SOC245: Visualizing Data
Precept 9: Characterizing Associations

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Outline

1. Linear Regression using OLS
    - Reviewing OLS
    - Computing OLS
2. Probability
    - Probability review
3. Computing probability of events in `R`

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3. Computing probability of events in `R`

Reviewing OLS

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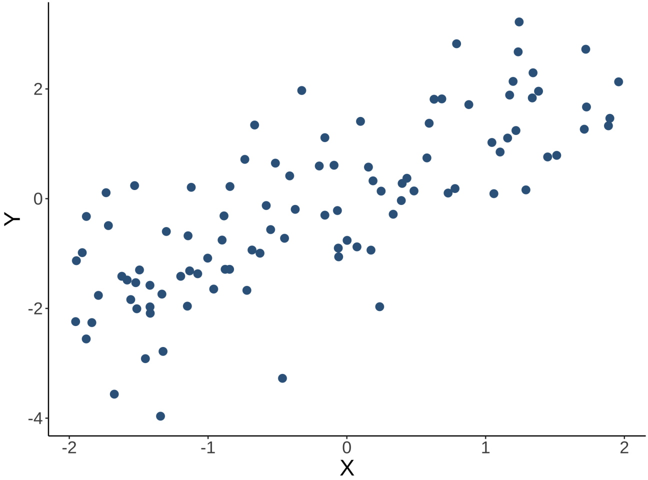

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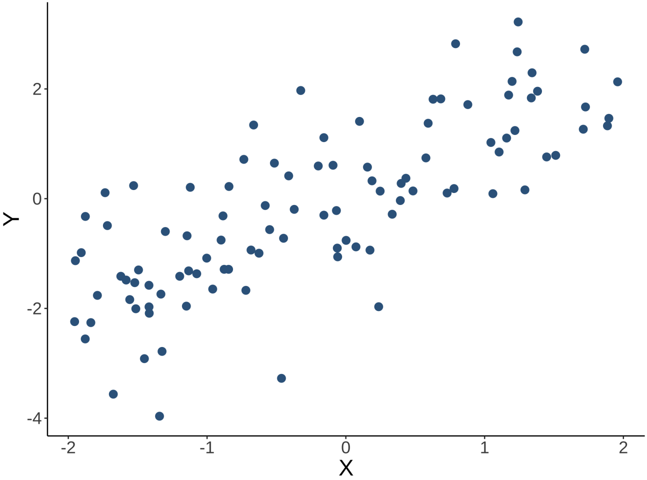
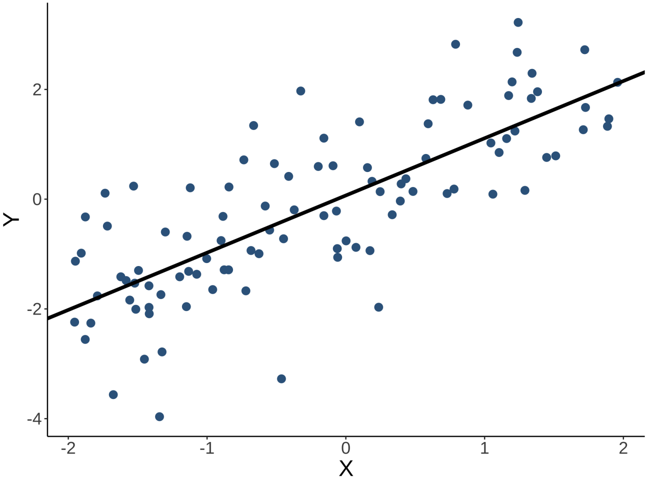


How do we find a line that best describes this data?

Reviewing OLS

Want to minimize the **sum of squared residuals (SSR)**.



