  $D$  be a cutset with no blue edges. Select an uncolored edge in  $D$  of min weight and color it blue.

apply the blue rule to the cutset

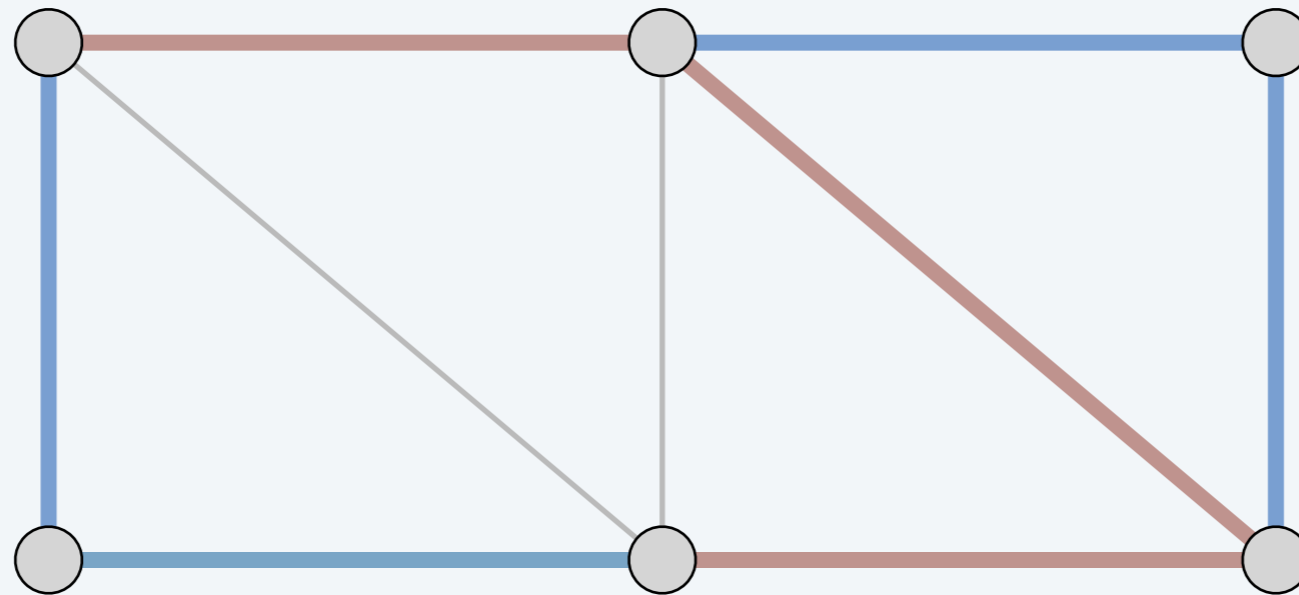




# Red-rule blue-rule demo

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current set of red and blue edges

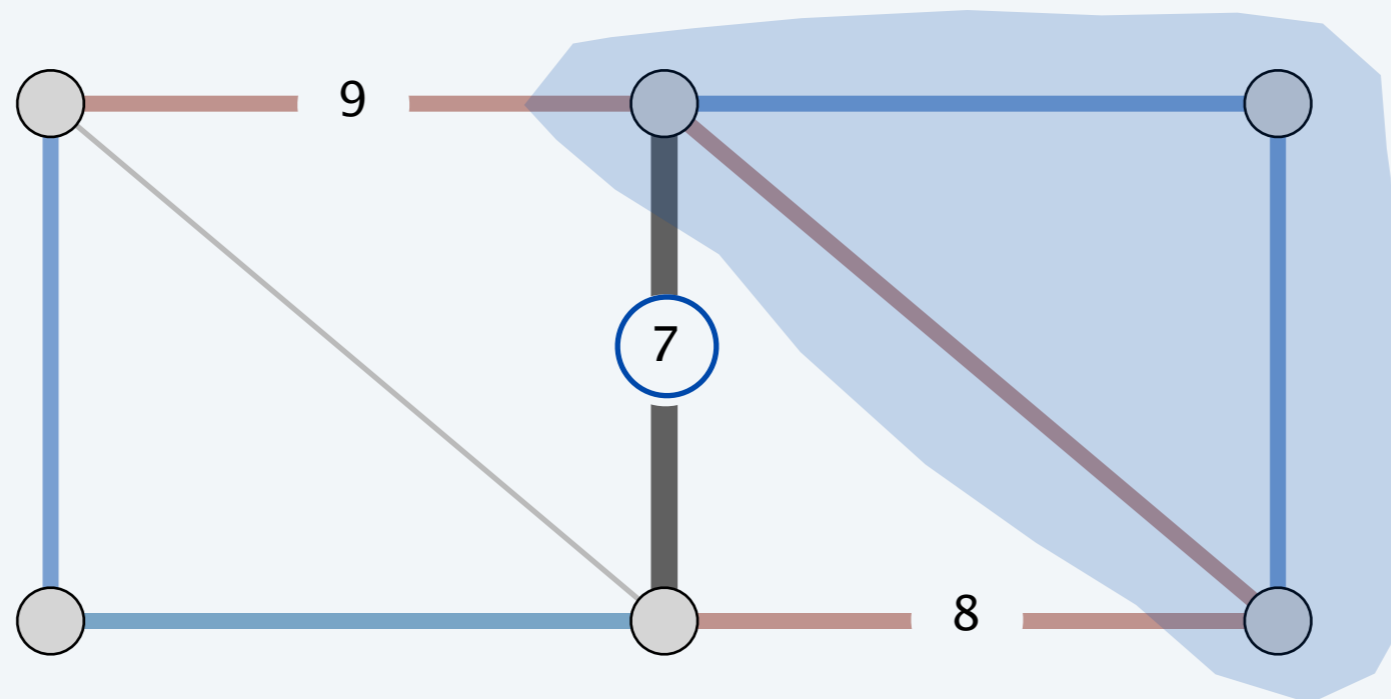


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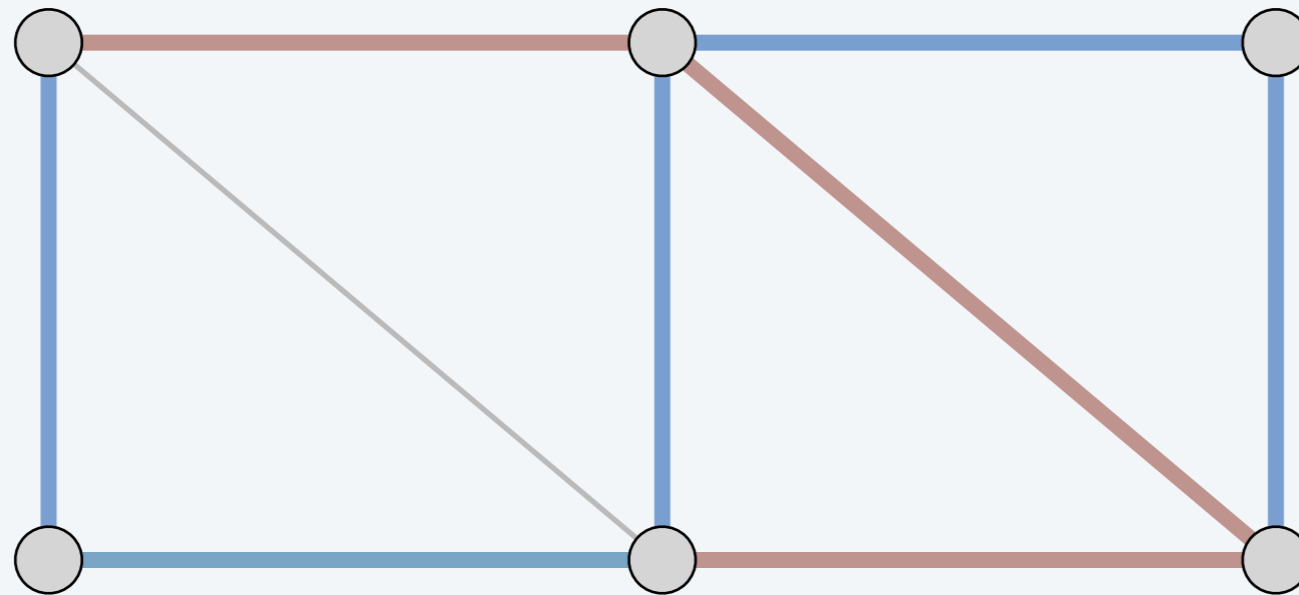
apply the blue rule to the cutset



# Red-rule blue-rule demo

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current set of red and blue edges

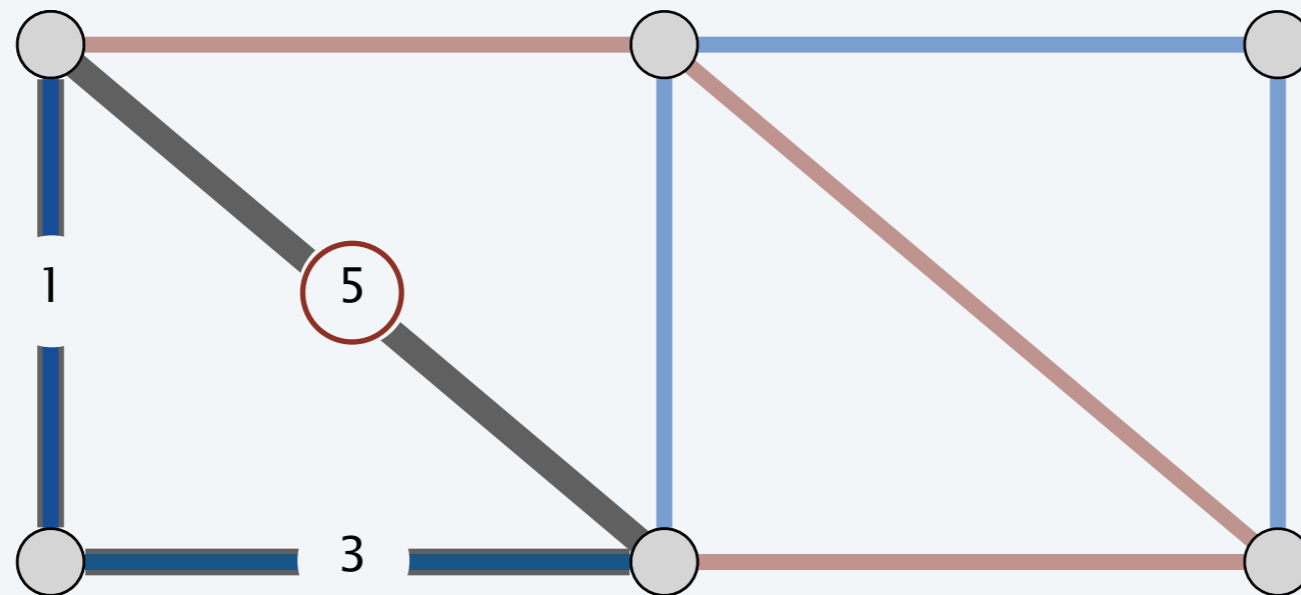


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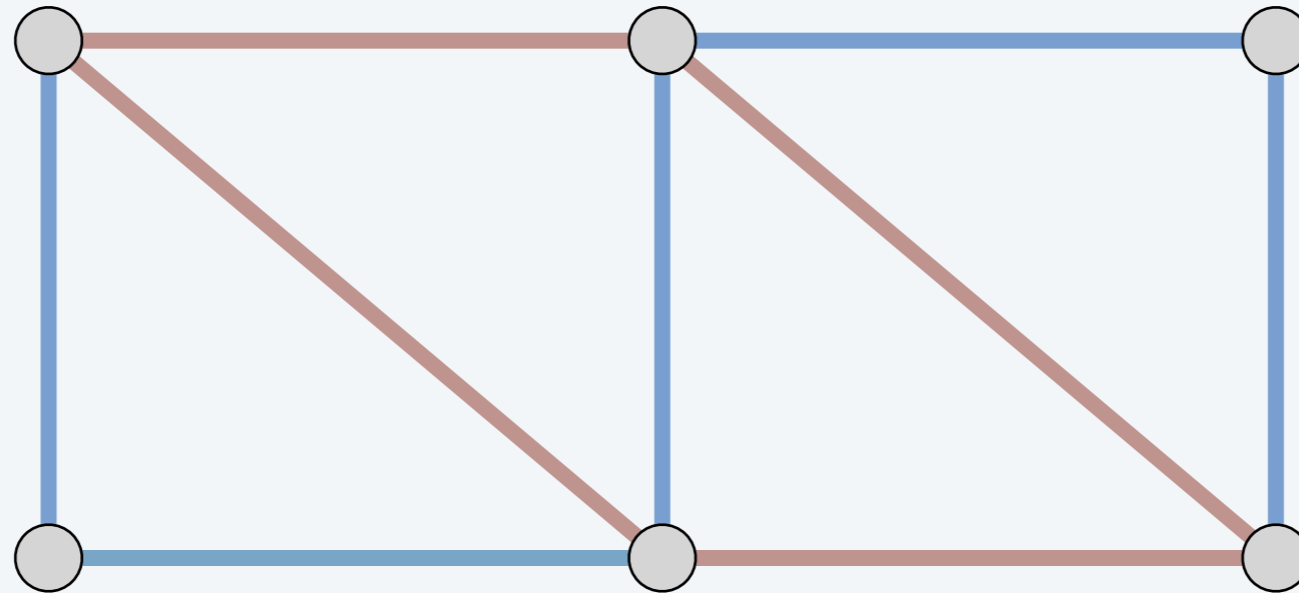
apply the red rule to the cycle



# Red-rule blue-rule demo

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current set of red and blue edges

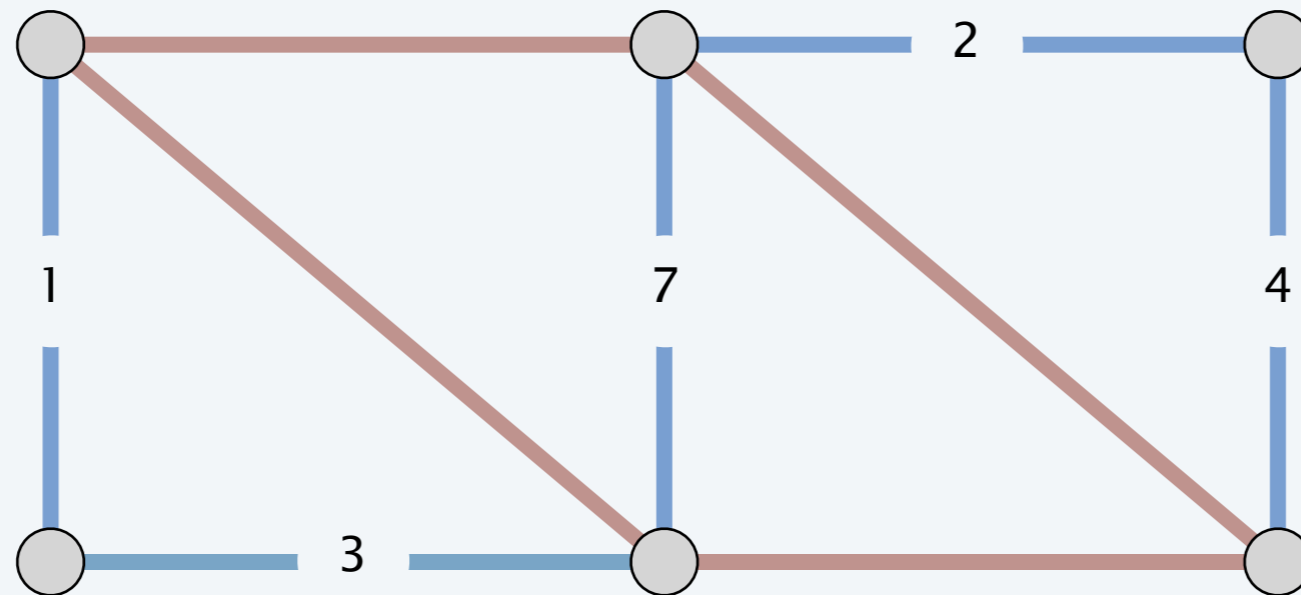


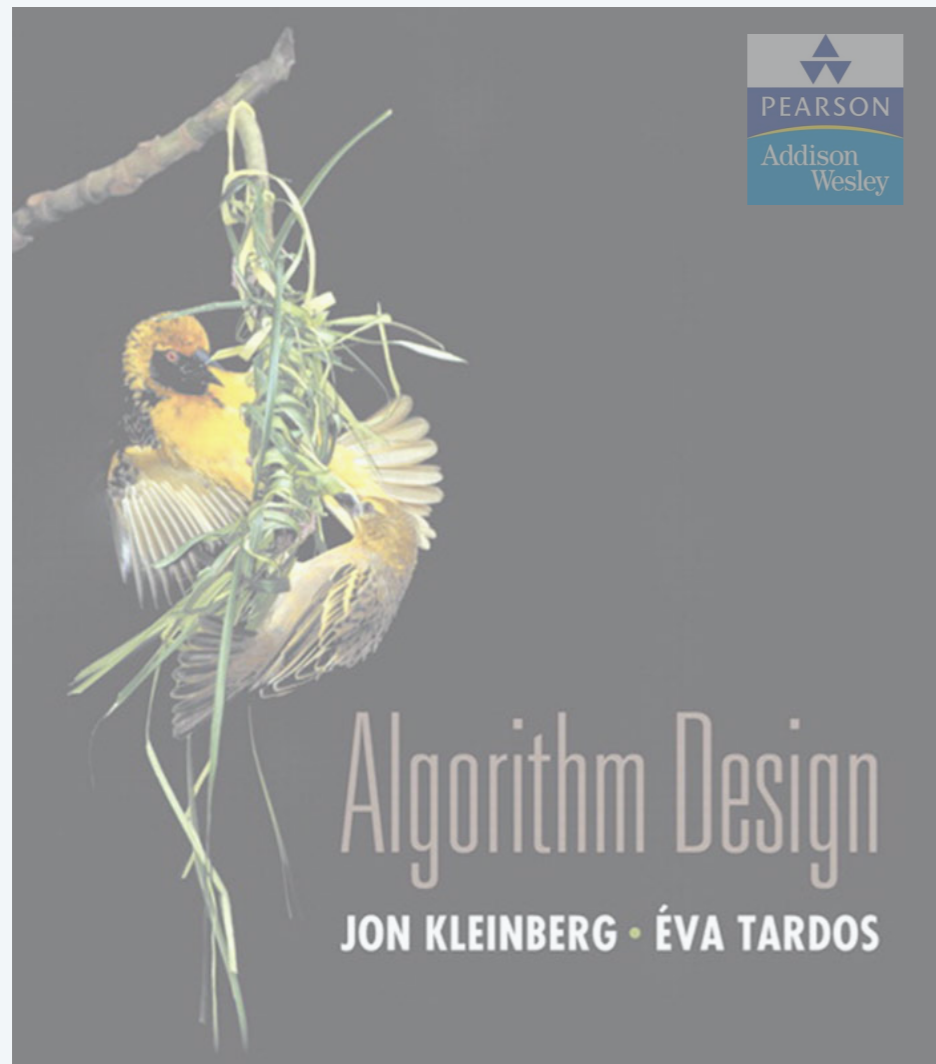
# Red-rule blue-rule demo

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Greedy algorithm. Upon termination, the blue edges form a MST.

a minimum spanning tree





## SECTION 4.5

# 4. GREEDY ALGORITHMS II

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- ▶ *red-rule blue-rule demo*
- ▶ ***Prim's algorithm demo***
- ▶ *Kruskal's algorithm demo*
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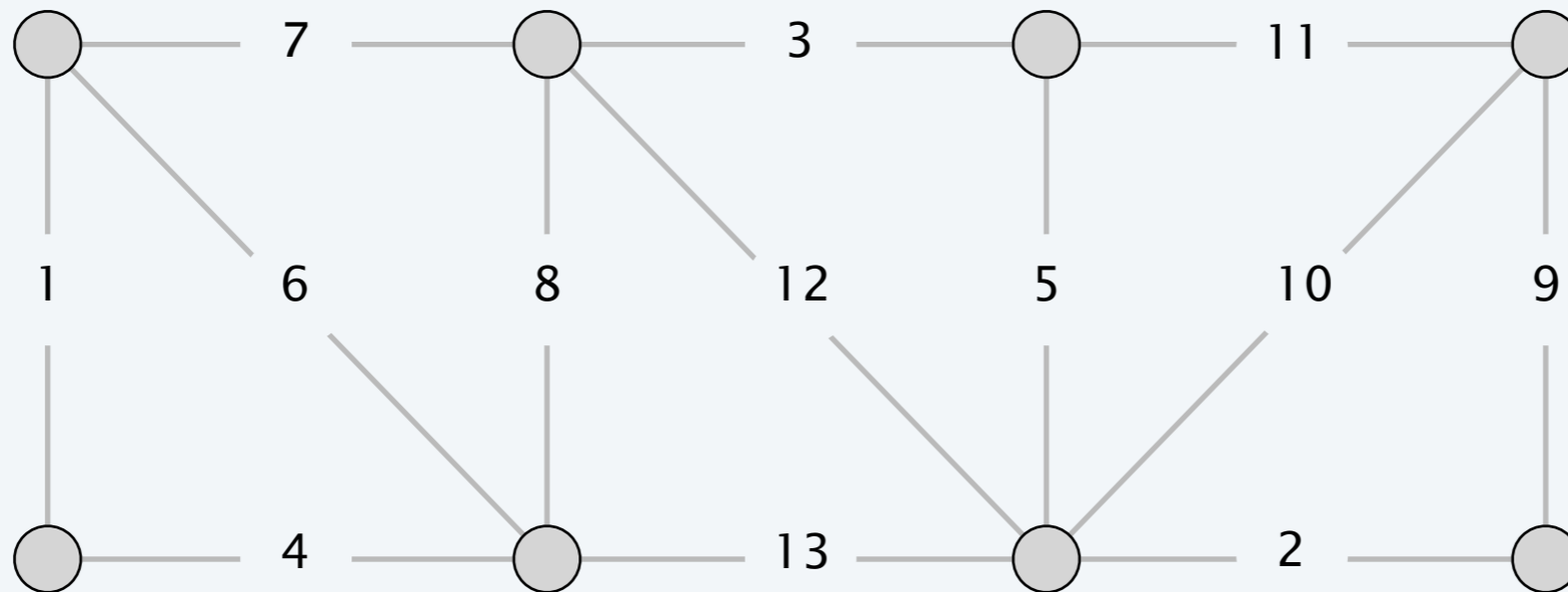
# Prim's algorithm demo

---

Initialize  $S = \{ s \}$  for any node  $s$ ,  $T = \emptyset$ .

