SOC245: Visualizing Data
Lecture 8: Uncertainity

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Aug 1, 2022

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- But, we don't really care about *just* the respondents of this survey:
- What can we say about the general population of the United States from this?

Where We've Been and Where We're Going

- Understanding central tendency and spread of a sample

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Going from the sample to the population: How confident are we about our estimates?

Outline

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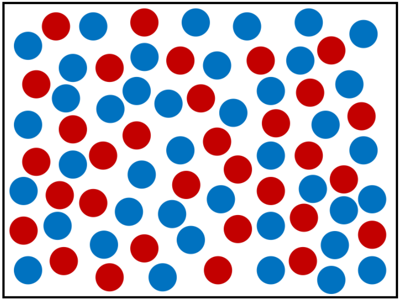
Motivating Question

How certain can we be that the Democratic party candidate will win the popular vote?

Population distribution



Suppose this is our population, with blue representing a Democrat and red representing that a Republican.

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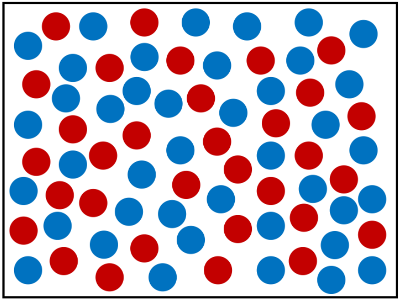

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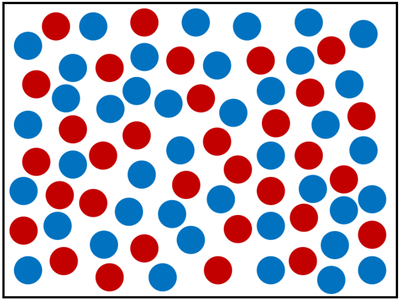

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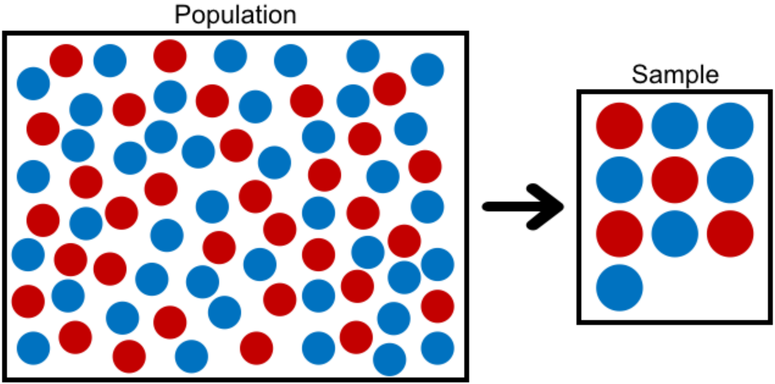
We can ask every single citizen (the total **population**) how they are going to vote . . .
But this is very expensive and time-consuming.

Getting an estimate for the population

We can ask a subset of the total population.



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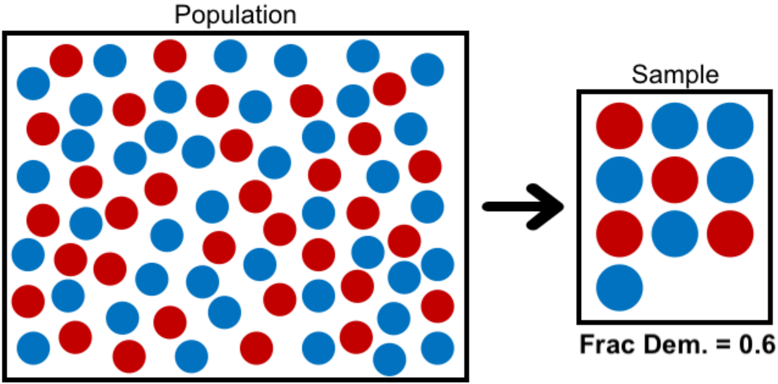

Now, we can compute the fraction of Democrat votes within the sample.

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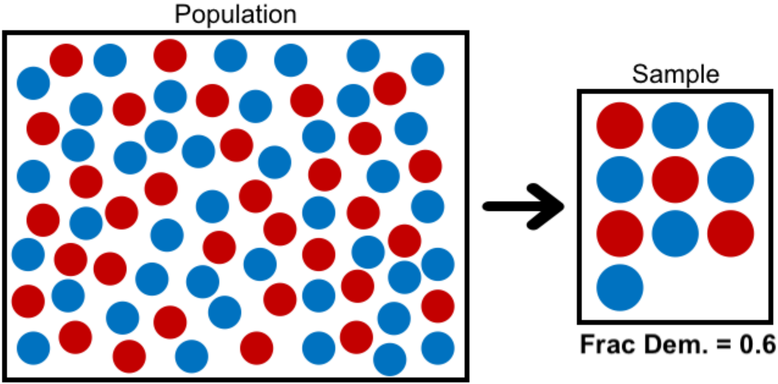

What can we say about the population from the sample?

