SOC245: Visualizing Data
Lecture 9: Central Limit Theorem

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Where we are and where we're going

Before:

- Computing estimates on a sample.

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Today:

- Getting guarantees about confidence intervals (with more assumptions)

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- We now have this set of values, and we can analyze the distribution.
- This is called the **sampling distribution**

Pop Quiz!

Can we compute the sampling distribution in practice?

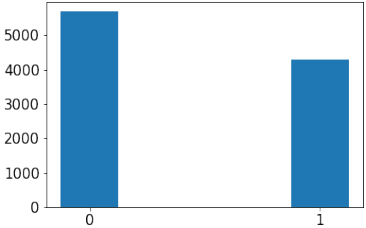
Pop Quiz!

Can we compute the sampling distribution in practice?

No! It's too time consuming and expensive to keep picking samples from the population.

Outline

What do sampling distributions look like?



Population distribution

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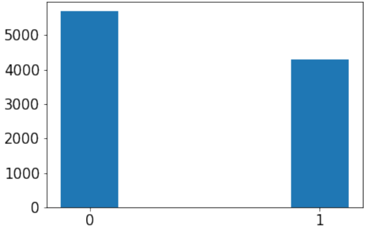

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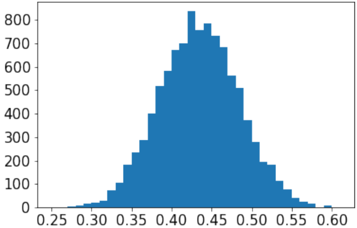

Distribution of sample mean

What do sampling distributions look like?

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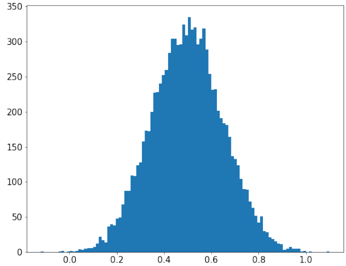

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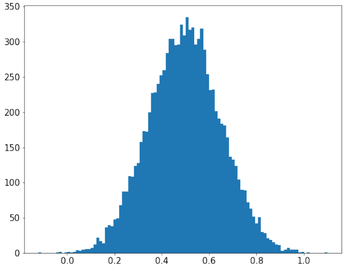

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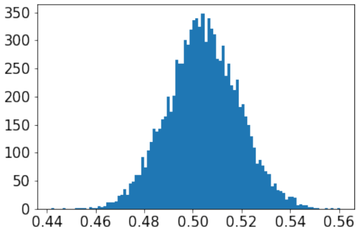

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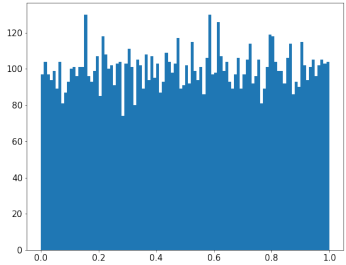

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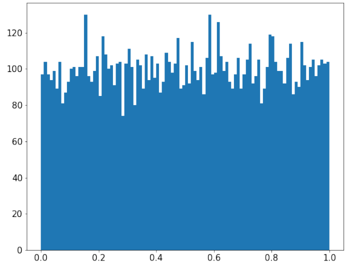

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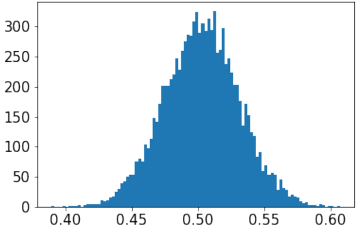

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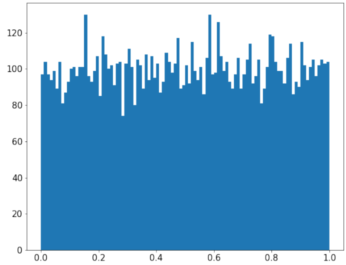

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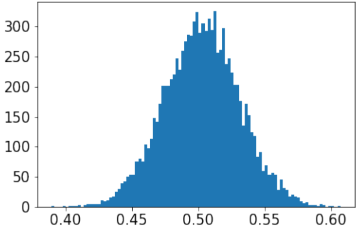

Distribution of sample mean

What do you notice about these sampling distributions?