Repeat  $n - 1$  times:

- Add to  $T$  a min-weight edge with exactly one endpoint in  $S$ .
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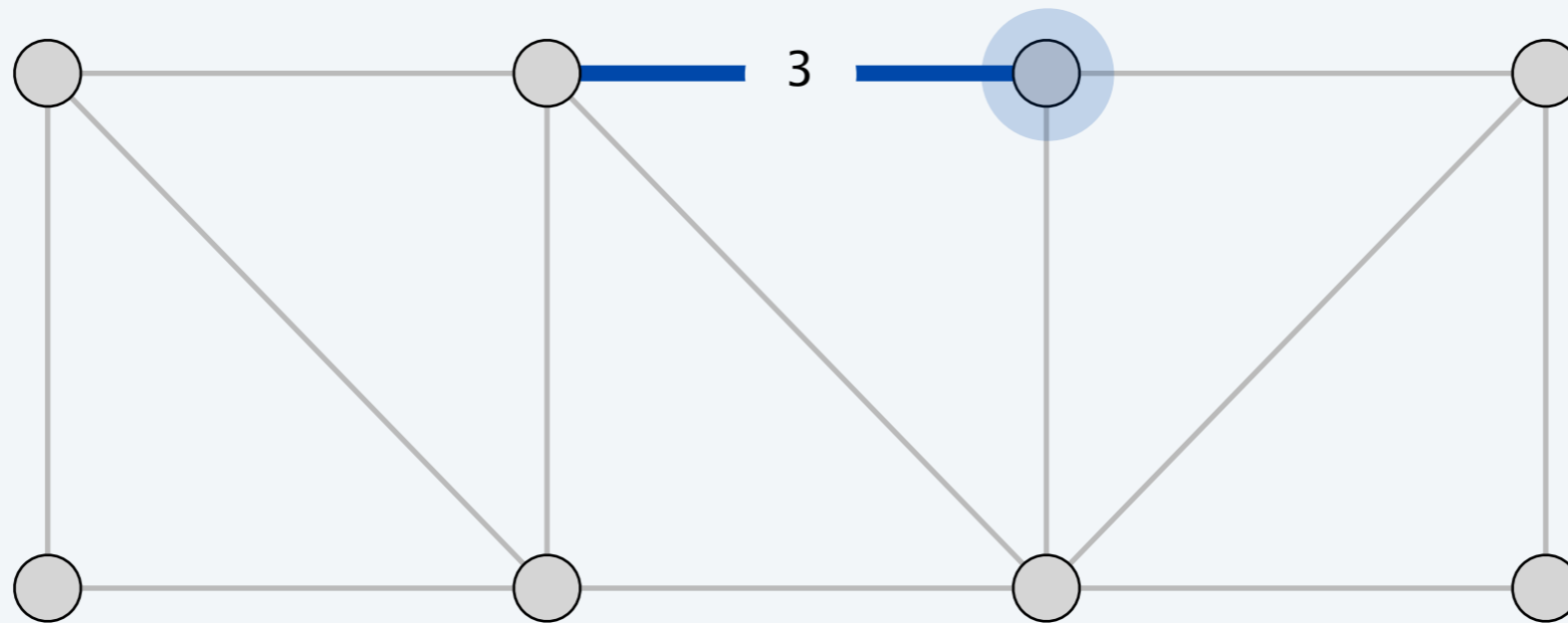
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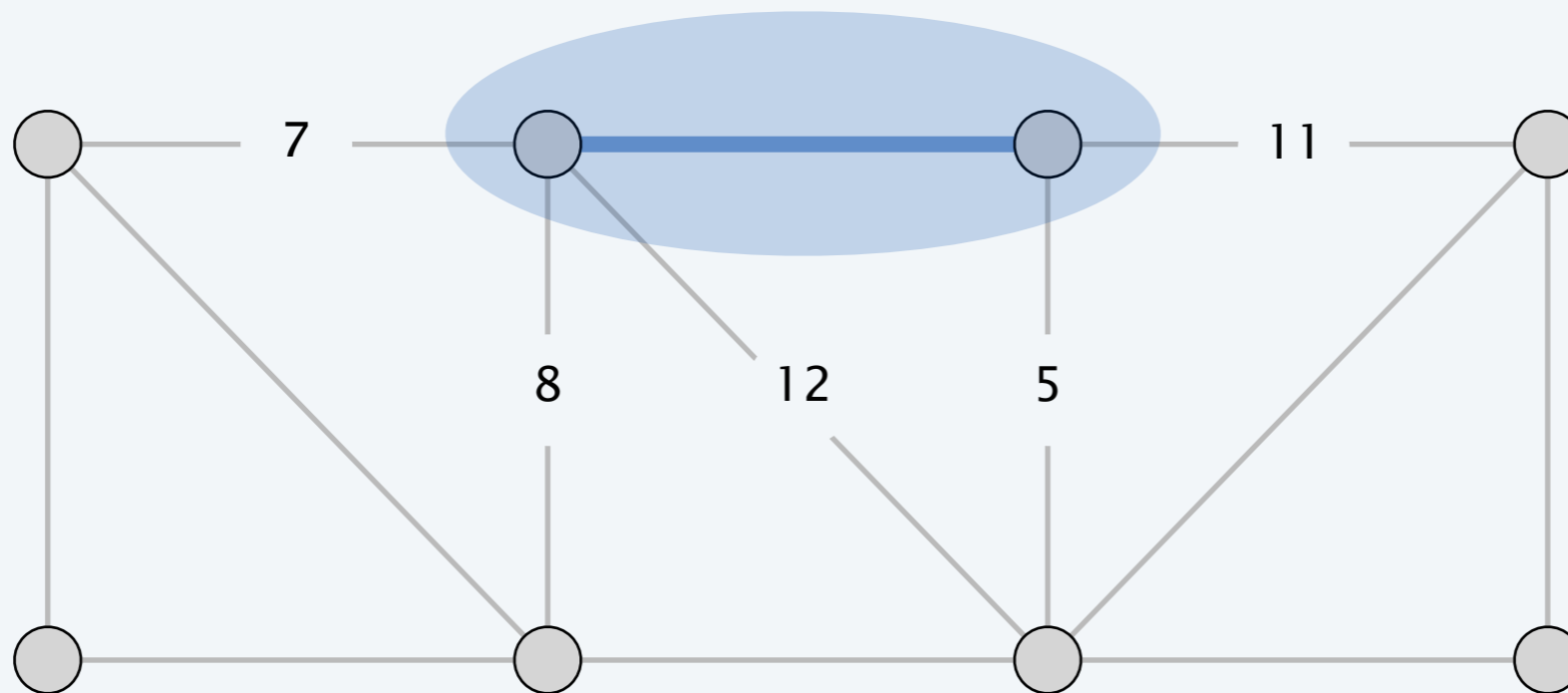
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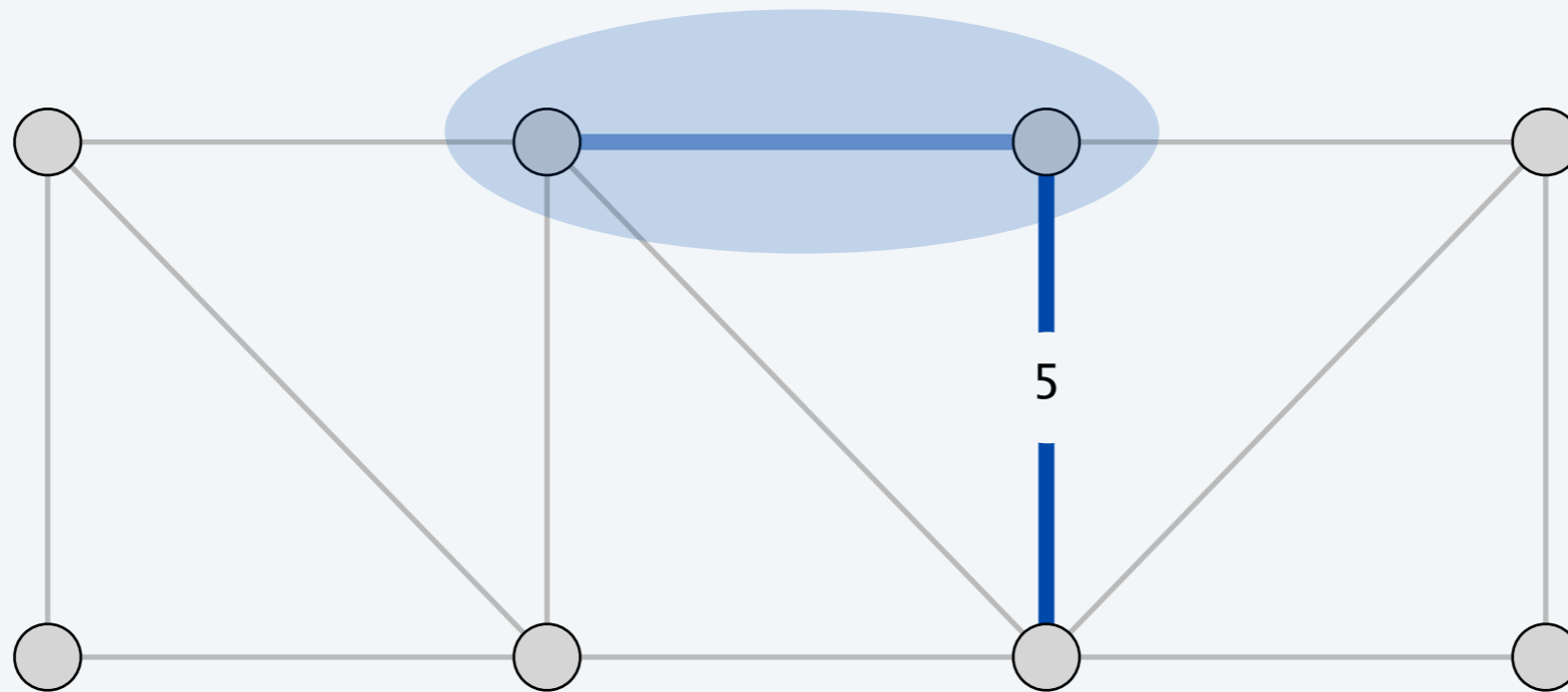
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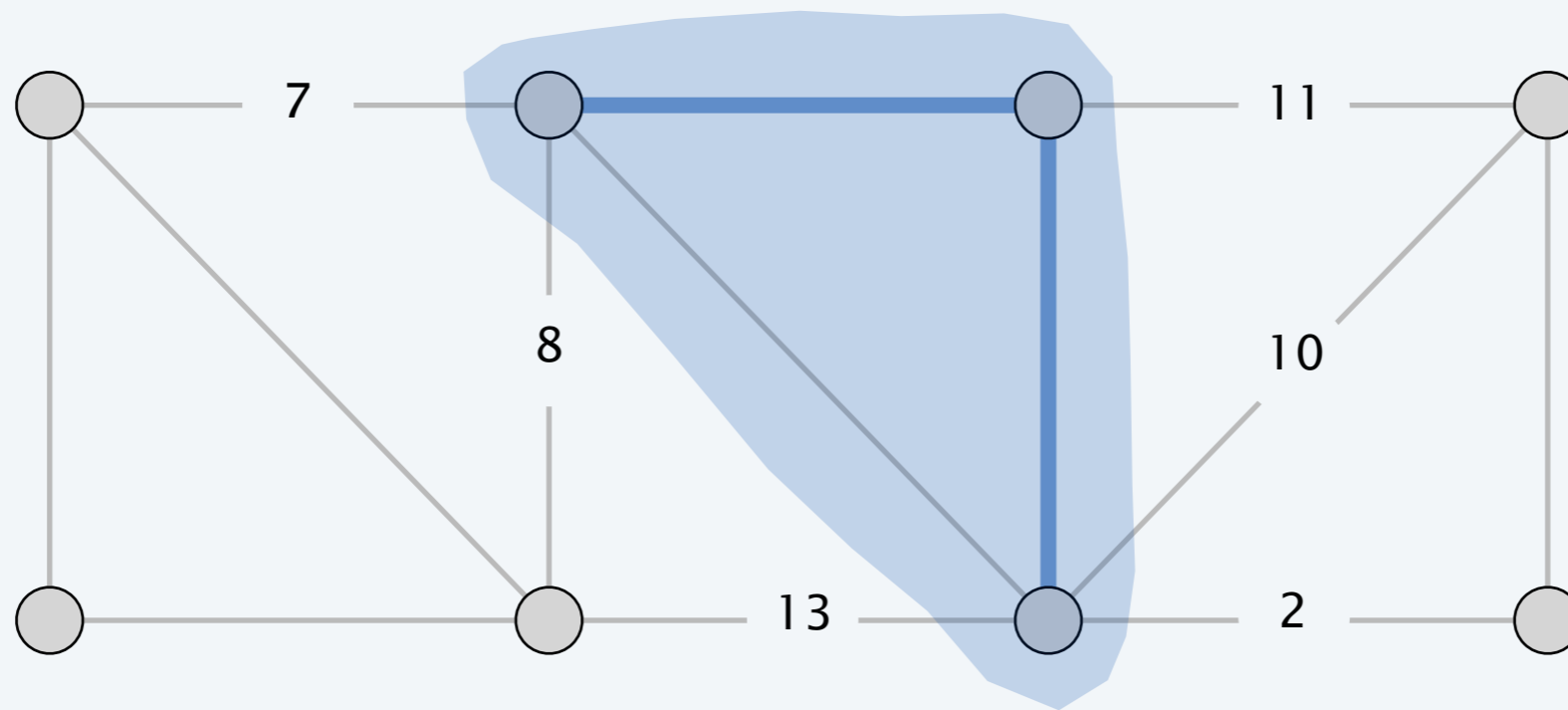
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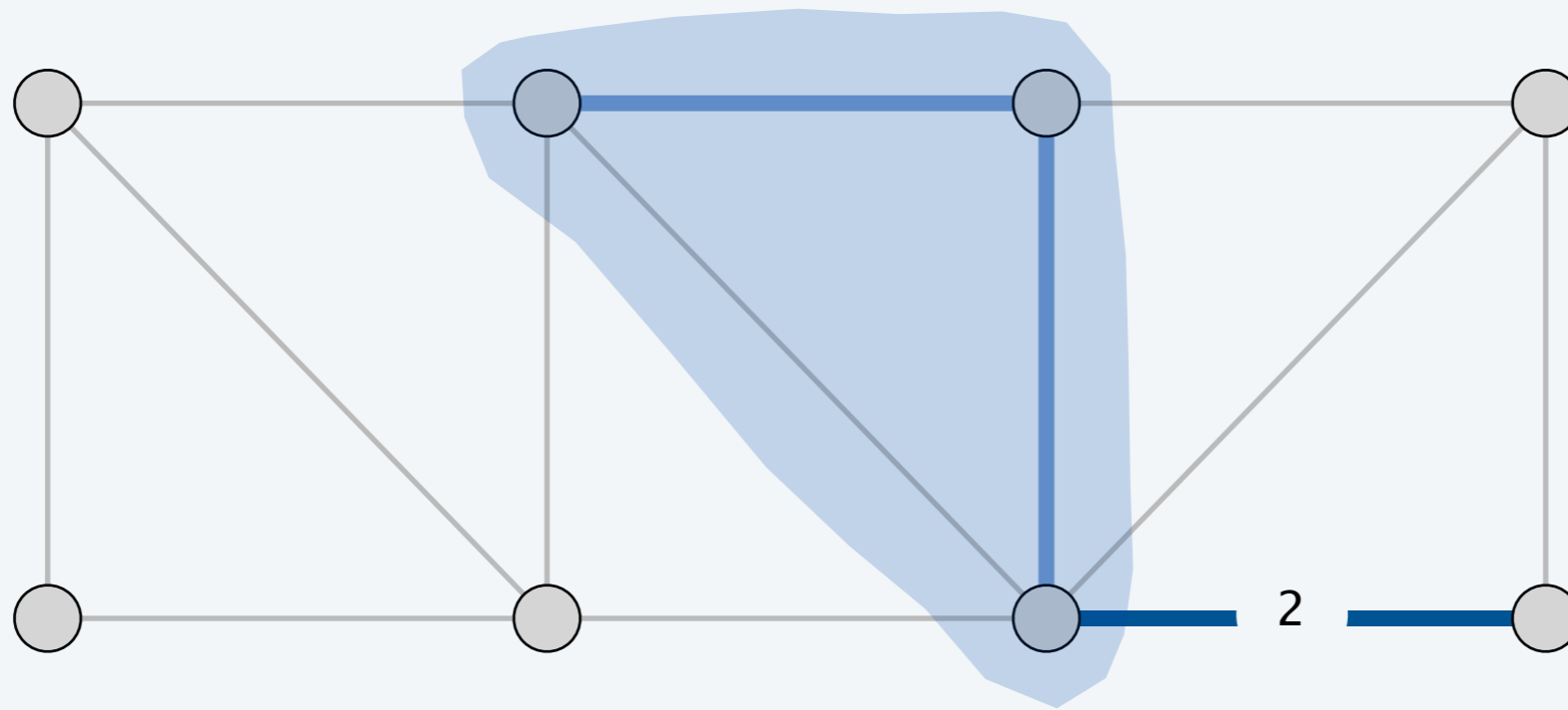
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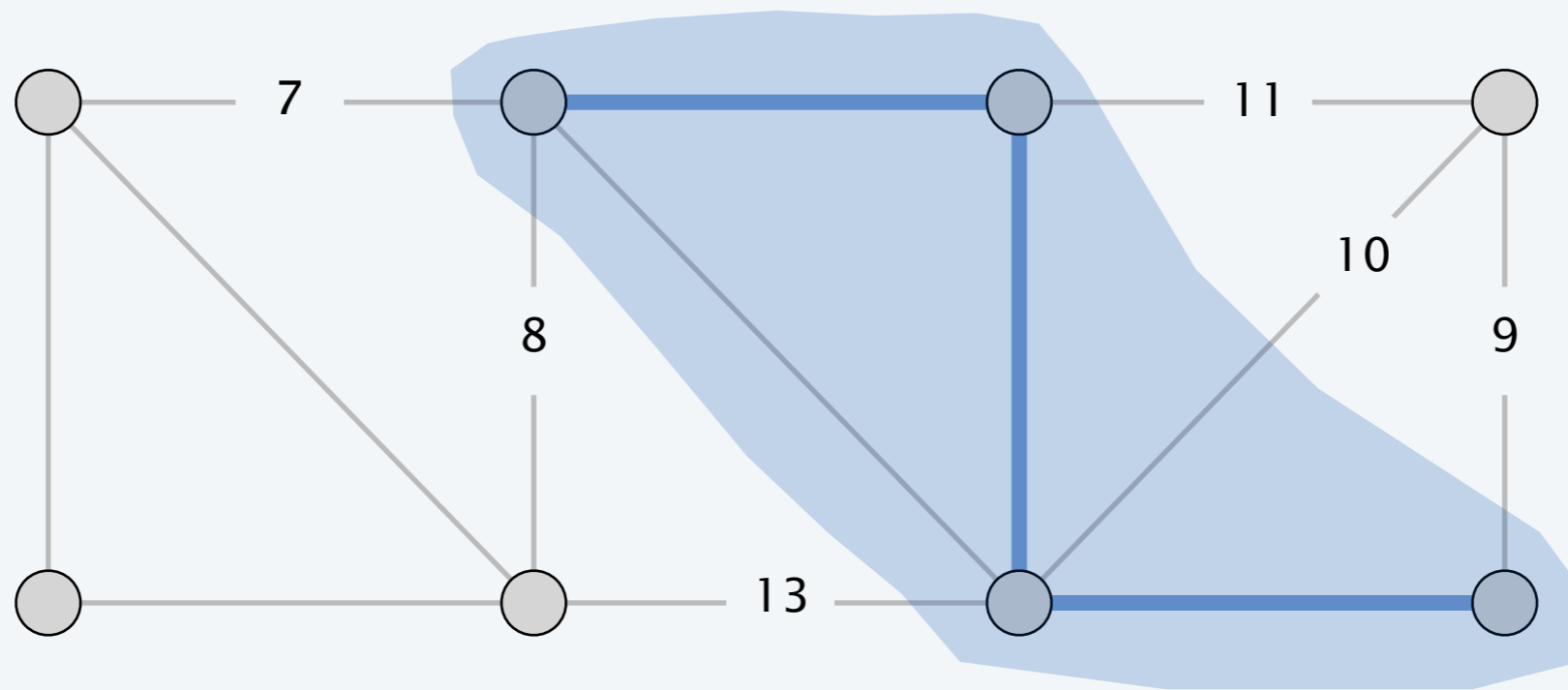
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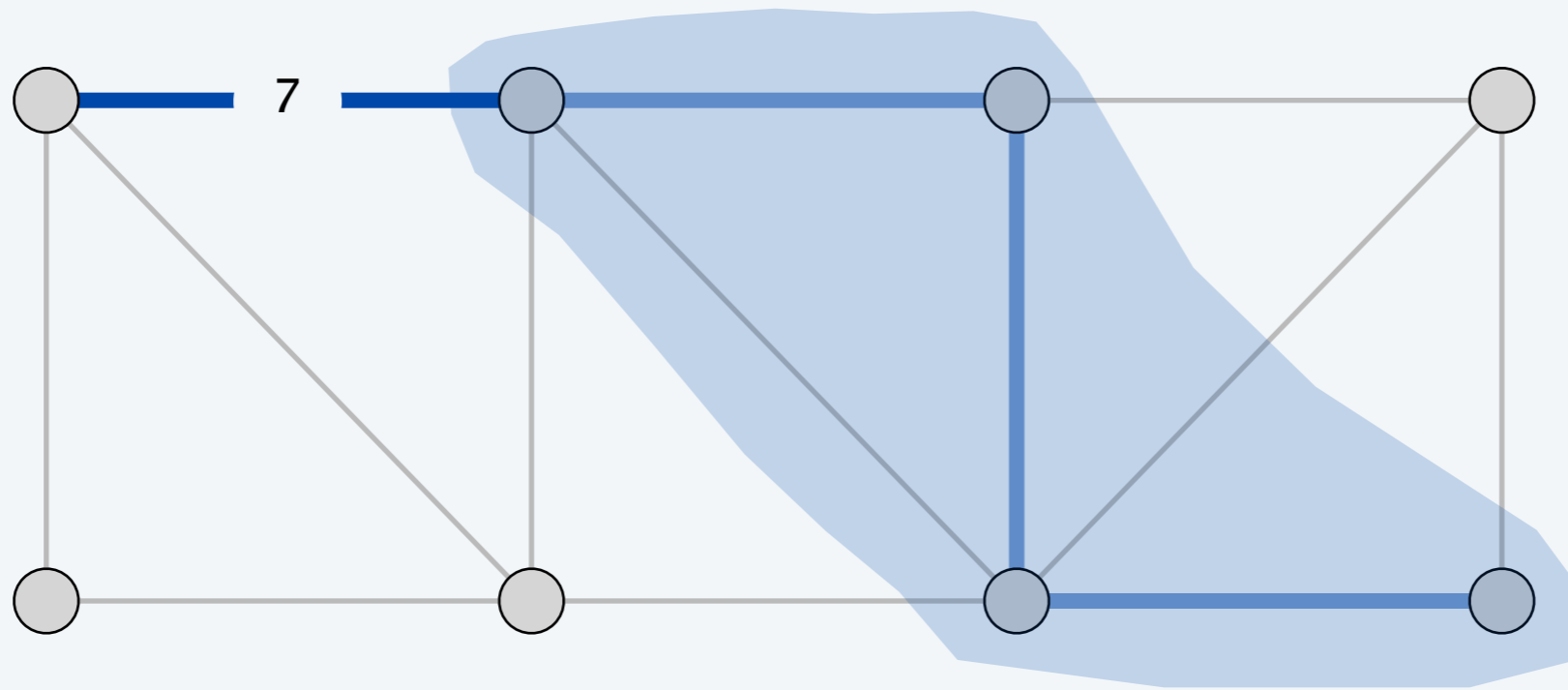