Estimates depend on the sample

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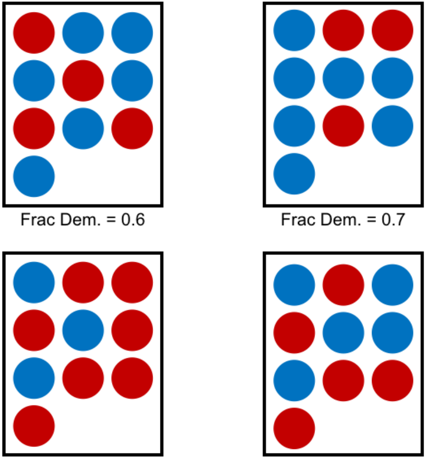

Frac Dem. = 0.6

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Frac Dem. = 0.3

Frac Dem. = 0.5

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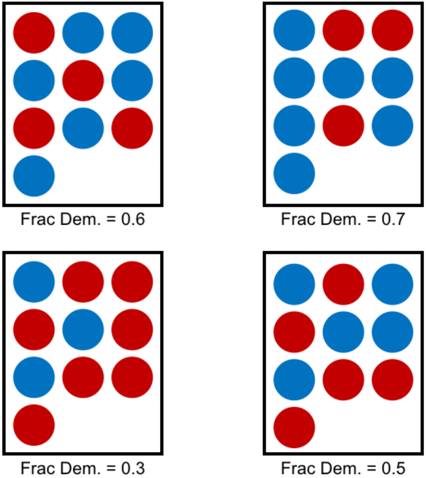

But once we pick a sample, the estimate is fixed.

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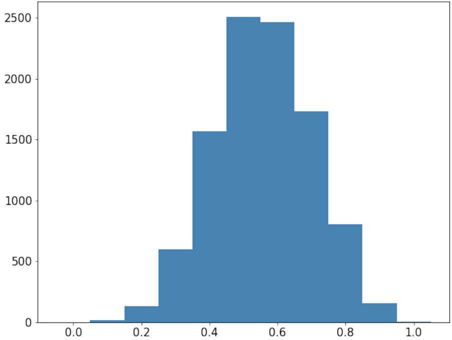
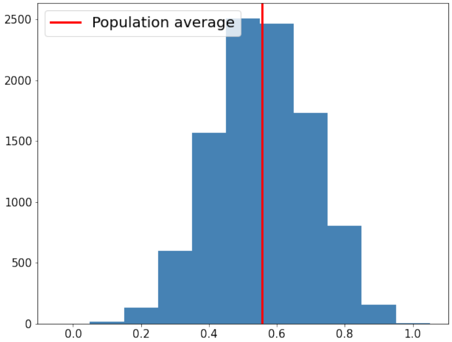
- This is called the **sampling distribution**

Visualizing the sampling distribution



Visualizing the sampling distribution

Here is the true value for the fraction of Democrat votes (if we were able to compute this for the whole population).



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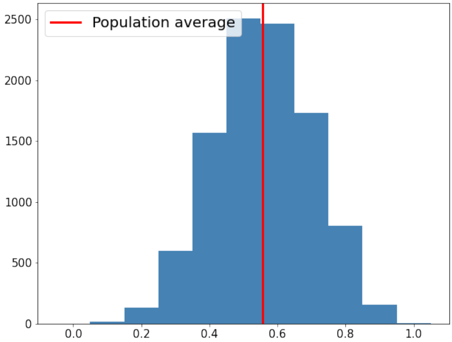

How can we reason about how close our estimate is to the actual?

Ideal goal

Given a statistic (i.e, a sample), what is the probability of (a parameter) of the population?

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This is called **statistical inference**: using a sample to learn about the underlying population.

Problem

We don't know the full population and we only have access to a single sample (so we can't compute the sampling distribution.)