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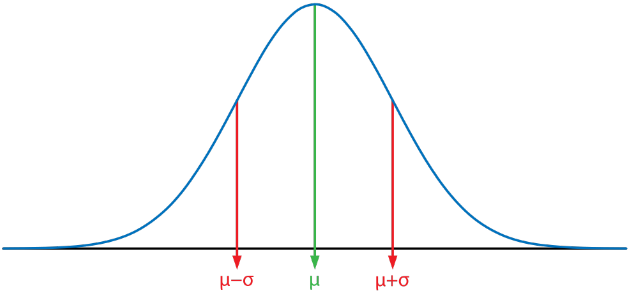
- This shape is called a **Normal** or a **Gaussian** distribution.
- You might have also seen it called a bell-curve (but that covers other distributions as well).

Normal distributions

Suppose we have a Normal distribution with mean μ and standard deviation σ.



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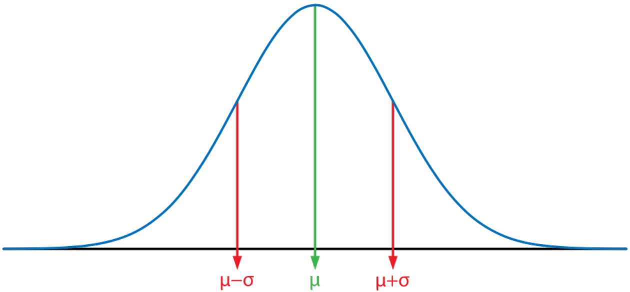

- It's symmetric around the mean value

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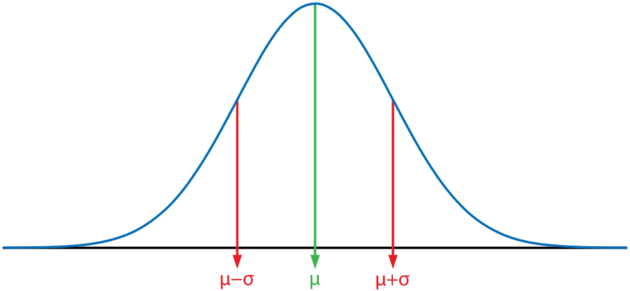

- The mean, median and mode of this distibution is μ

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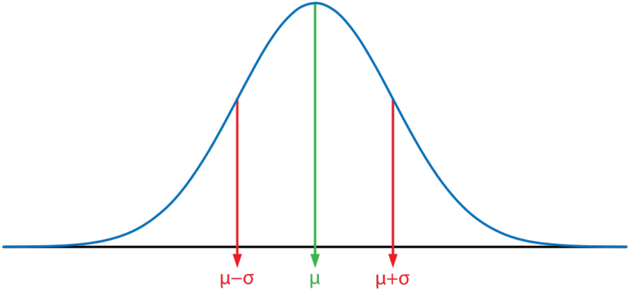

- This distribution occurs naturally pretty often: the height of people is a Normal distribution within each gender.

An amazing theorem in Statistics

The sampling distribution of the sample means of n independent, identically distributed random variables from a population with mean μ and variance σ^2 approaches a Normal distribution with mean μ and standard deviation $\frac{\sigma}{\sqrt{n}}$ as the sample size gets larger

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This is called the **Central Limit Theorem**.

An amazing theorem in Statistics

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This is defining how we sample: drawing an observation should not affect any observations we draw after that.

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This says that all my observations come from the same underlying population.

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Notice that we have no other constraints on the population: the distribution of the population can be anything!