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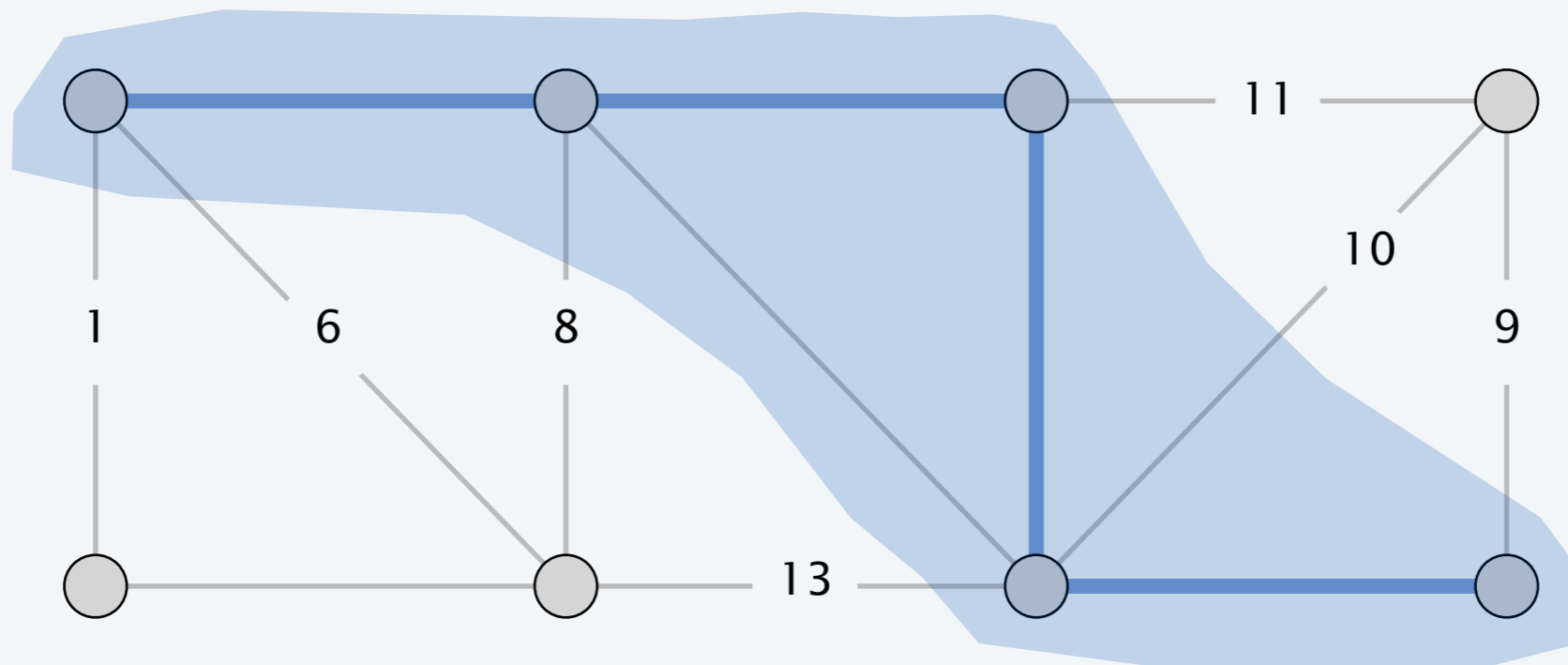
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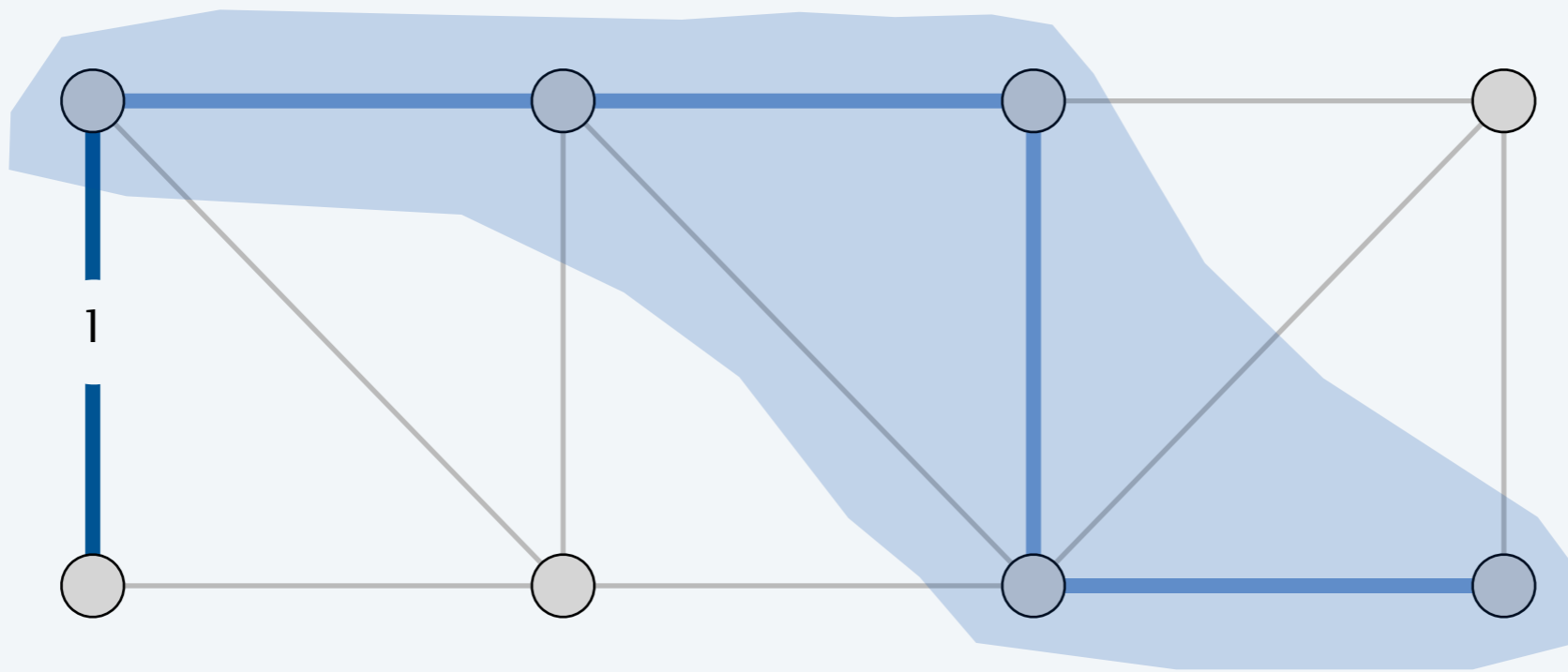
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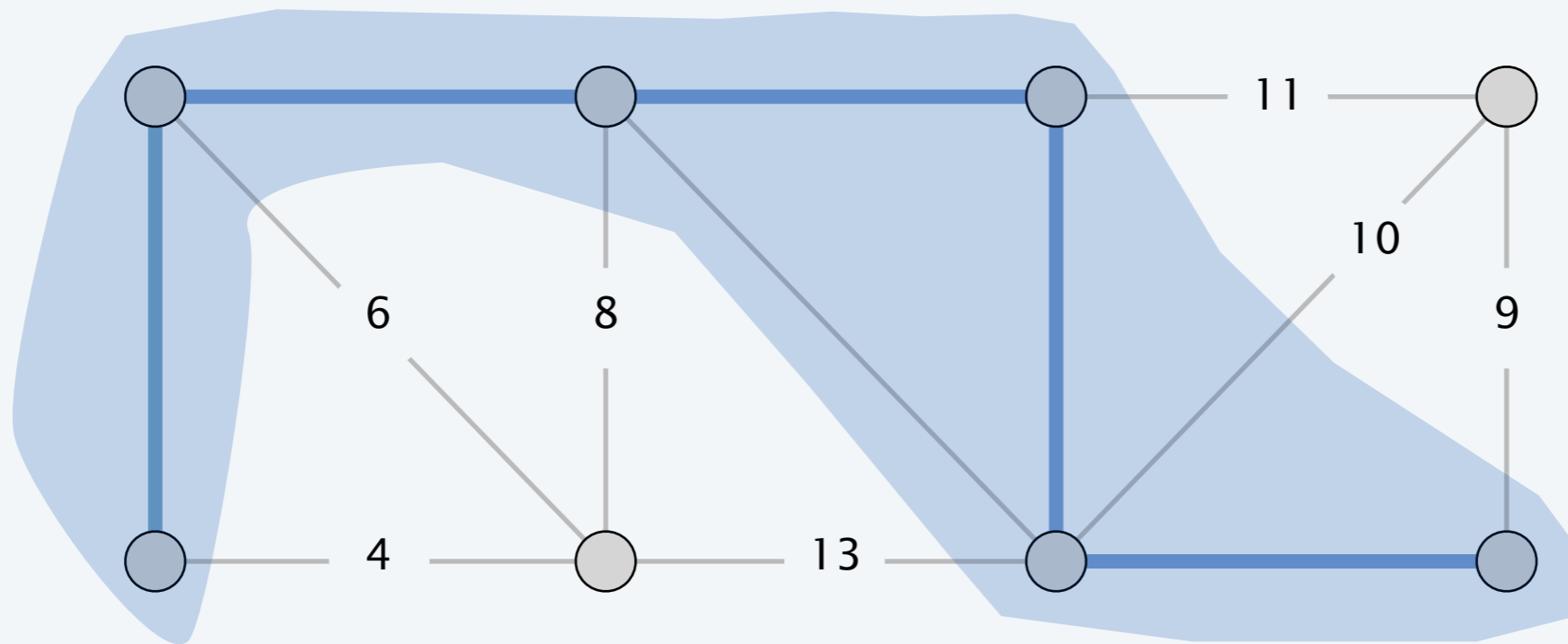
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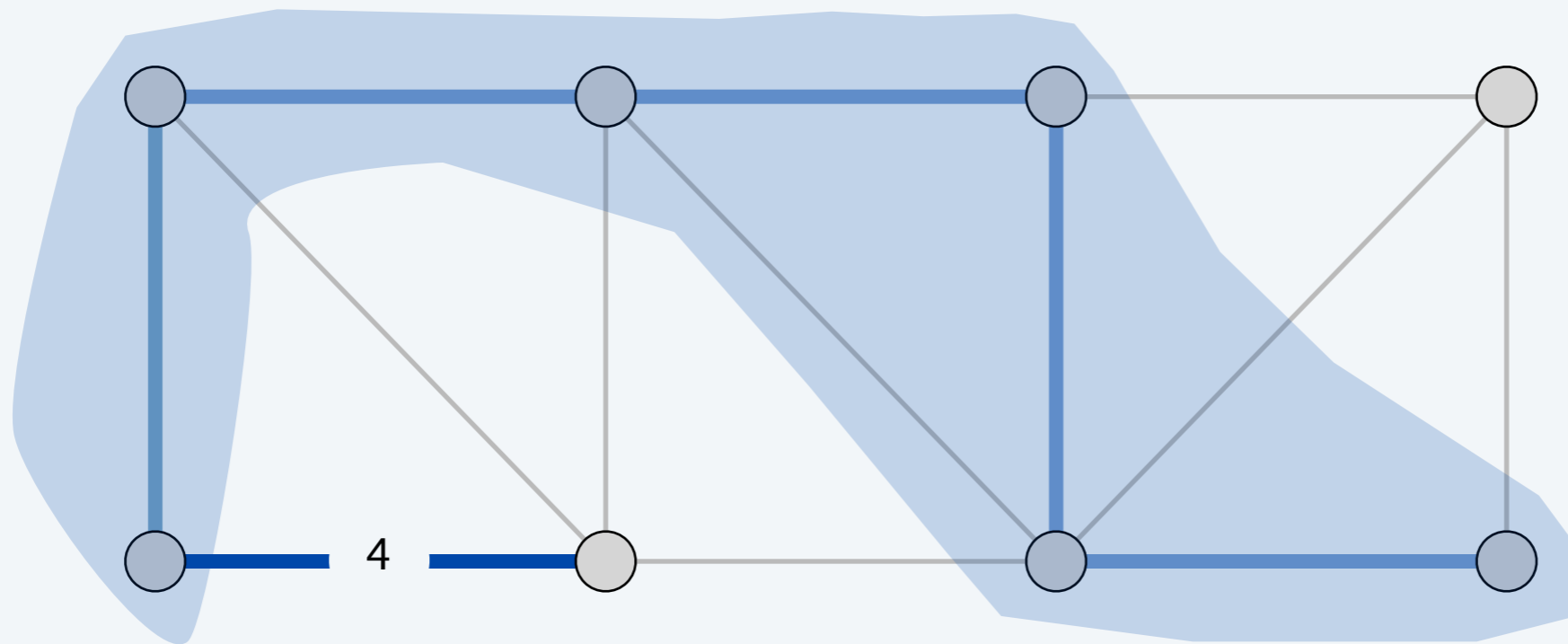
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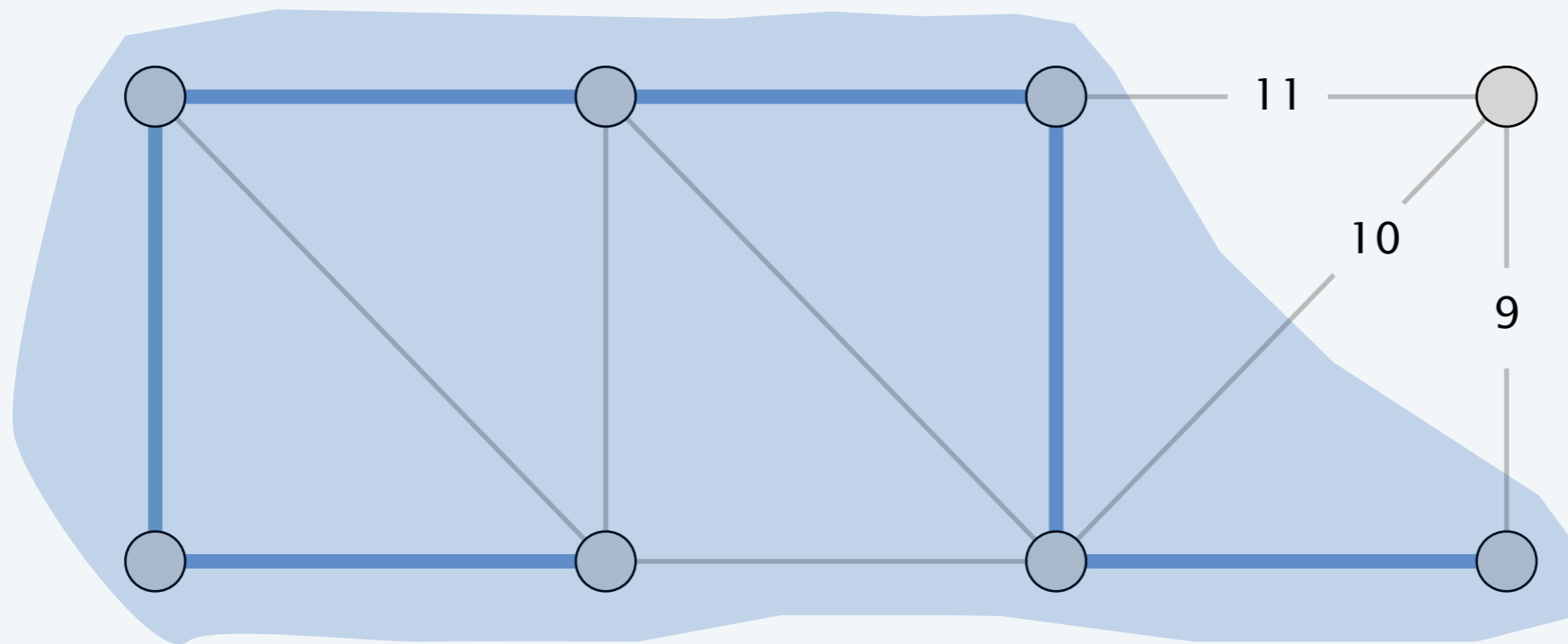
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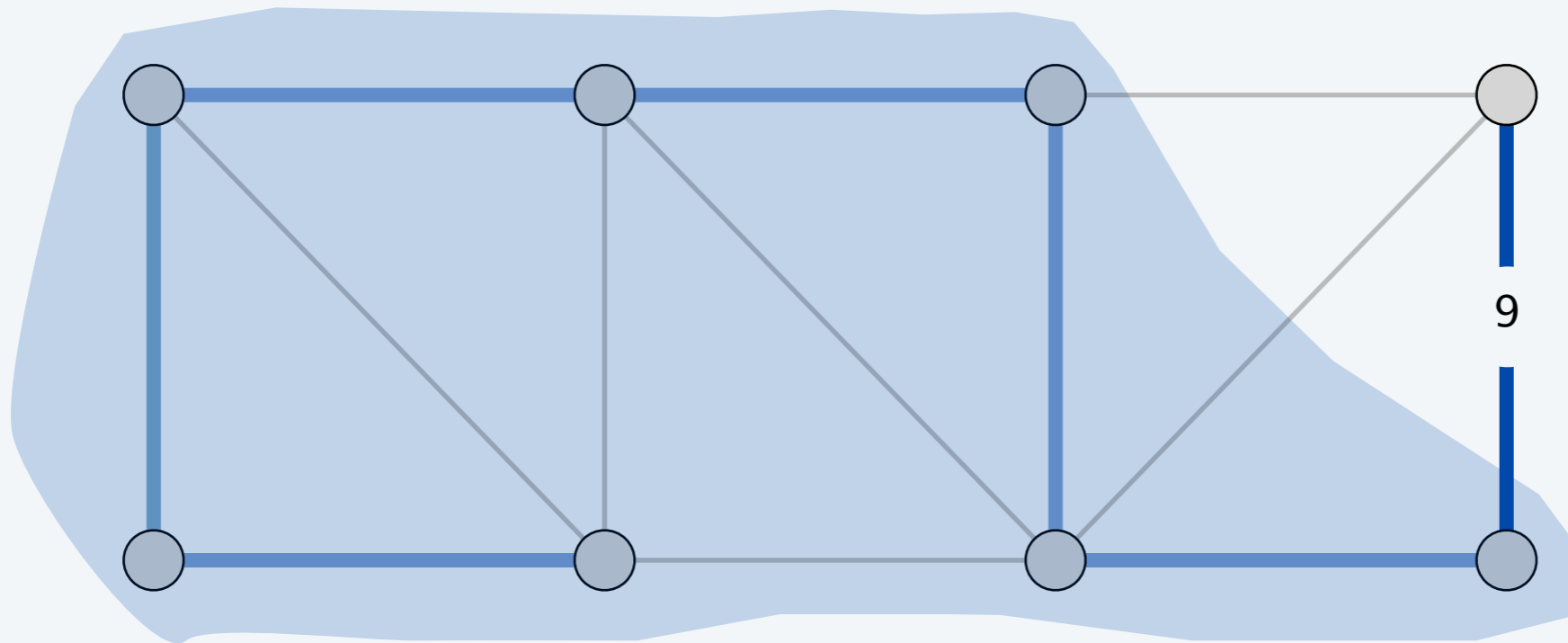
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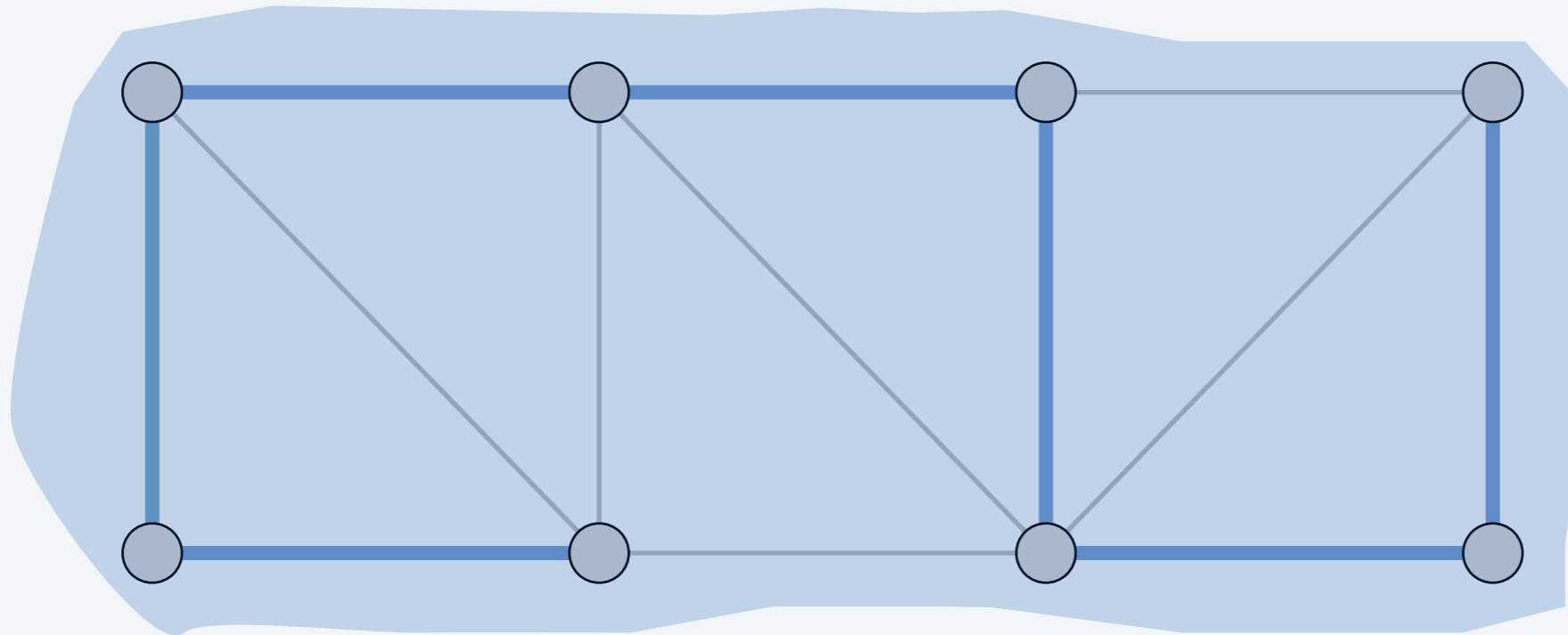
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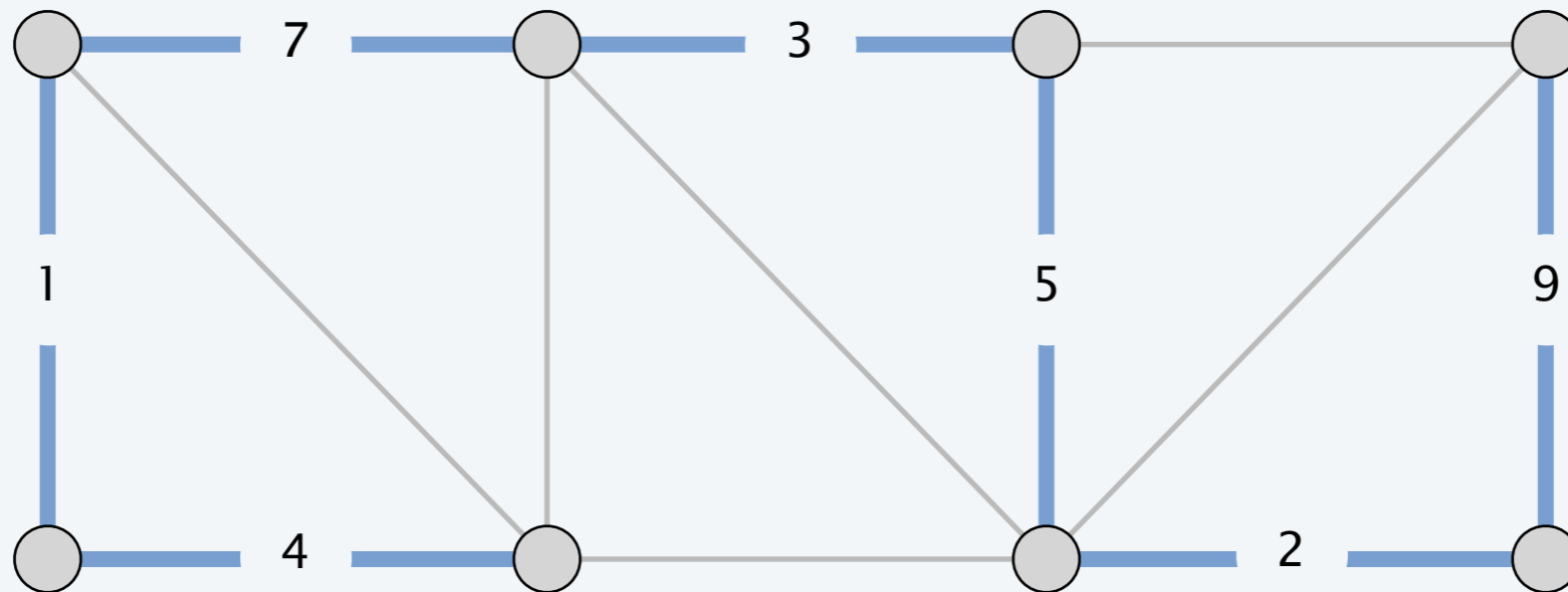
# Prim's algorithm demo