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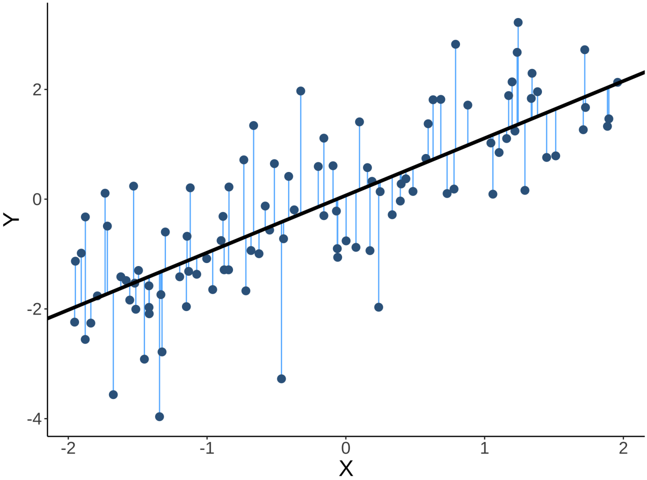

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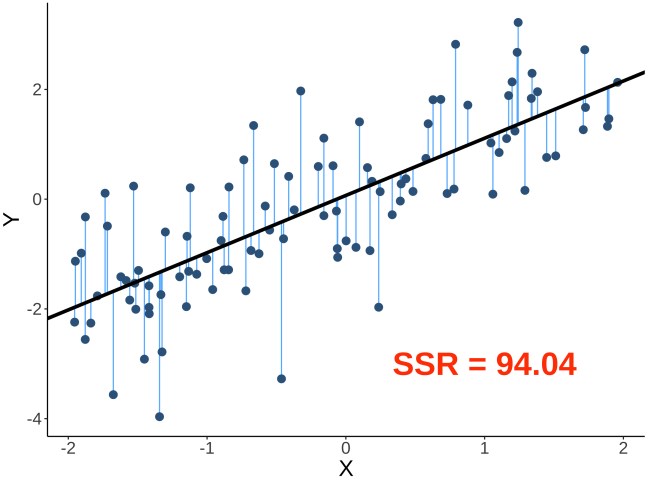

Reviewing OLS

- Our line is of the form $\widehat{Y} = a + bX$.
- Values for a and b that minimize the SSR are:

$$b = \frac{\widehat{\text{Cov}}(X, Y)}{\hat{\sigma}_X^2}$$

$$a = \overline{Y} - \frac{\widehat{\text{Cov}}(X, Y)}{\hat{\sigma}_X^2}\overline{X}$$

Let's work with the `oppotunity2.csv` dataset, that contains colleges along with the median income of children who graduate from it and the median income of their parents.

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Load `tidyverse` and read in this dataset.

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- Note: `cov(X, Y)` finds the covariance between variables `X` and `Y`.

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- Note: `cov(X, Y)` finds the covariance between variables `X` and `Y`.
- What's the formula for variance?

Computing OLS in R

```
b <- cov(col$par_median, col$k_median )/
        (var(col$par_median))
a <- mean(col$k_median) - b*mean(col$par_median)
```

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Let's plot this association along with the linear regression

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ggplot(col) +
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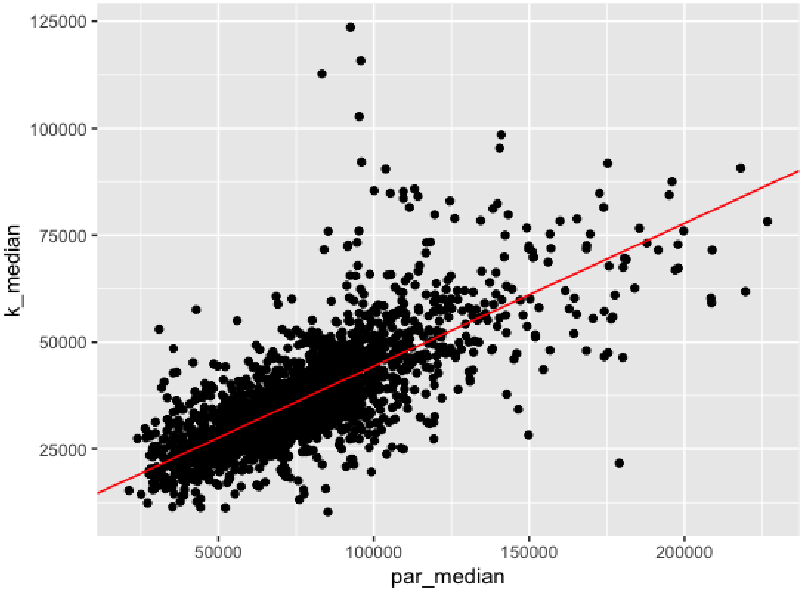
Let's plot this association along with the linear regression

```
ggplot(col) +
  geom_point(mapping =aes(x=par_median, y=k_median)) +
  geom_abline(intercept=a, slope=b, color="red")
```