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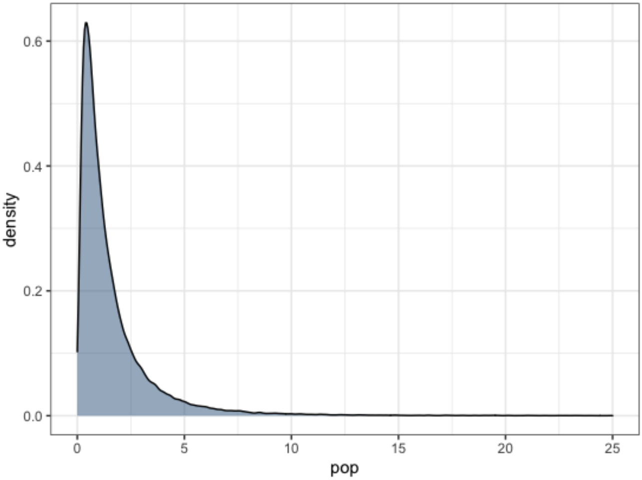
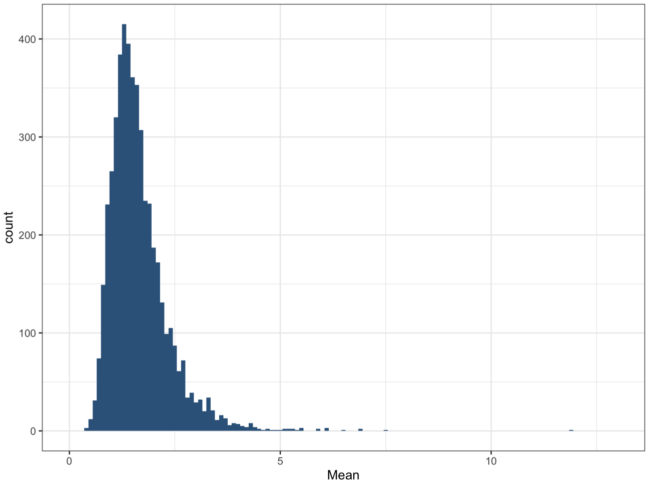
The text is very important. Let's see some examples.

Sample size is important for the Central limit Theorem

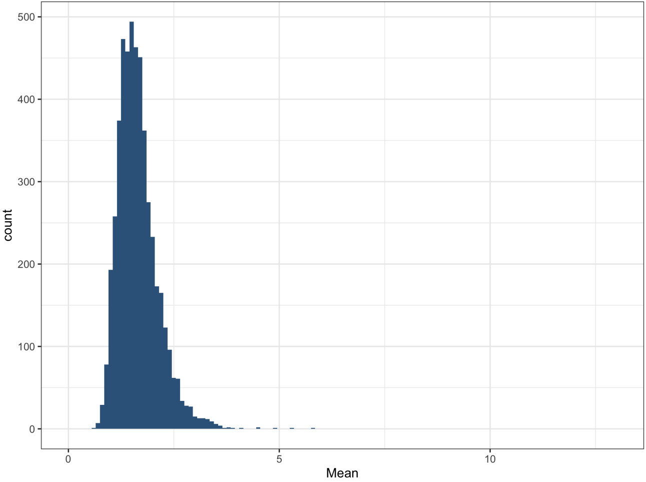
Suppose we're drawing samples from a population that looks like this, and we want to compute the mean.