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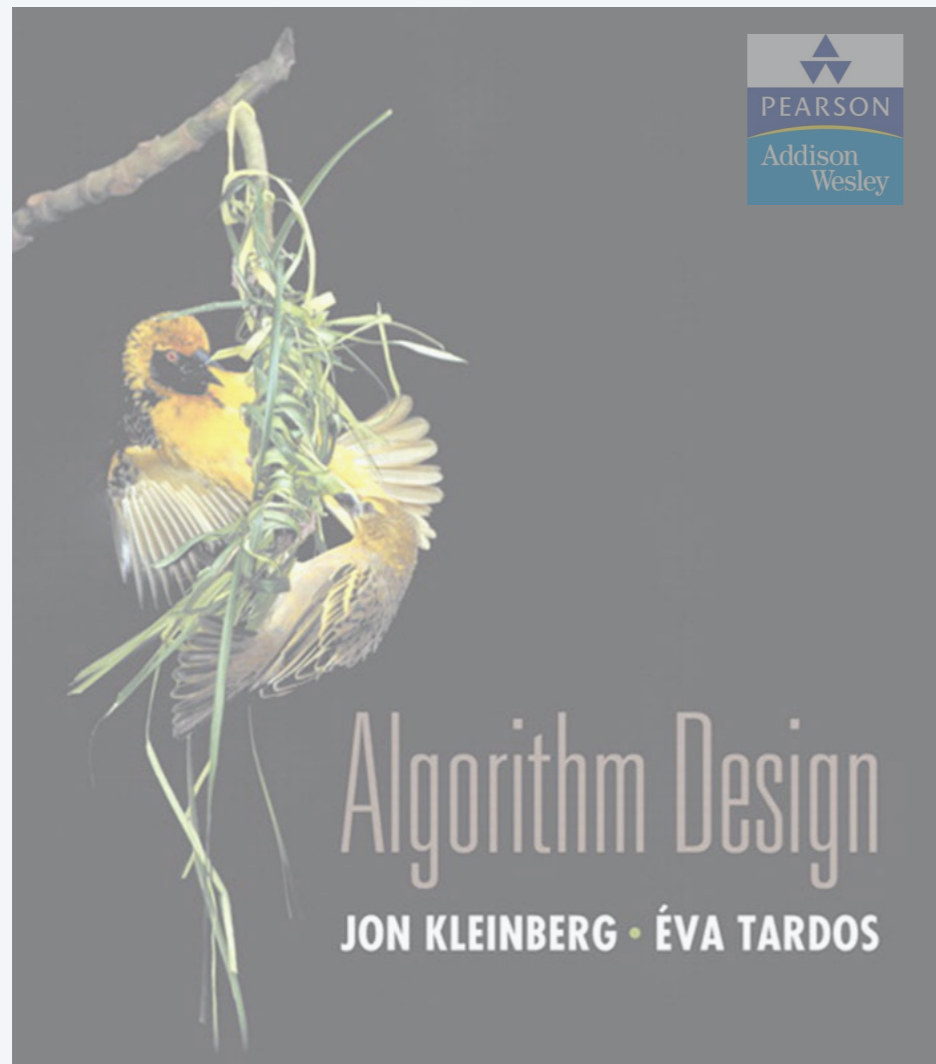
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## SECTION 4.5

# 4. GREEDY ALGORITHMS II

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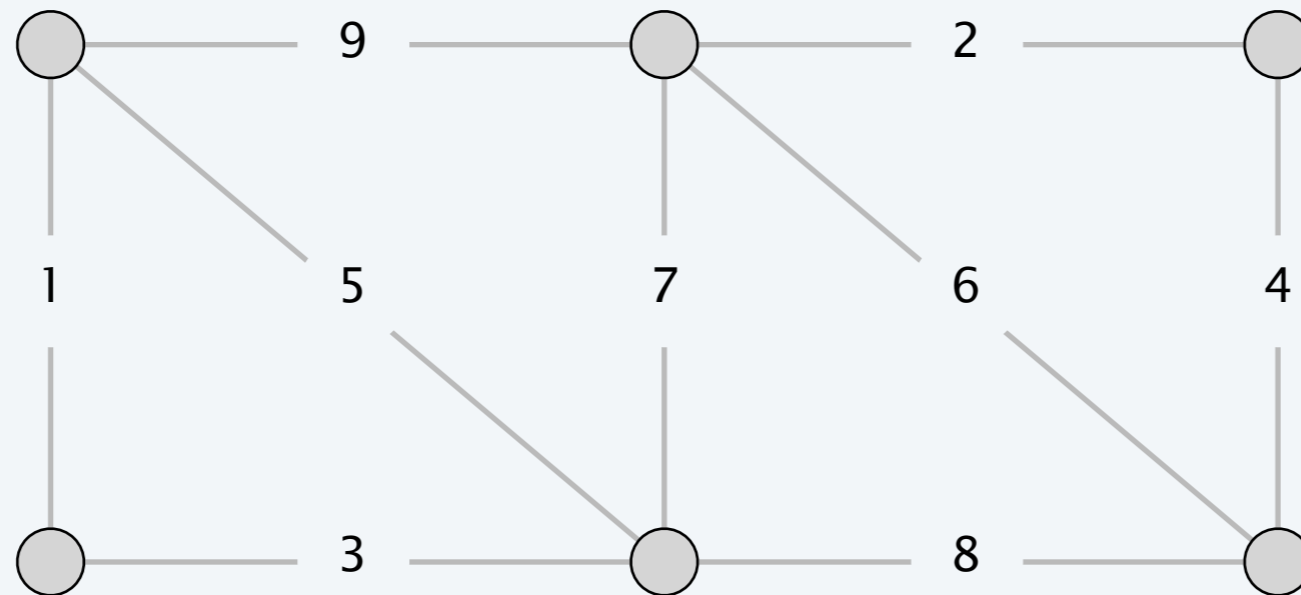
- ▶ *red-rule blue-rule demo*
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- ▶ ***Kruskal's algorithm demo***
- ▶ *reverse-delete algorithm demo*
- ▶ *Boruvka's algorithm demo*

# Kruskal's algorithm demo

---

Consider edges in ascending order of weight:

- Add to  $T$  unless it would create a cycle.

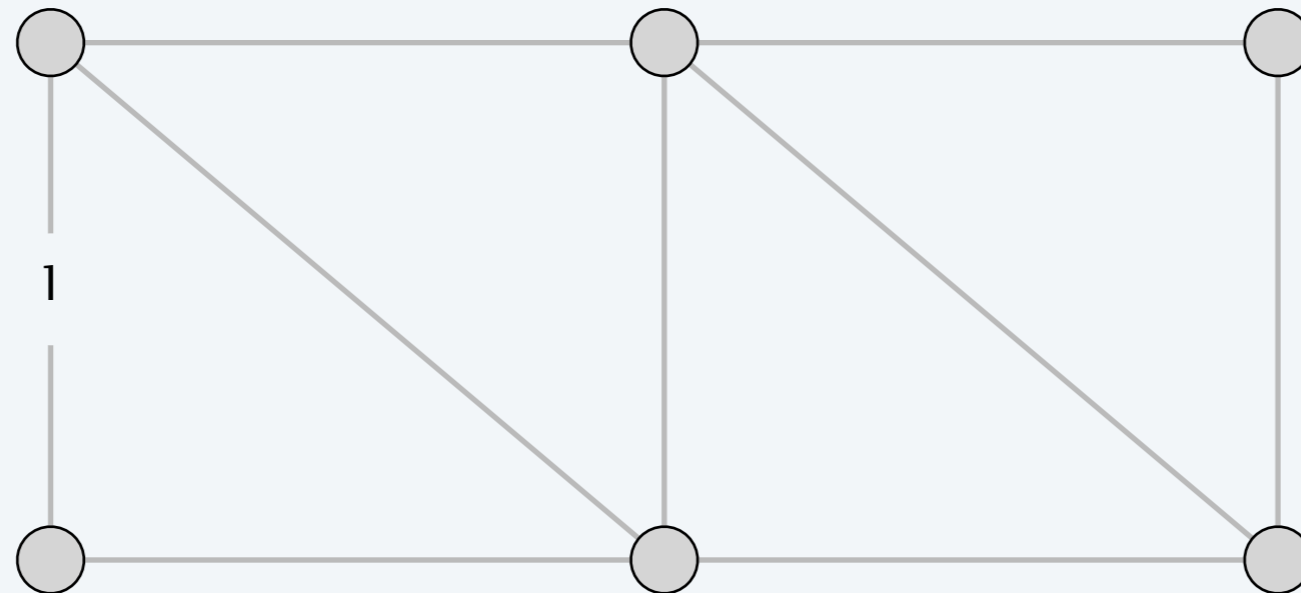


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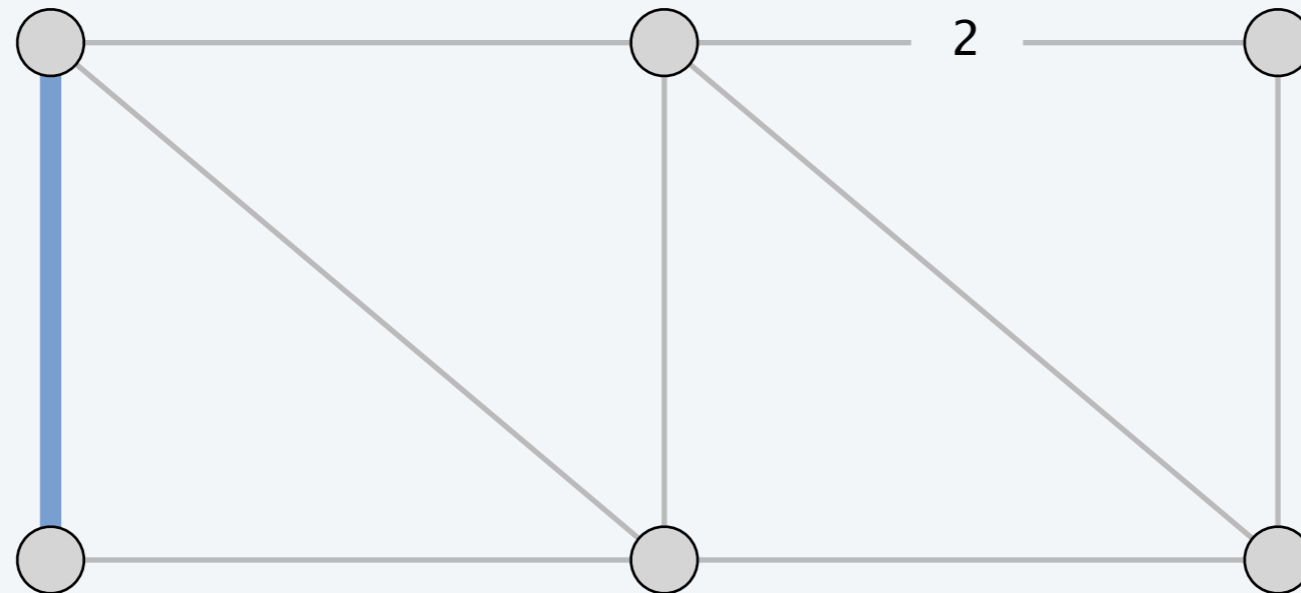