125000
100000
75000
50000
25000
k_median
50000
100000
150000
200000
par_median

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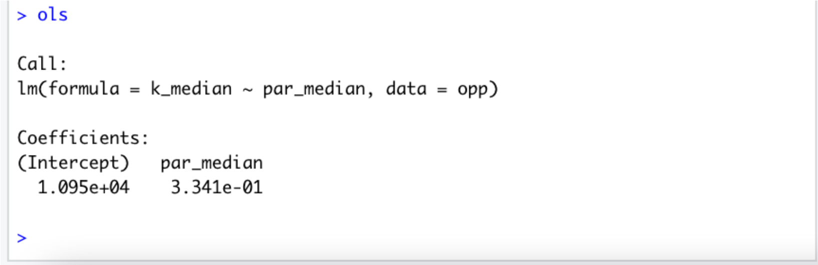
Look at `ols`. What does it contain?

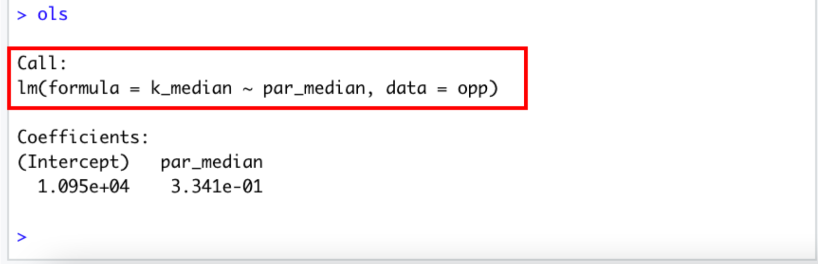


```
> ols

Call:
lm(formula = k_median ~ par_median, data = opp)

Coefficients:
(Intercept)   par_median
  1.095e+04    3.341e-01

>
```