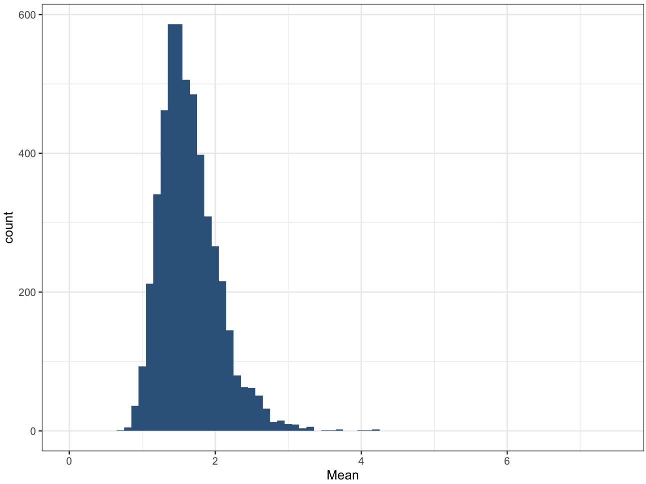
Sample size = 10



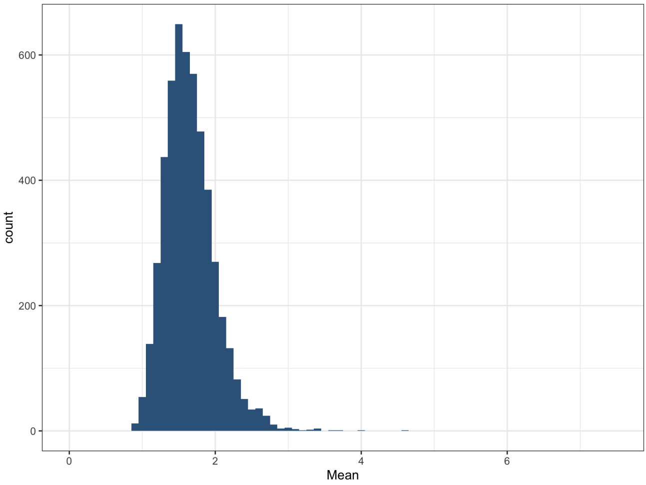
Sample size = 20



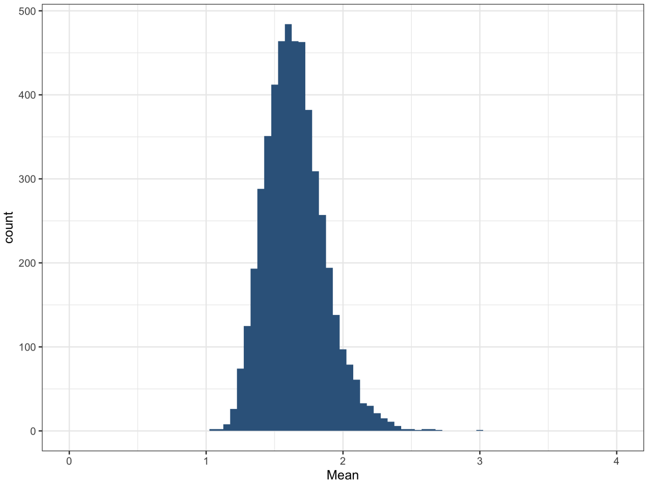
Sample size = 30



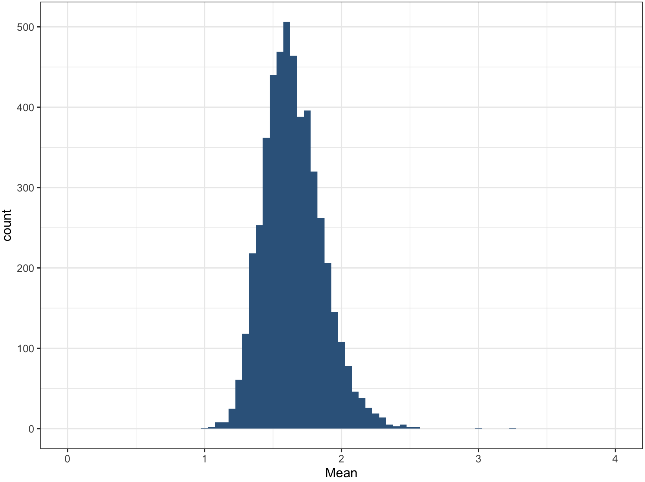
Sample size = 40



Sample size = 50



Sample size = 100



What do you notice?

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- The spread of the sampling distribution reduces.

Pop quiz

True or False: As we increase the sample size, the distribution of the sample becomes Normal.

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True or False: As we increase the sample size, the distribution of the sample becomes Normal.

False. The **sampling distribution** of the sample means approximates a Normal.

Outline

Key Assumption

Our sample size is large enough so that the central limit theorem holds, i.e, the sampling distribution is close to being a Normal distribution.

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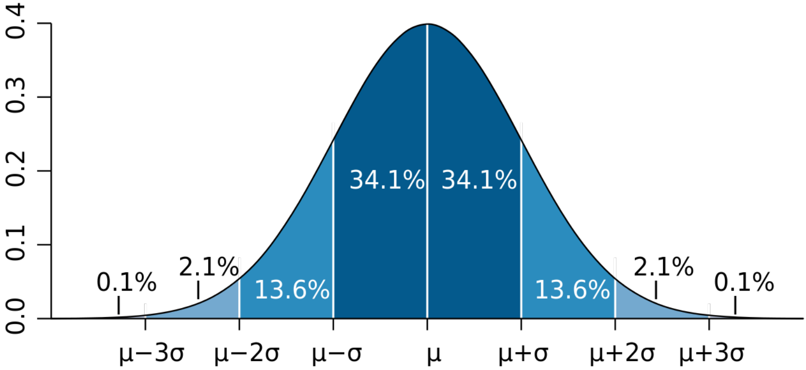
We can compute percentiles of the assumed distribution to get confidence bounds.