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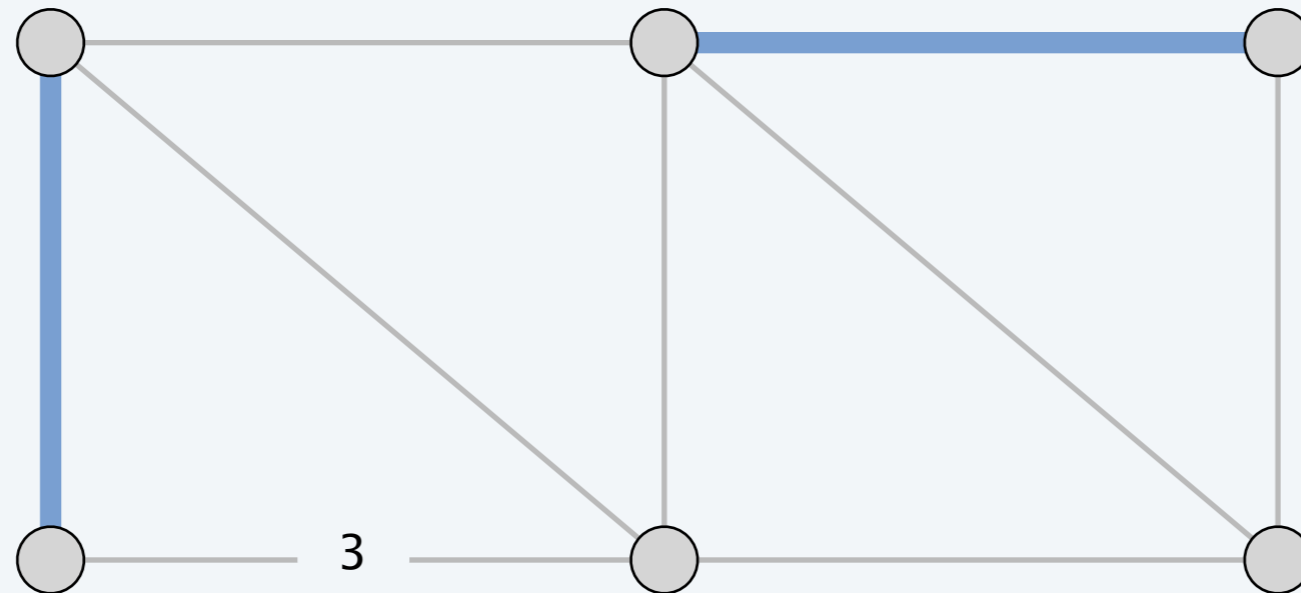


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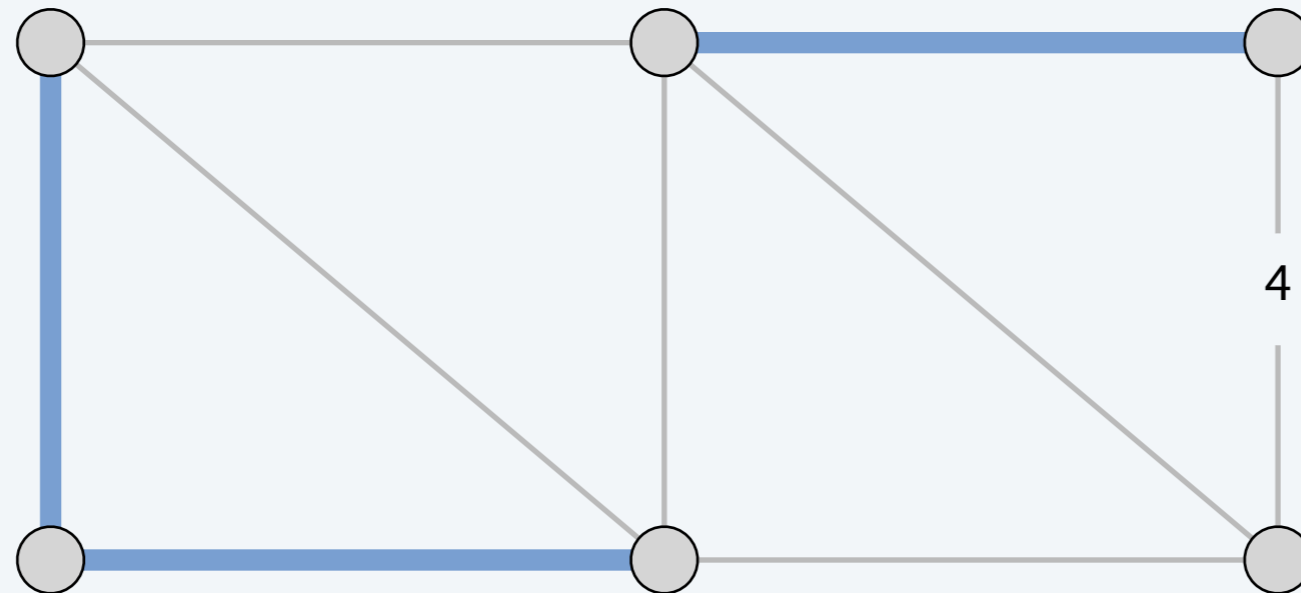


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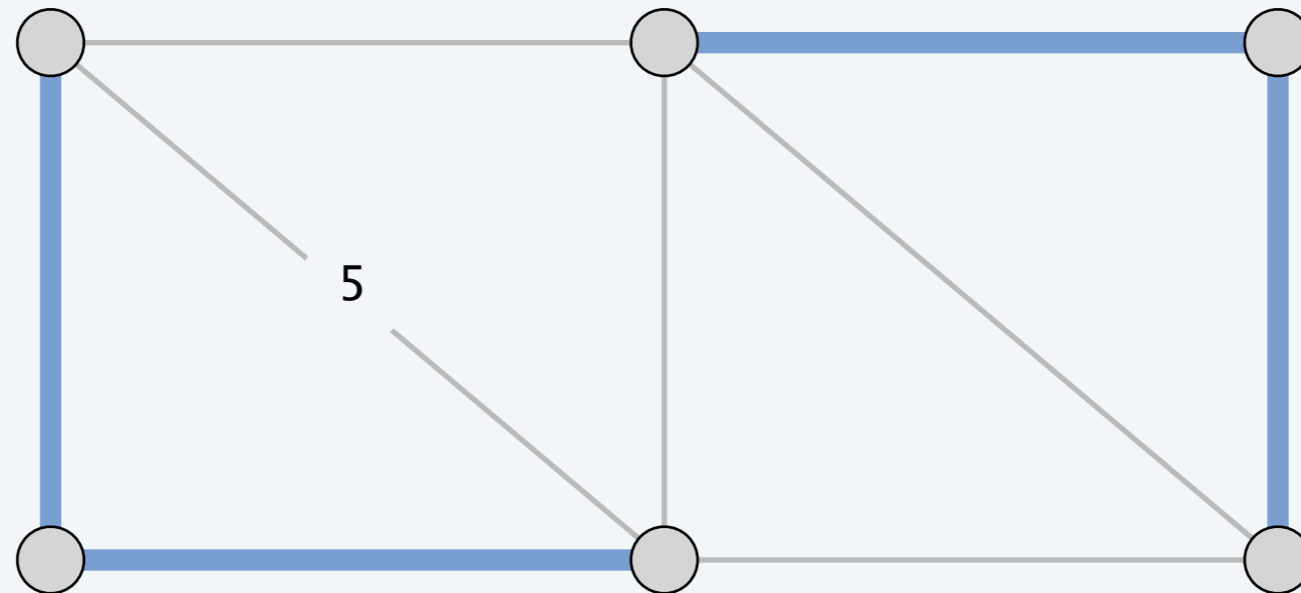


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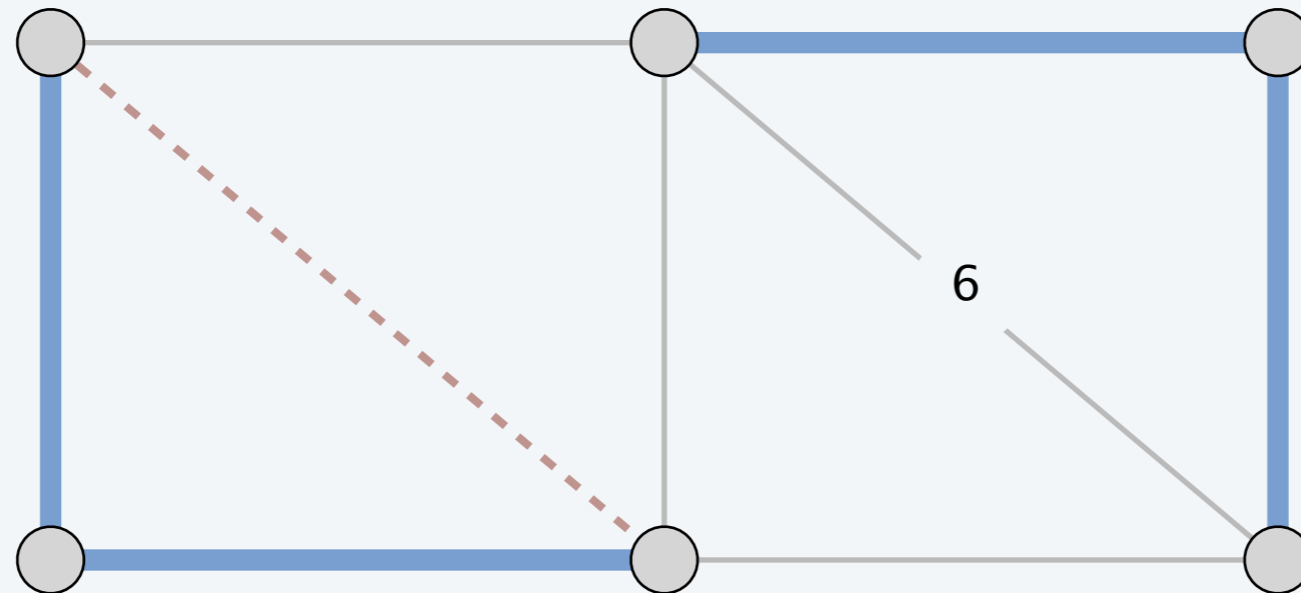


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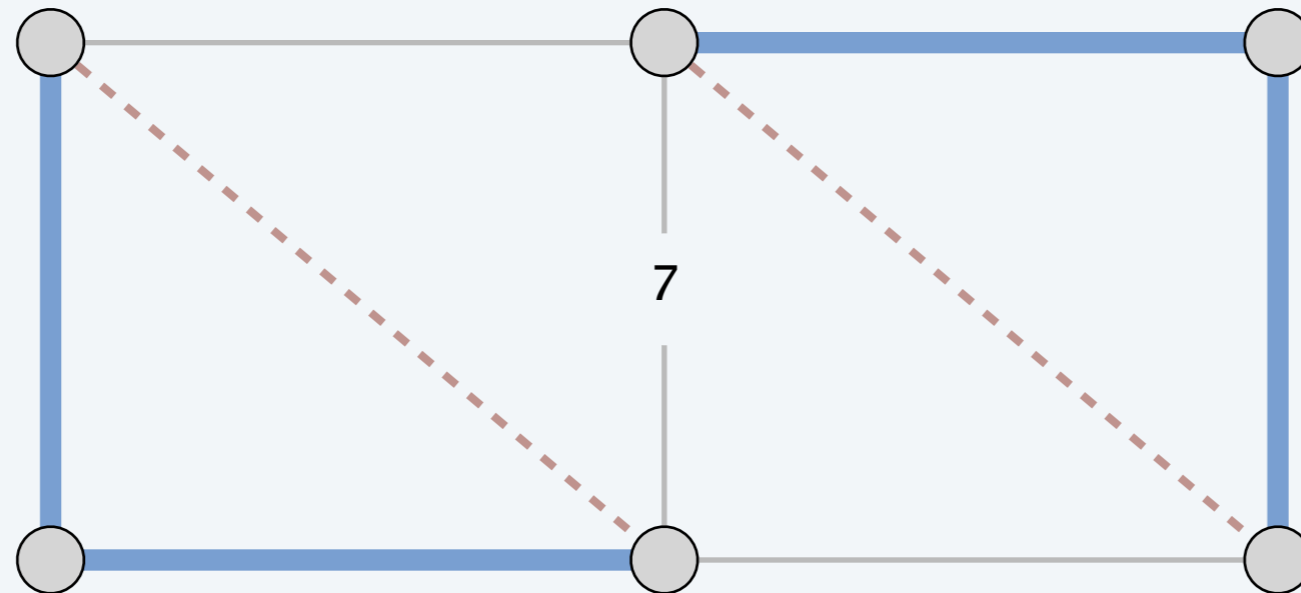


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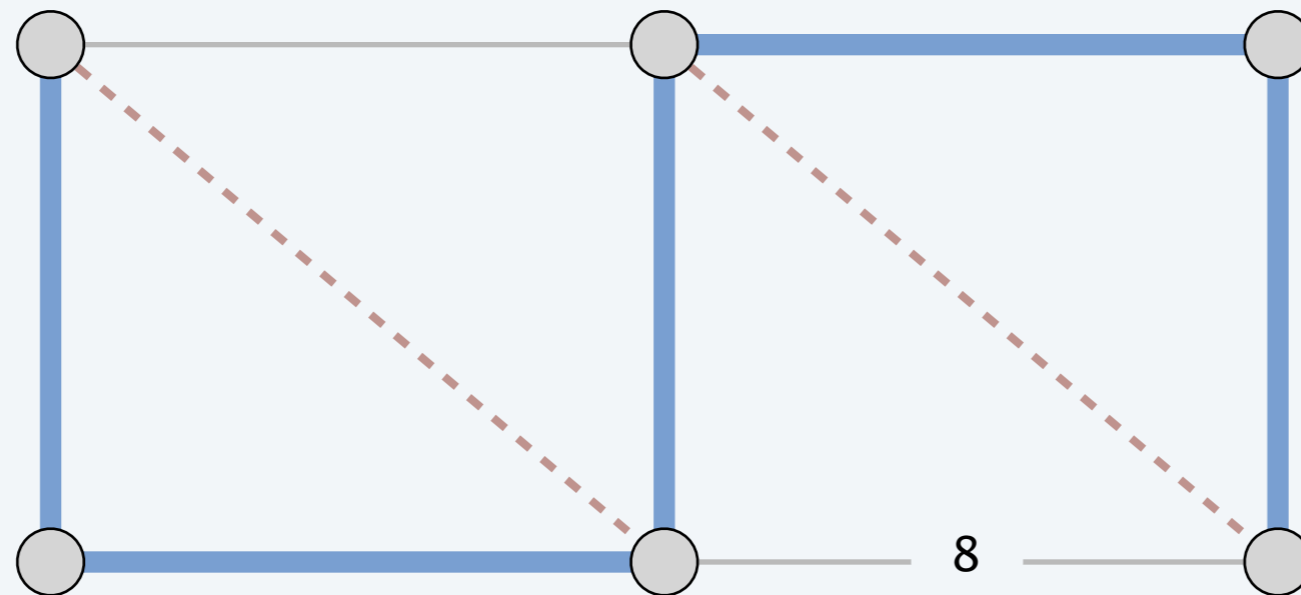


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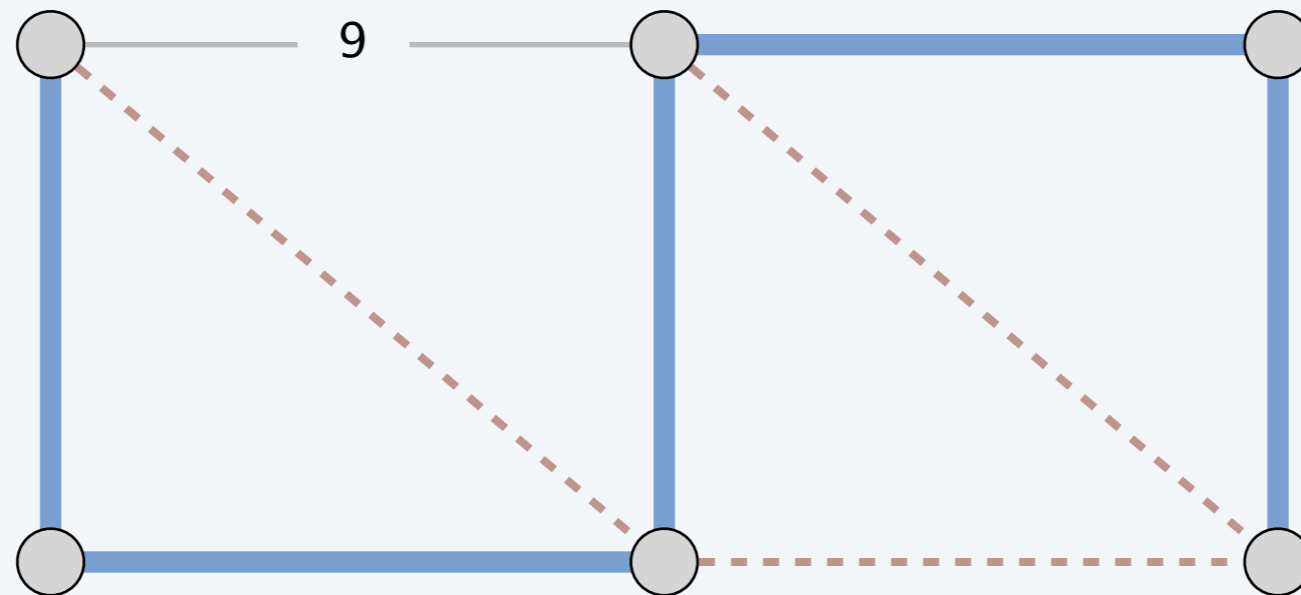


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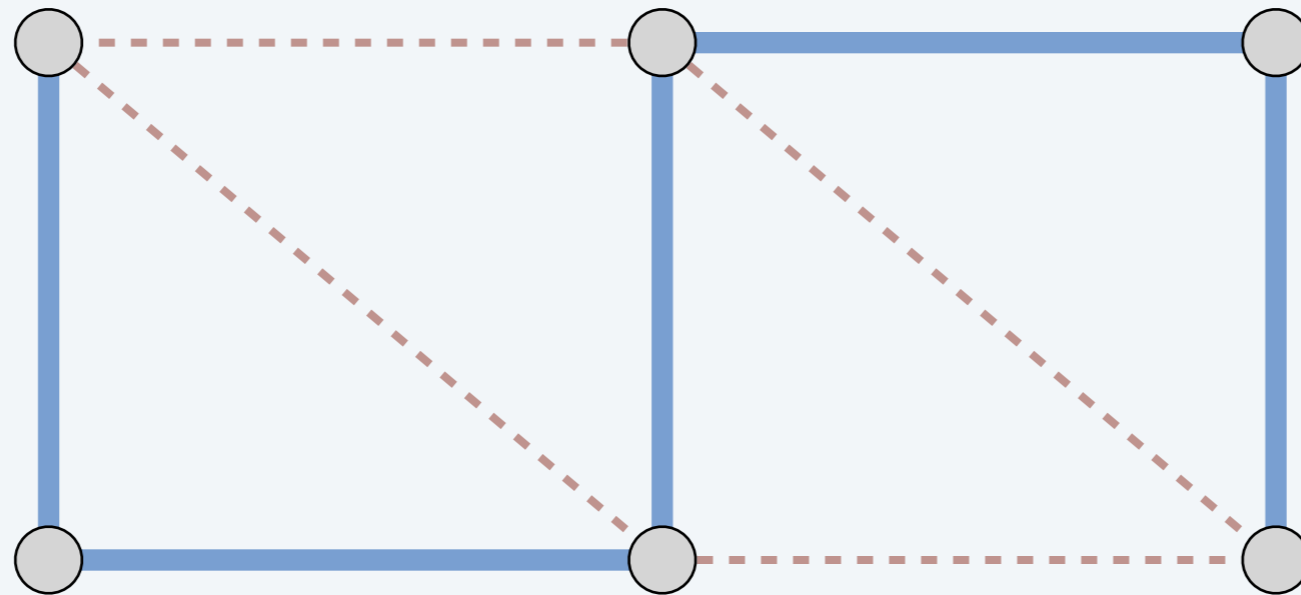