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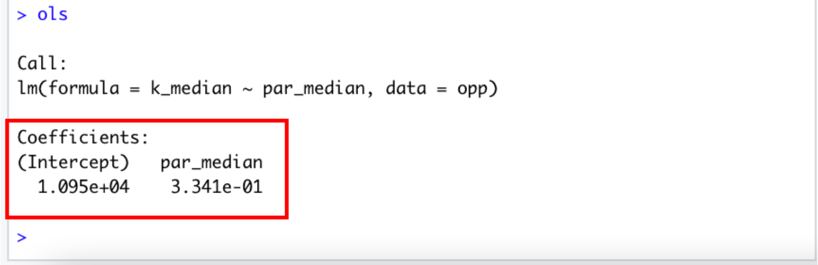
```

We have what the regression model was trained on . . .



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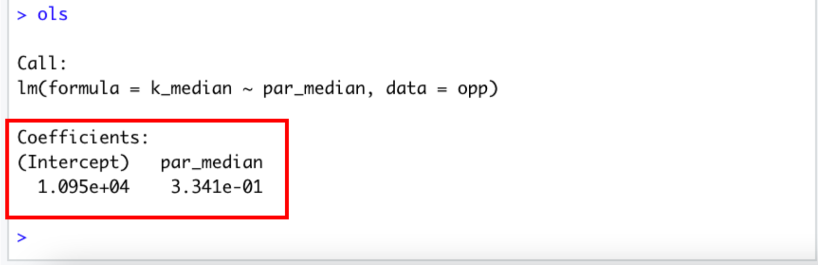
. . . and the coefficients.



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. . . and the coefficients. (Are these what you found before?)

Now, try running `summary(ols)`

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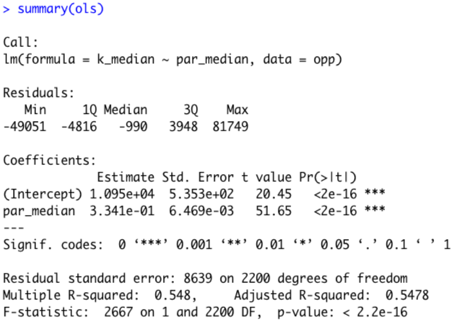

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> summary(ols)

Call:
lm(formula = k_median ~ par_median, data = opp)

Residuals:
   Min     1Q Median     3Q    Max 
-49051  -4816   -990   3948  81749 

Coefficients:
             Estimate Std. Error t value Pr(>|t|)    
(Intercept) 1.095e+04  5.353e+02   20.45   <2e-16 ***
par_median  3.341e-01  6.469e-03   51.65   <2e-16 ***
---
Signif. codes:  0 ‘***’ 0.001 ‘**’ 0.01 ‘*’ 0.05 ‘.’ 0.1 ‘ ’ 1

Residual standard error: 8639 on 2200 degrees of freedom
Multiple R-squared:  0.548,     Adjusted R-squared:  0.5478 
F-statistic:  2667 on 1 and 2200 DF,  p-value: < 2.2e-16
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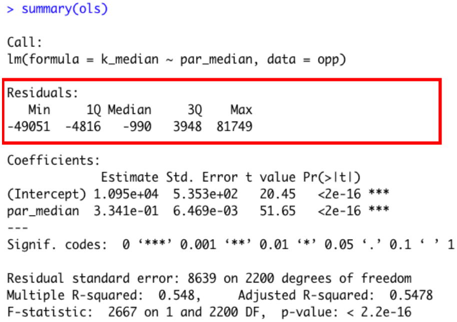

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We also have some statistics about the residuals.

Accessing the residuals

You can access the residuals using `ols$residuals`.

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How would you compute the SSR of this line?

Computing R^2

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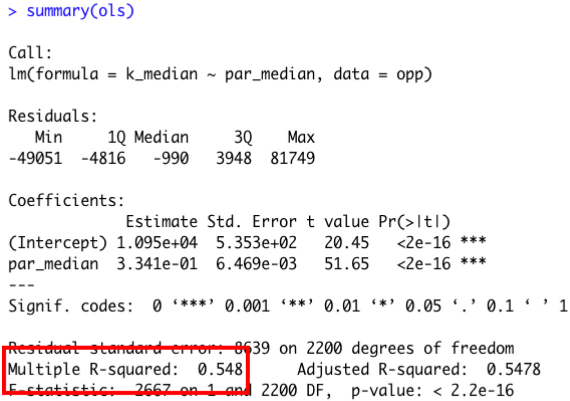

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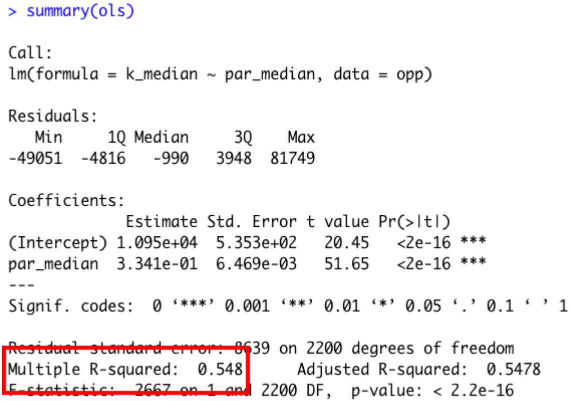

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We can access it using `summary(ols)$r.squared`.

Outline

Recall

- Probabilities indicate our uncertainity about events.

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- Intuitively, we measure how likely an event is to occur.

Examples

Let's say I'm rolling a 6-sided die. What is the probability I get an even number?

Examples

Suppose I have a bag containing 5 white marbles, 6 red marbles, and 4 black marbles, and I pick a marble without looking. What is the probability I pick a white marble?

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Read in the dataset `masc_raw-responses.csv`. This is a dataset containing responses from roughly 6000 men about what they think it means to be a man.

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The full list of questions is in the `masculinity-survey.pdf`

We'll consider question 7(b) in particular to analyze today.

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This is the question: How often would you say you ask a friend for personal advice?

Answer options:

1. Often
2. Sometimes
3. Rarely
4. Never, but open to it
5. Never, and not open to it

What is the probability that the survey respondant said “Rarely”?

Computing probabilities in R

- Let R denote the event that the response is “Rarely”

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- How can we do this?

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- What should our numerator be?
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```
num <- nrow(filter(masc, q0007_0002 == "Rarely"))
```

You try!

What is the probability that the survey respondant said either “Never, but open to it” or “Never, and not open to it”?

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nrow(filter(masc,
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      q0007_0002 == "Never, and not open to it")) /
  nrow(masc)
```

How can we find

$$\Pr(R|\text{Respondant is over 65})$$