Percentiles of a Normal Distribution



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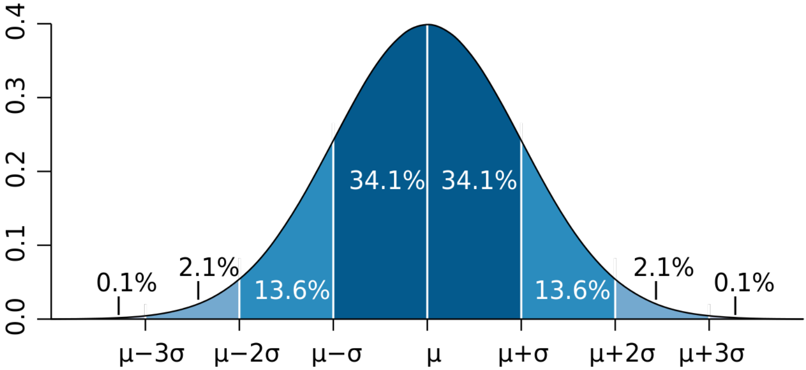

For a Normal distribution with mean μ and variance σ,

$$\Pr(X \geq \mu - \sigma \text{ and } X < \mu + \sigma) = 0.682$$

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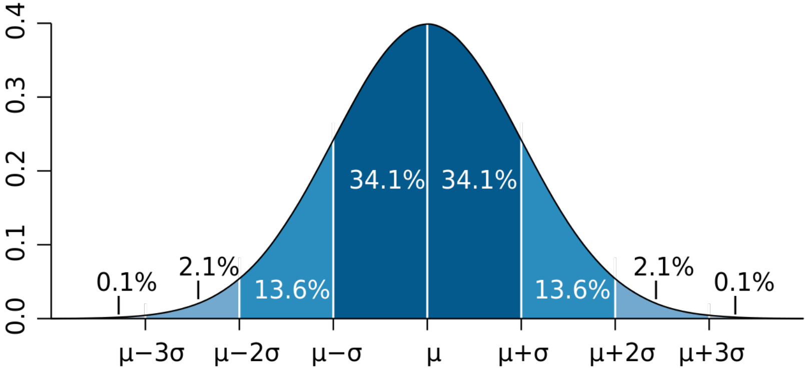

For a Normal distribution with mean μ and variance σ,

$$\Pr(X \geq \mu - 2\sigma \text{ and } X < \mu + 2\sigma) = 0.9558$$

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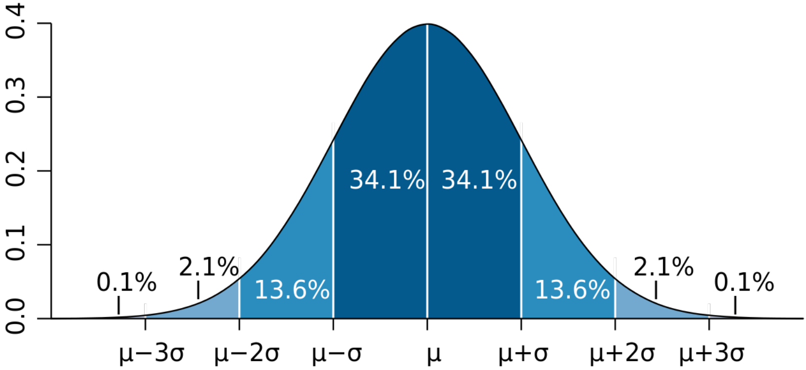
Percentiles of a Normal Distribution



For a Normal distribution with mean μ and variance σ,

$$\Pr(X \geq \mu - 3\sigma \text{ and } X < \mu + 3\sigma) > 0.99$$

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- The 97.5 percentile for this distribution is $\mu + 1.96\frac{\sigma}{\sqrt{n}}$.
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Problem: We actually don't know σ (the standard deviation of the population).

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- What is the formula for the sample variance?

$$\hat{\sigma^2} = \frac{1}{n-1}\sum_i (X_i - \overline{X})^2$$

- Thus, we get a 95% confidence interval of :

$$[\overline{X} - 1.96\frac{\hat{\sigma}}{\sqrt{n}}, \overline{X} + 1.96\frac{\hat{\sigma}}{\sqrt{n}}]$$

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- Let's assume that the sample size is large enough for the sampling distribution to approximate a Normal.