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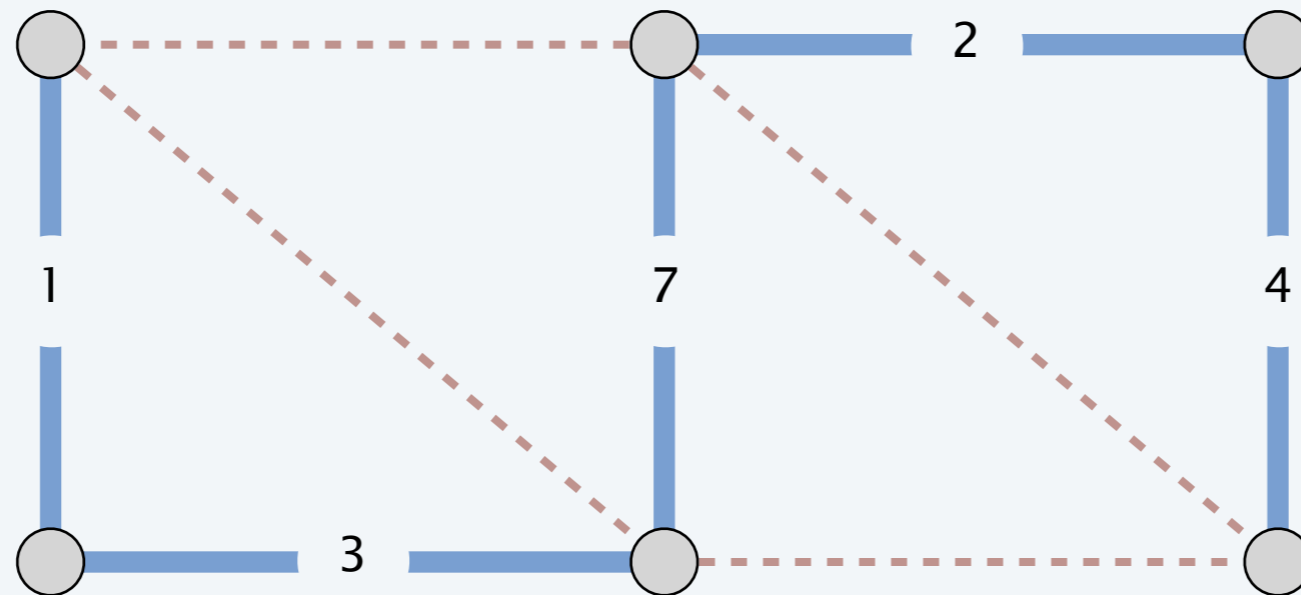


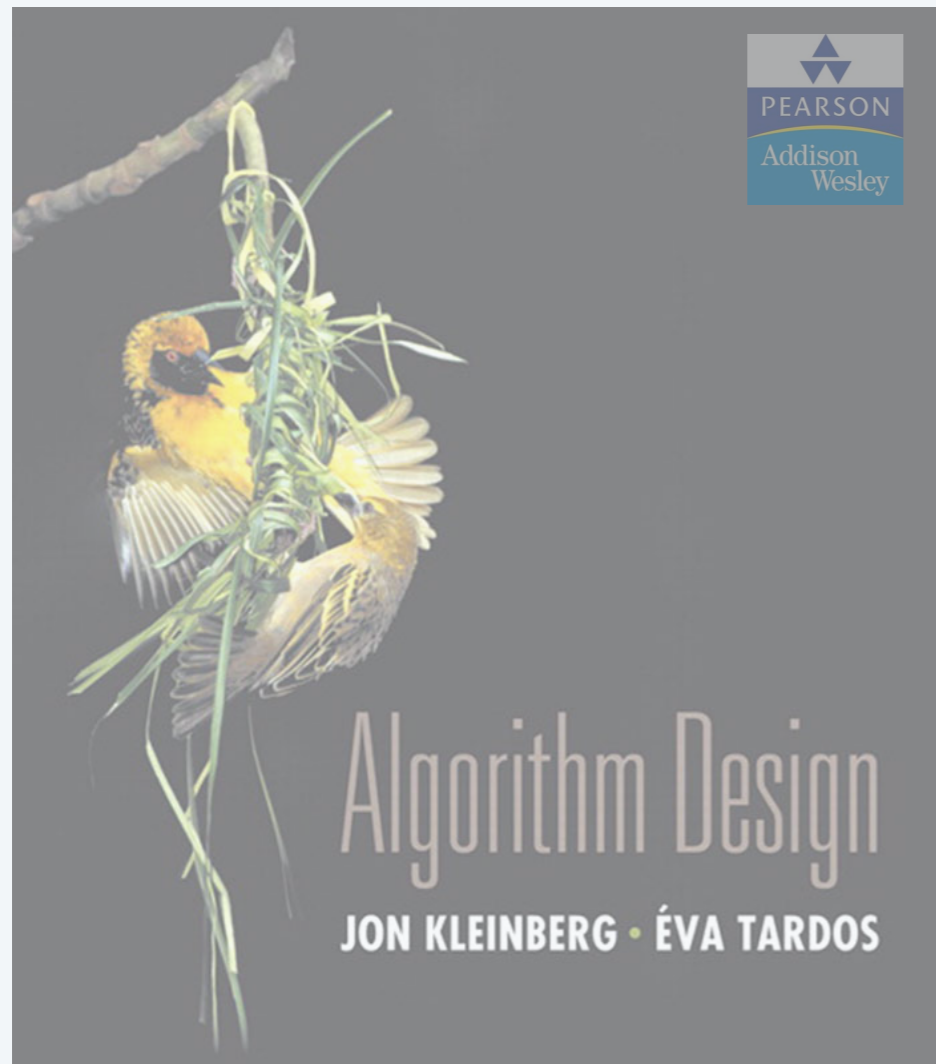
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## SECTION 4.5

# 4. GREEDY ALGORITHMS II

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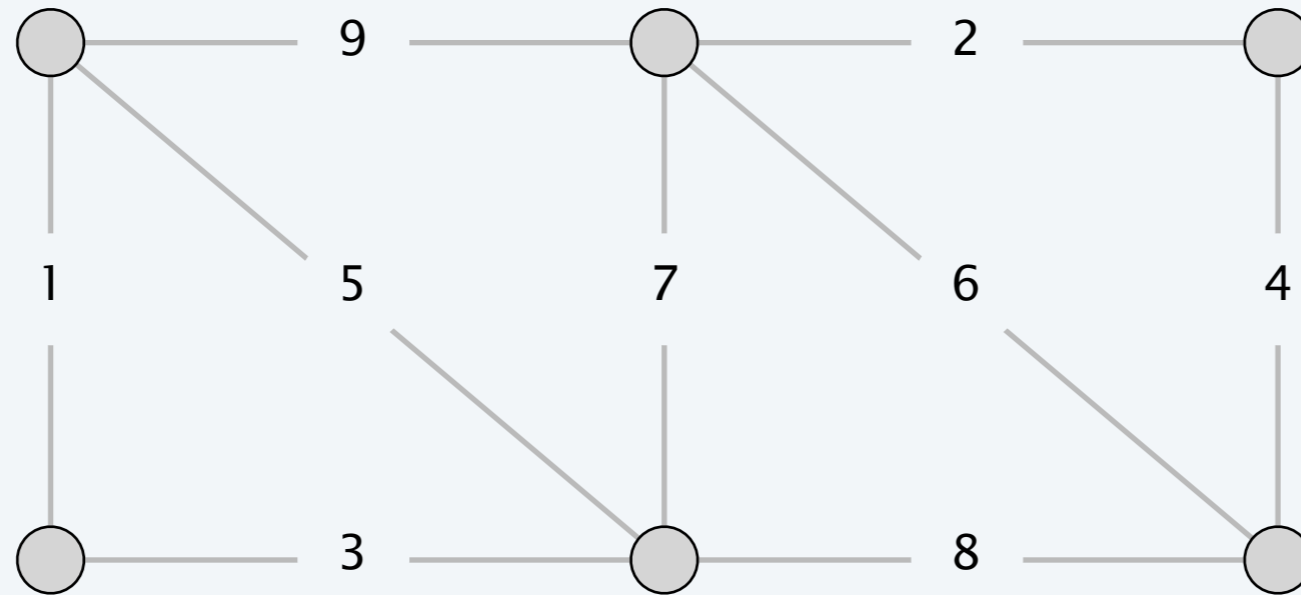
- ▶ *red-rule blue-rule demo*
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- ▶ ***reverse-delete algorithm demo***
- ▶ *Boruvka's algorithm demo*

# Reverse-delete algorithm demo

---

Start with all edges in  $T$  and consider them in descending order of weight:

- Delete edge from  $T$  unless it would disconnect  $T$ .

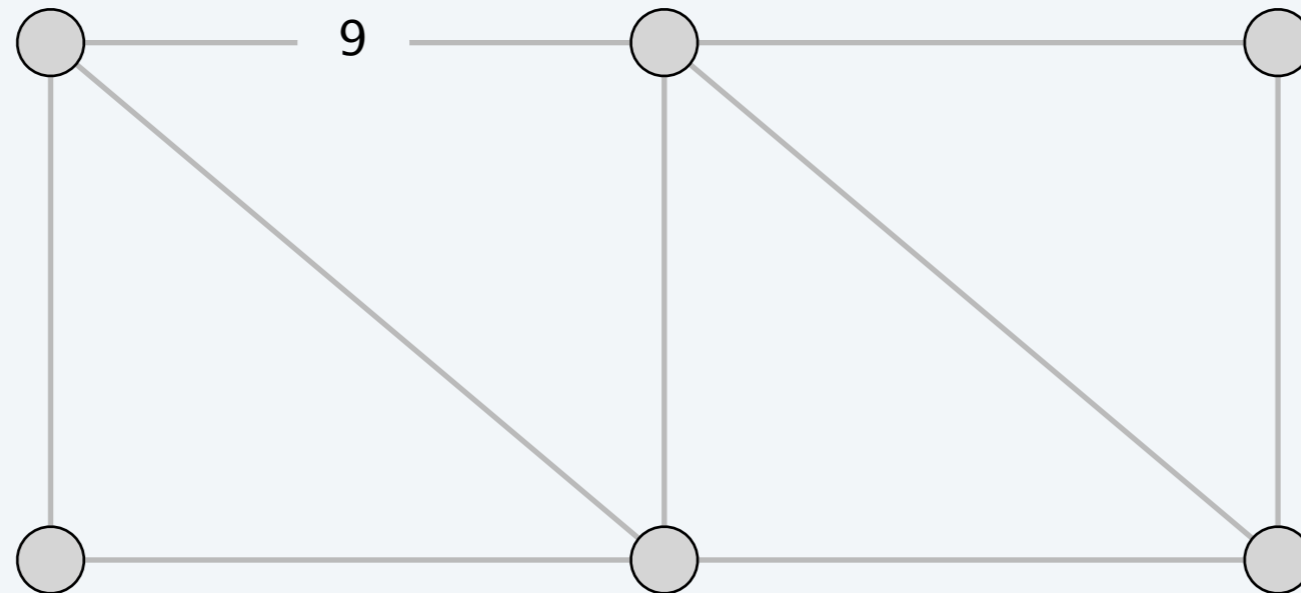


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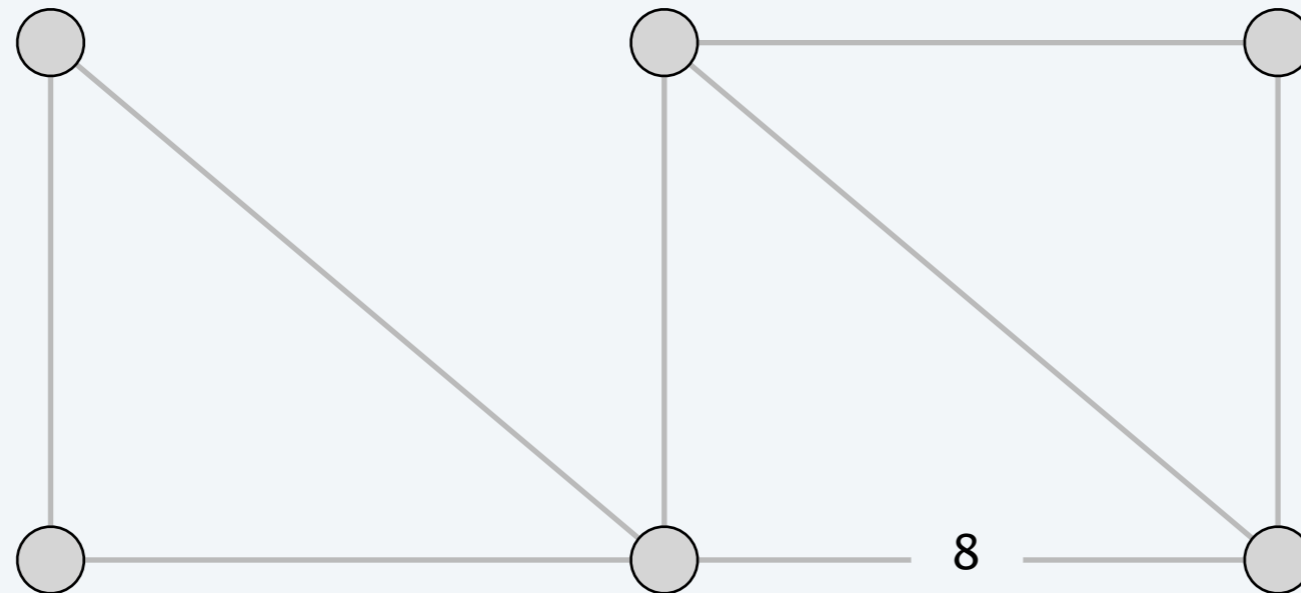


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Start with all edges in  $T$  and consider them in descending order of weight:

- Delete edge from  $T$  unless it would disconnect  $T$ .

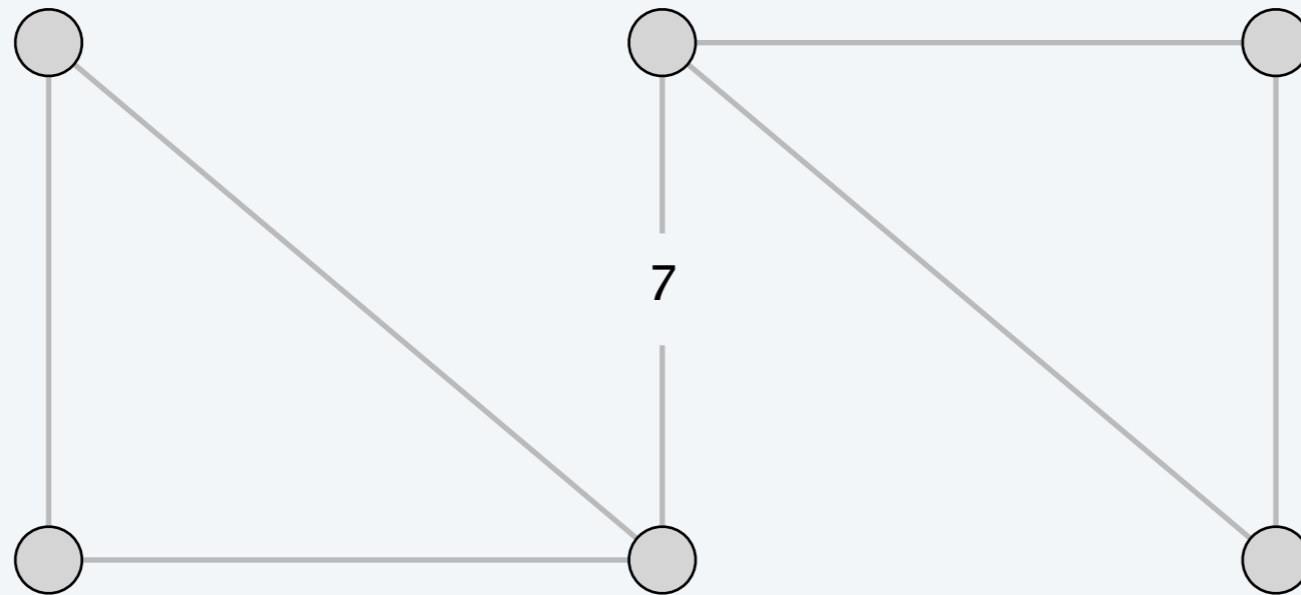


# Reverse-delete algorithm

---

Start with all edges in  $T$  and consider them in descending order of weight:

- Delete edge from  $T$  unless it would disconnect  $T$ .

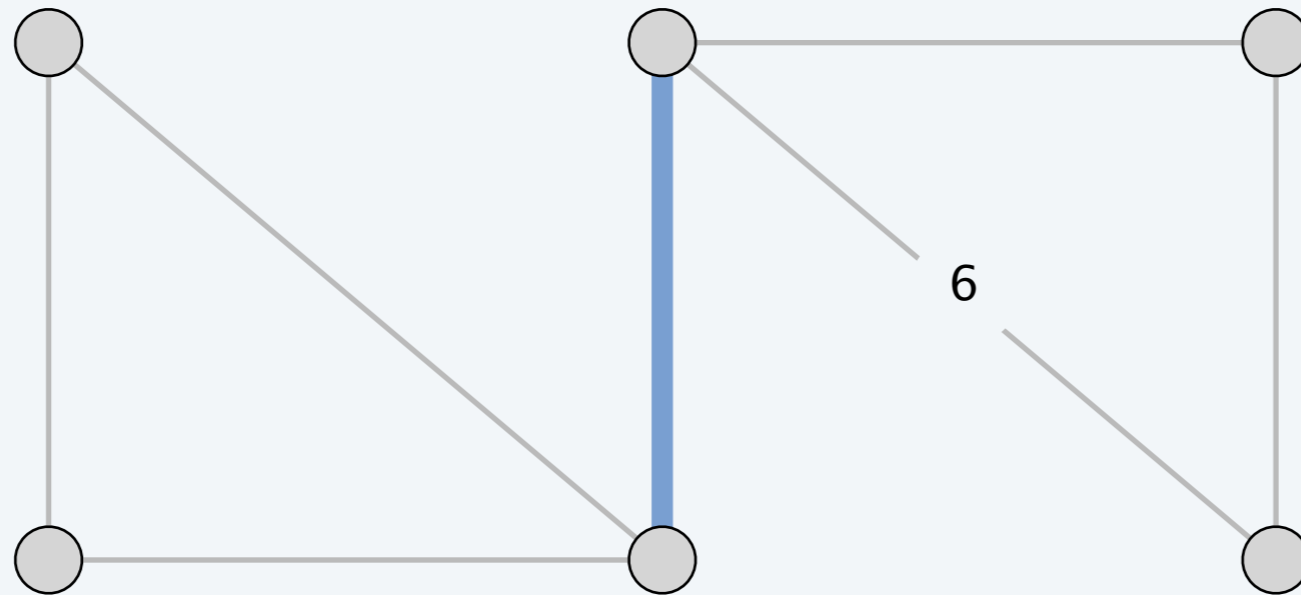


# Reverse-delete algorithm

---

Start with all edges in  $T$  and consider them in descending order of weight:

- Delete edge from  $T$  unless it would disconnect  $T$ .

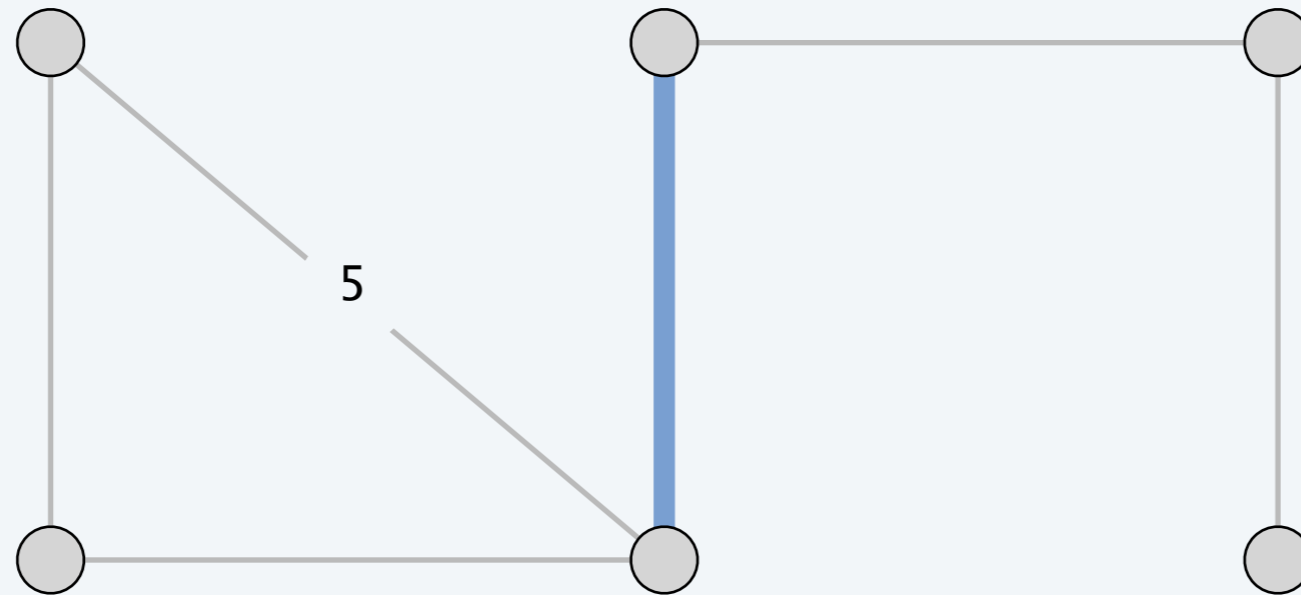


# Reverse-delete algorithm

---

Start with all edges in  $T$  and consider them in descending order of weight:

- Delete edge from  $T$  unless it would disconnect  $T$ .

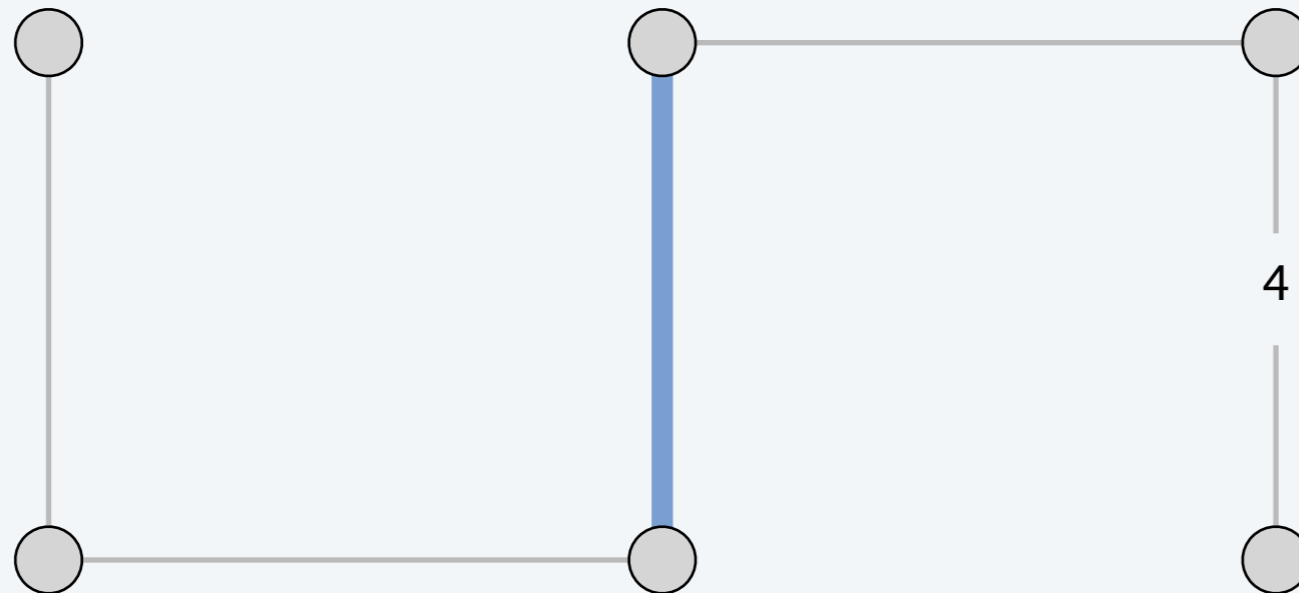


# Reverse-delete algorithm

---

Start with all edges in  $T$  and consider them in descending order of weight:

- Delete edge from  $T$  unless it would disconnect  $T$ .

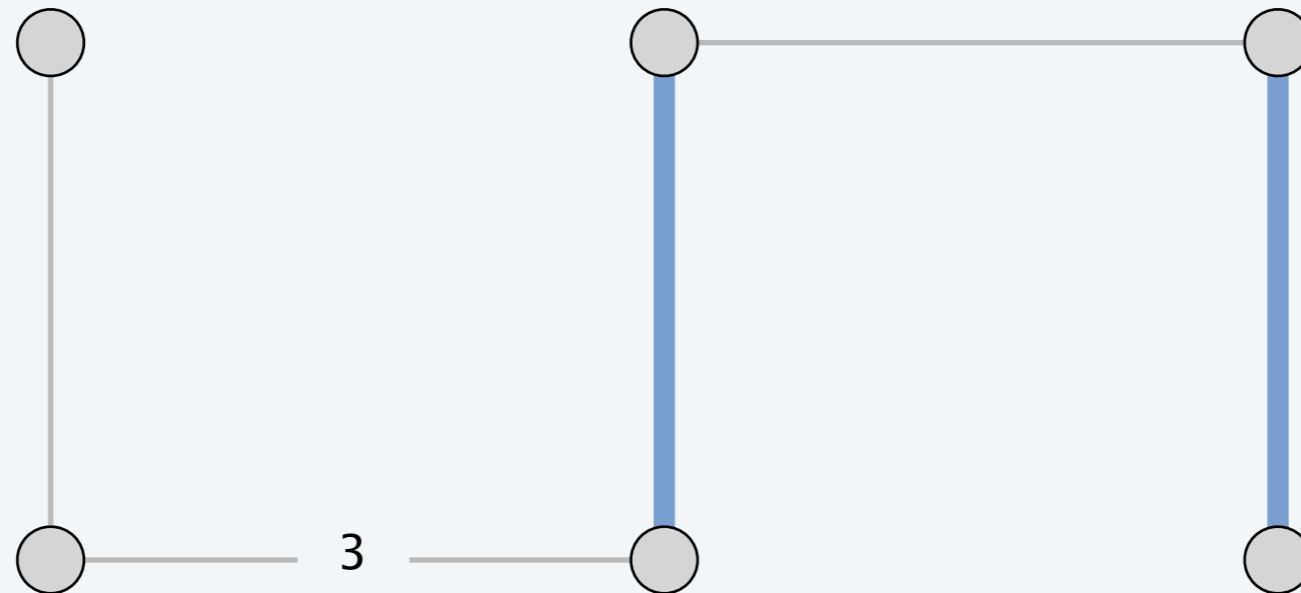


# Reverse-delete algorithm

---

Start with all edges in  $T$  and consider them in descending order of weight:

- Delete edge from  $T$  unless it would disconnect  $T$ .

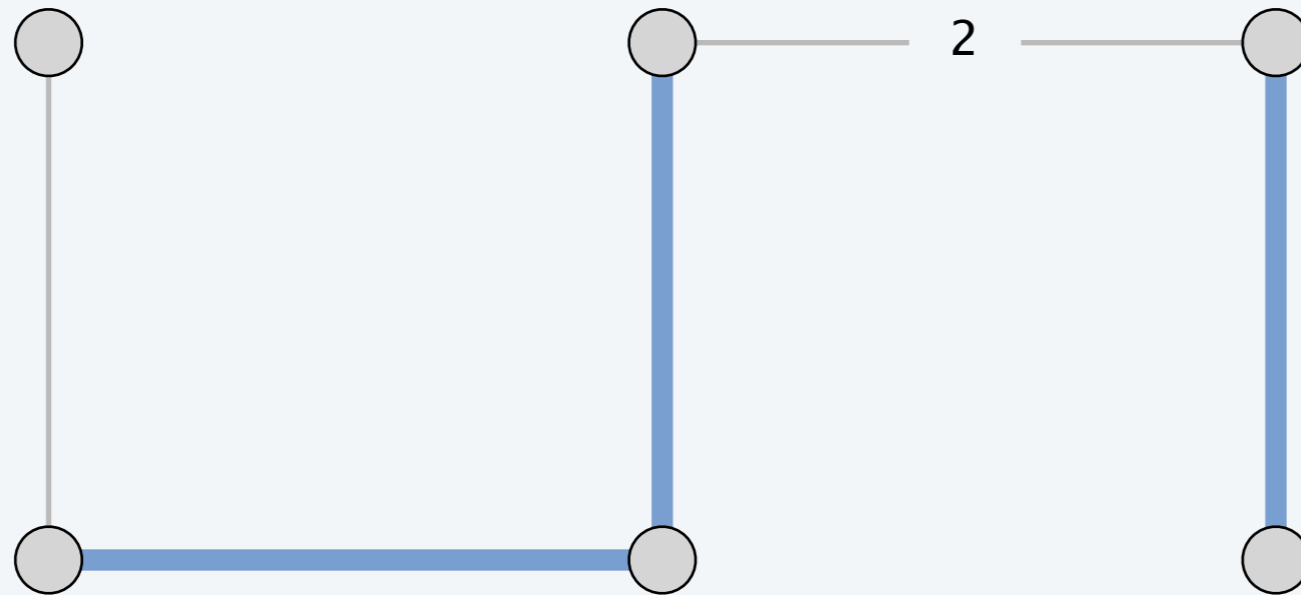


# Reverse-delete algorithm

---

Start with all edges in  $T$  and consider them in descending order of weight:

- Delete edge from  $T$  unless it would disconnect  $T$ .

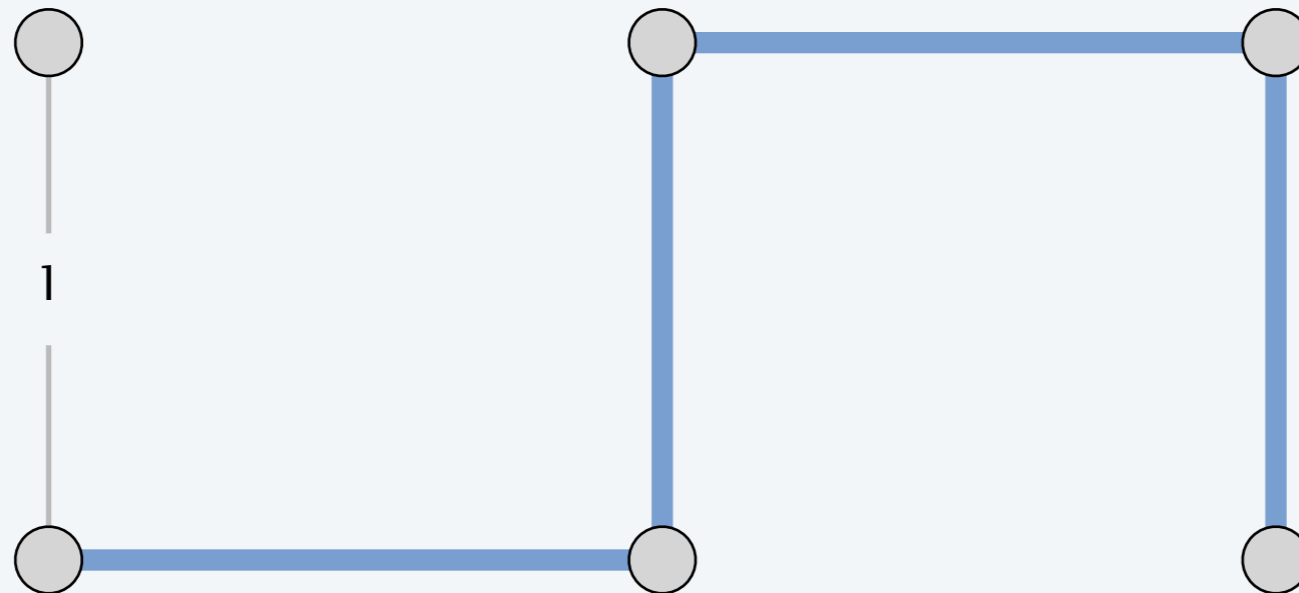


# Reverse-delete algorithm

---

Start with all edges in  $T$  and consider them in descending order of weight:

- Delete edge from  $T$  unless it would disconnect  $T$ .



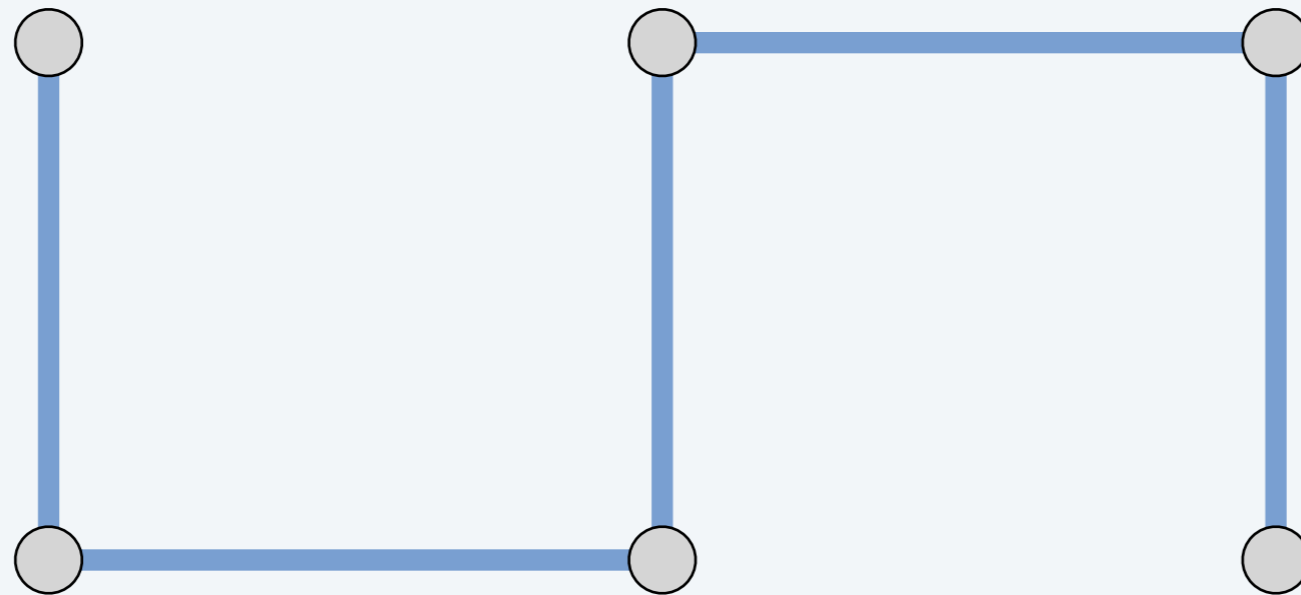


# Reverse-delete algorithm

---

Start with all edges in  $T$  and consider them in descending order of weight:

- Delete edge from  $T$  unless it would disconnect  $T$ .

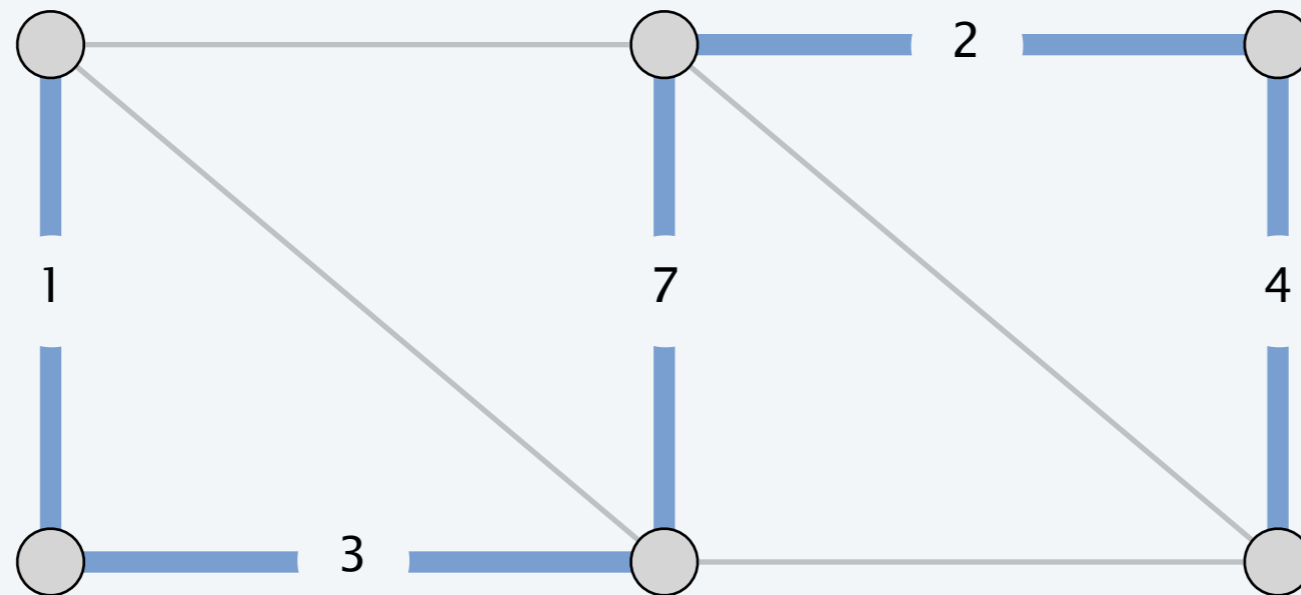


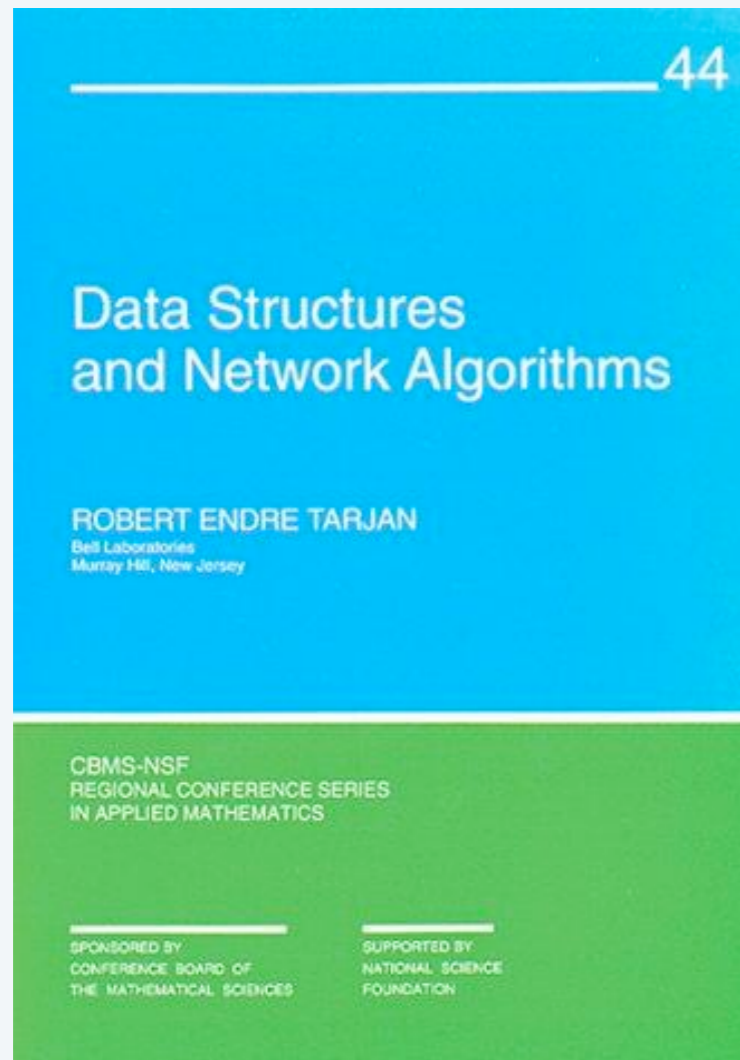
# Reverse-delete algorithm

---

Start with all edges in  $T$  and consider them in descending order of weight:

- Delete edge from  $T$  unless it would disconnect  $T$ .