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- Let's assume that the sample size is large enough for the sampling distribution to approximate a Normal.
- What's the standard deviation of the sampling distribution? We don't know. But we can estimate it using $\frac{\hat{\sigma}}{\sqrt{100}} = 0.04878$.
- Thus, we can estimate 95% confidence intervals as $[0.38 - 1.96 \times 0.04878, 0.38 + 1.96 \times 0.04878] = [0.2844, 0.4756]$.

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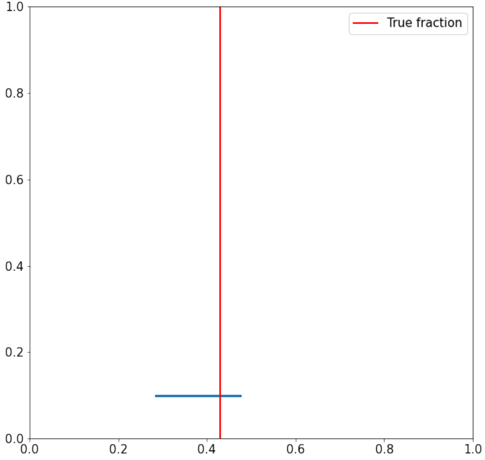

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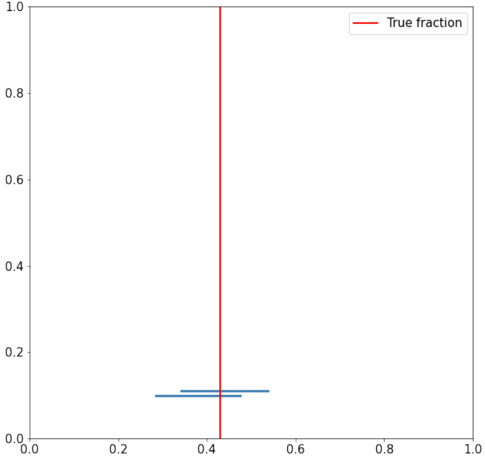

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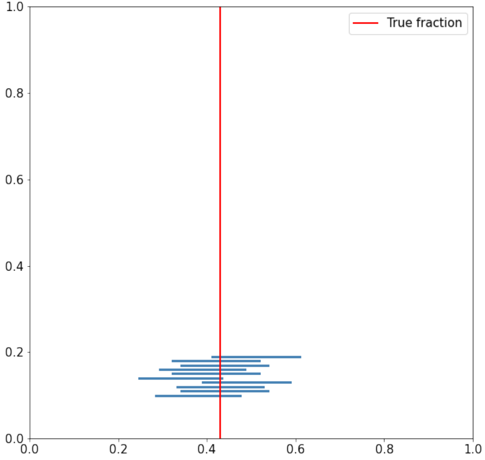

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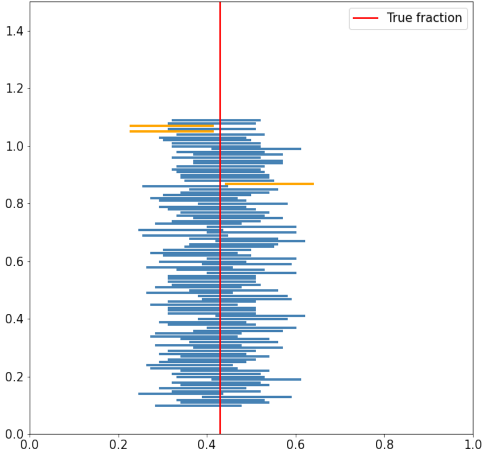

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This probability is called the p-value.

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- Using our conditional probability notation:

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- Much like picking 95% confidence intervals, the threshold for significance is arbitrary. Why $P < 0.05$ or $P < 0.01$? Why not $P < 0.04$?
- Notion of a simple yes-or-no answer for statistical significance doesn't really make sense : design choices like the size of the sample, how the data is collected, how the analysis is done is often a lot more important.

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- And thus, might still be useful to implement (Polit and Beck (2012))

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Next week:

- Causality