## SECTION 6.2

# 4. GREEDY ALGORITHMS II

---

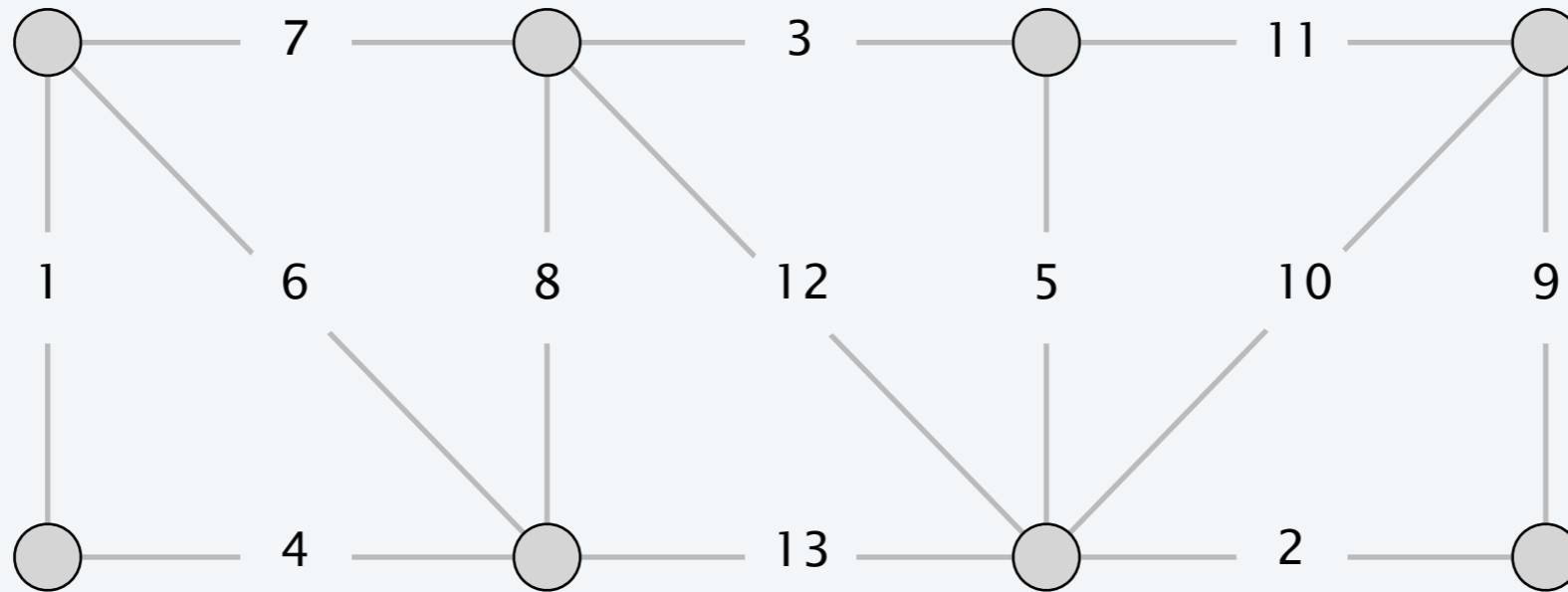
- ▶ *red-rule blue-rule demo*
- ▶ *Prim's algorithm demo*
- ▶ *Kruskal's algorithm demo*
- ▶ *reverse-delete algorithm demo*
- ▶ *Boruvka's algorithm demo*

# Borůvka's algorithm demo

---

Repeat until only one tree.

- Apply blue rule to cutset corresponding to **each** blue tree.
- Color **all** selected edges blue.

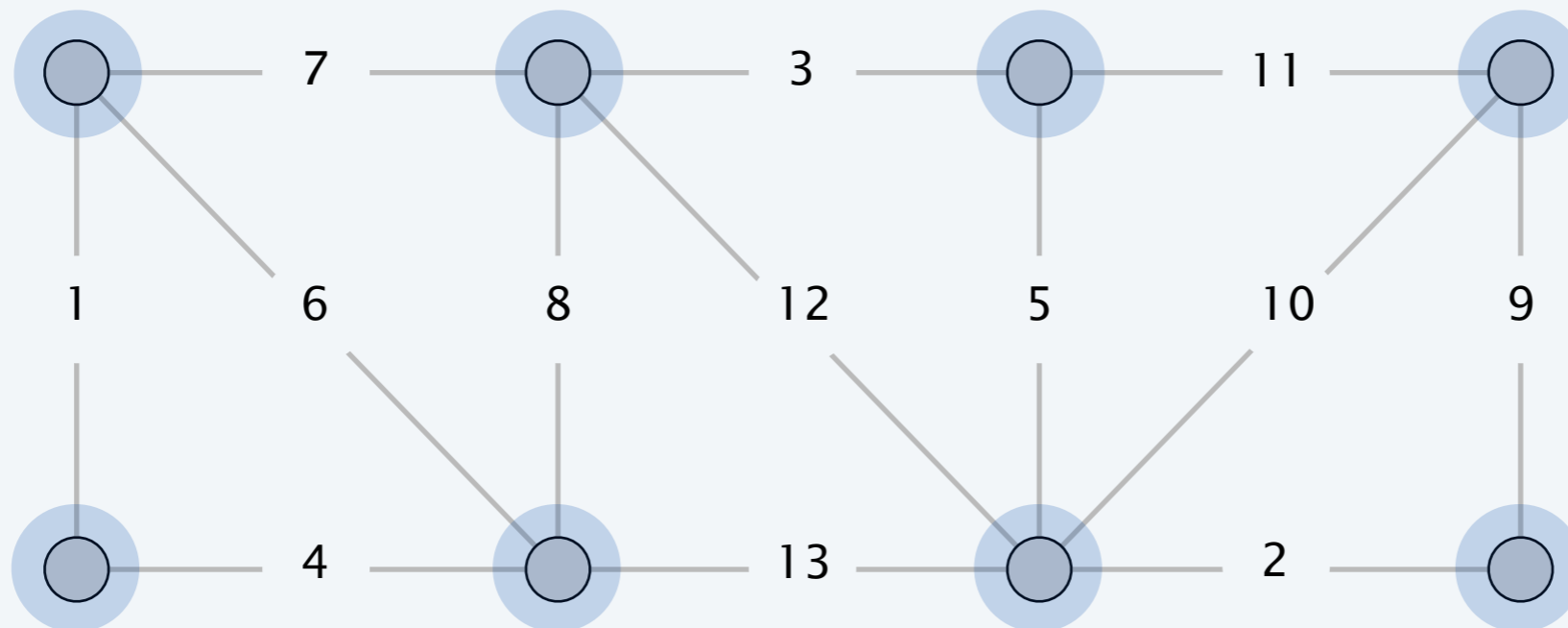


# Borůvka's algorithm demo

---

Repeat until only one tree.

- Apply blue rule to cutset corresponding to **each** blue tree.
- Color **all** selected edges blue.

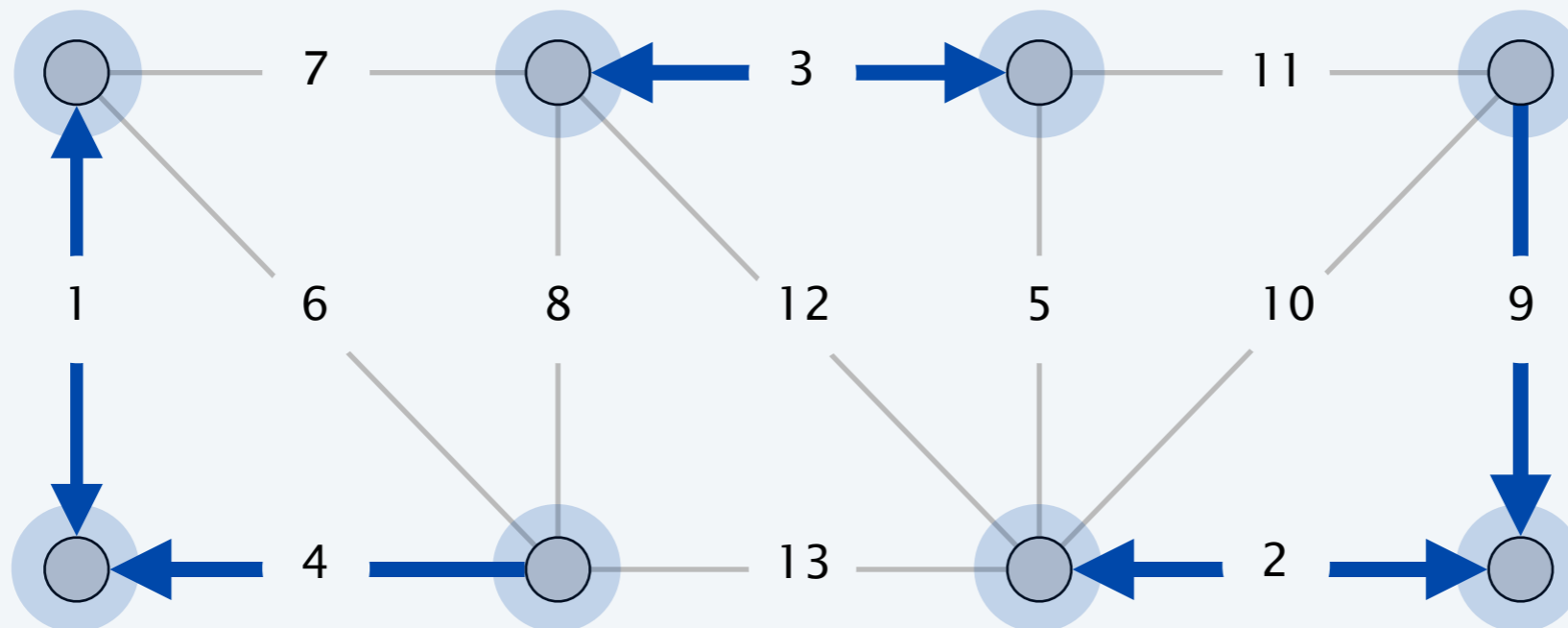


# Borůvka's algorithm demo

---

Repeat until only one tree.

- Apply blue rule to cutset corresponding to **each** blue tree.
- Color **all** selected edges blue.

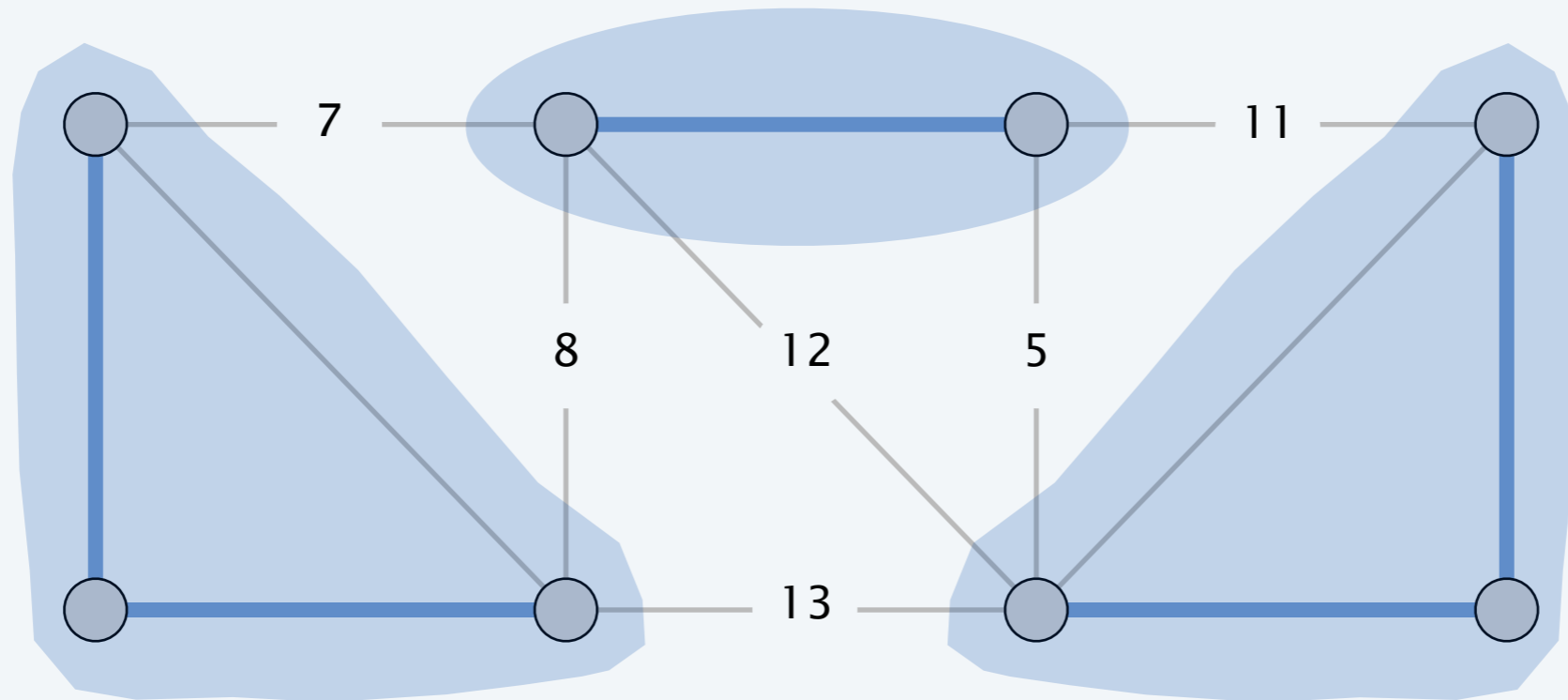


# Borůvka's algorithm demo

---

Repeat until only one tree.

- Apply blue rule to cutset corresponding to **each** blue tree.
- Color **all** selected edges blue.

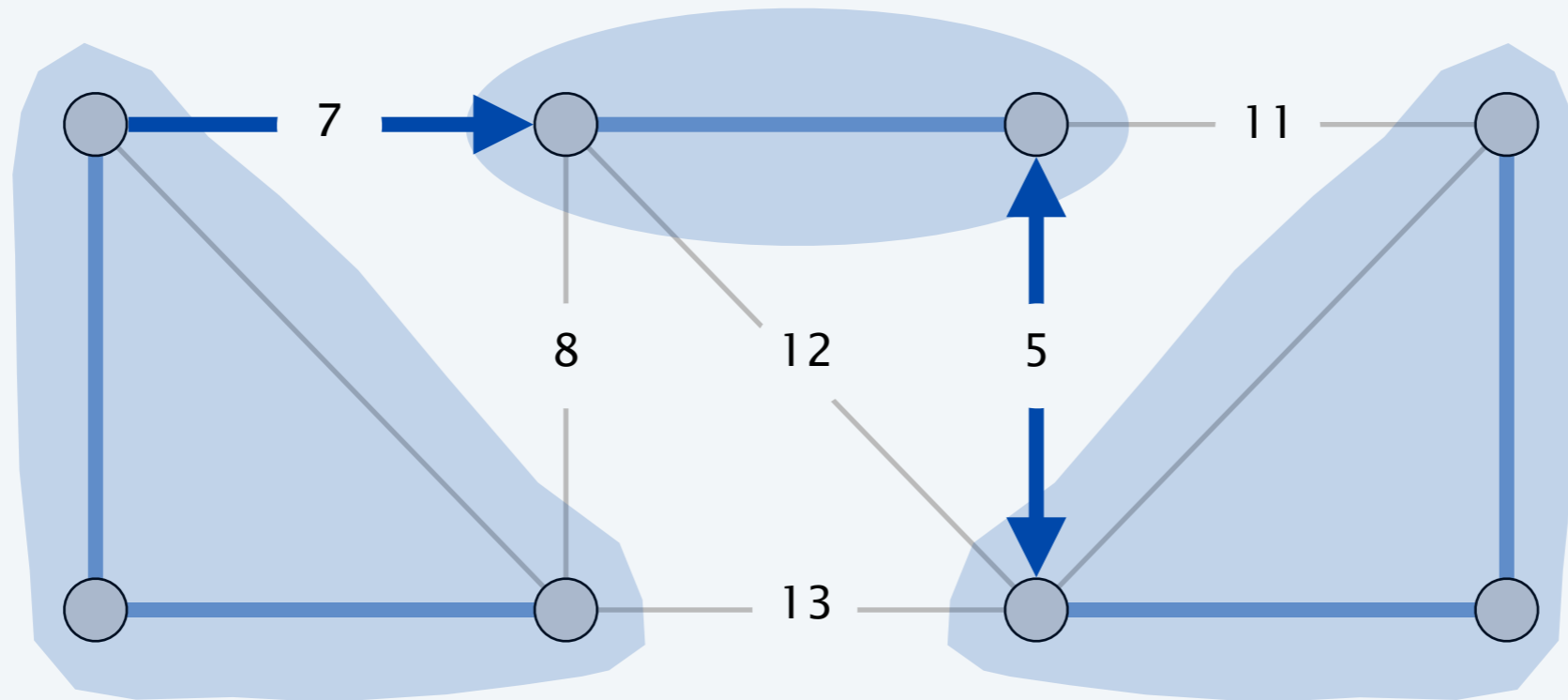


# Borůvka's algorithm demo

---

Repeat until only one tree.

- Apply blue rule to cutset corresponding to **each** blue tree.
- Color **all** selected edges blue.



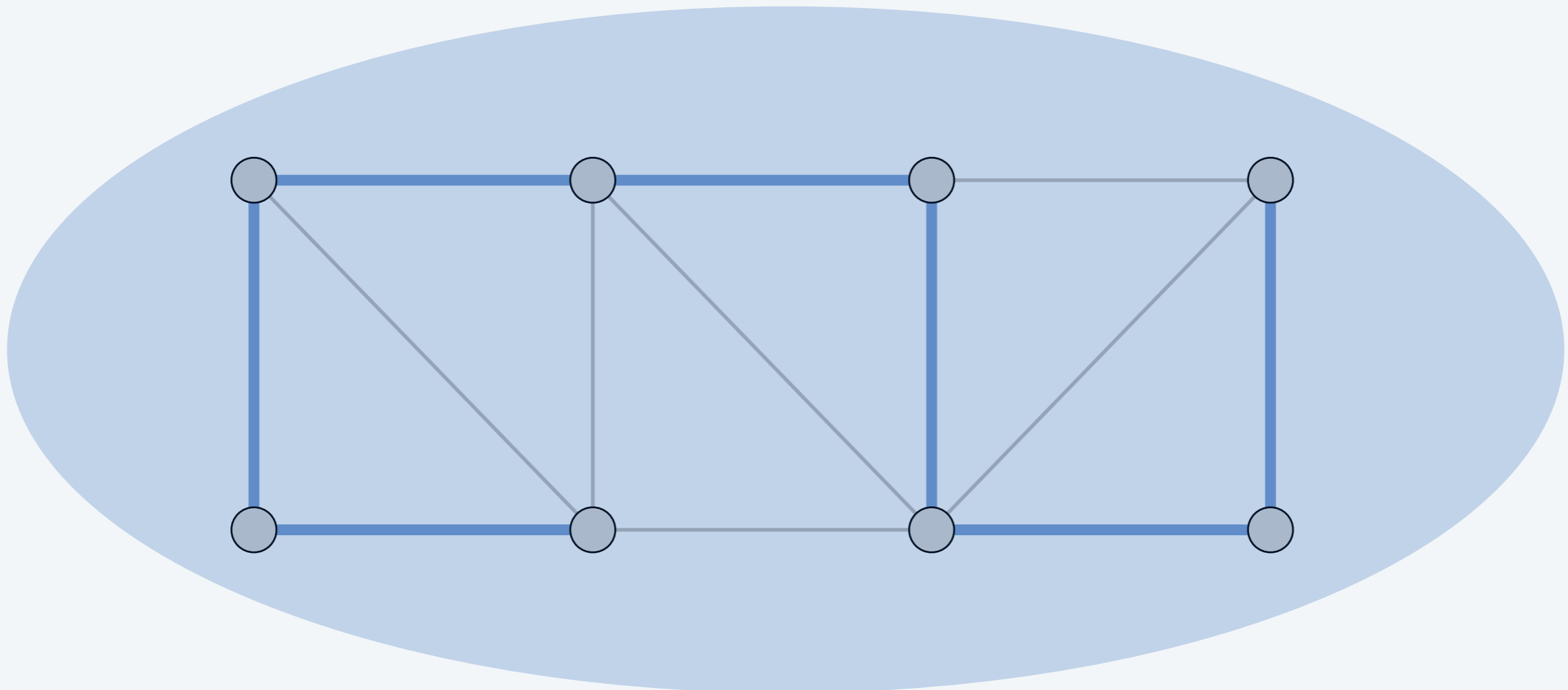


# Borůvka's algorithm demo

---

Repeat until only one tree.

- Apply blue rule to cutset corresponding to **each** blue tree.
- Color **all** selected edges blue.



# Borůvka's algorithm demo

---

Repeat until only one tree.

- Apply blue rule to cutset corresponding to **each** blue tree.
- Color **all** selected edges blue.