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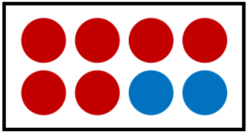
Sampling without replacement



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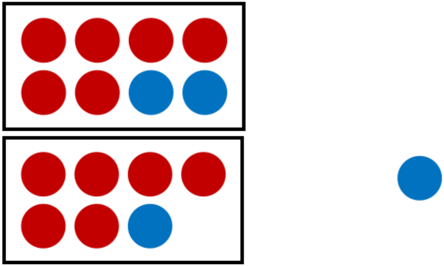

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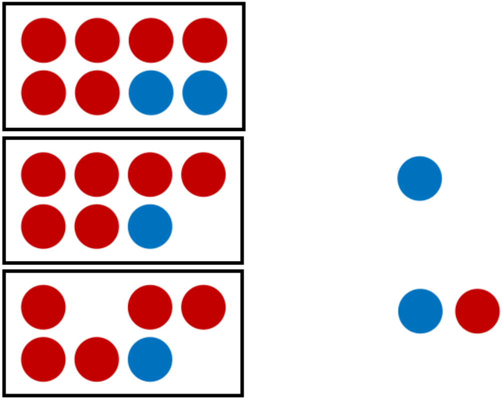

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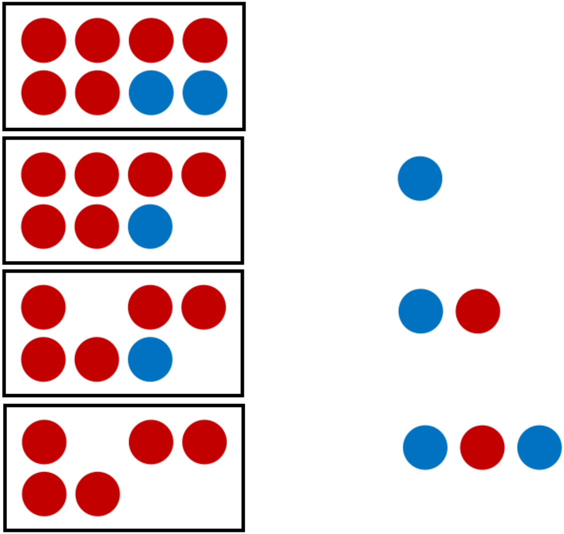

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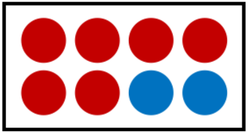

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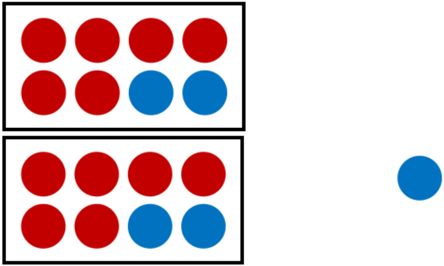

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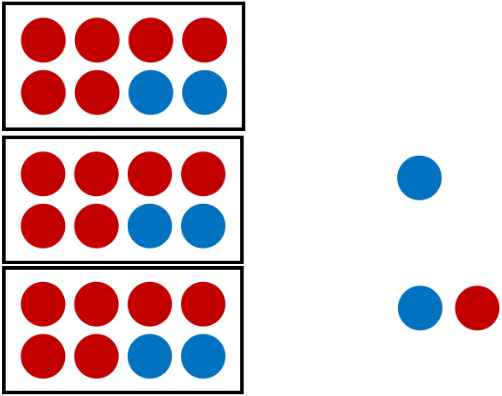

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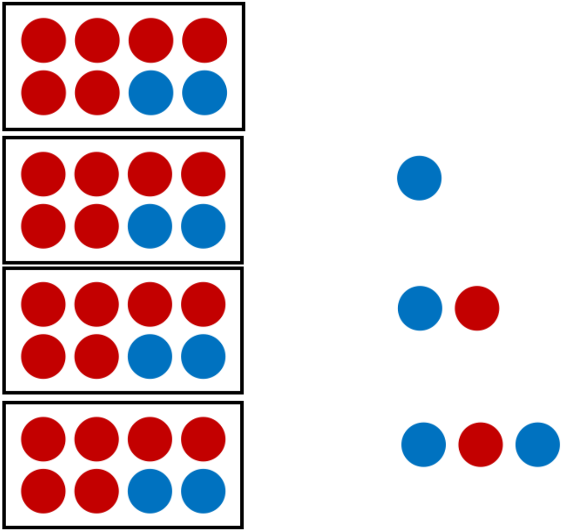
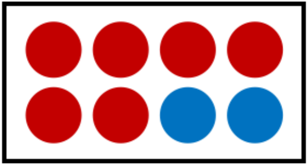


Why sampling with replacement

- Keeps the distribution the same across samples.



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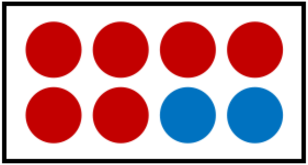

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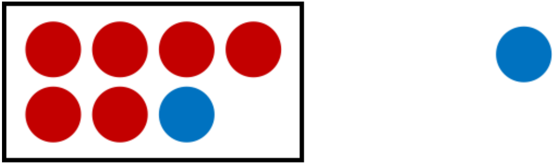

Original $\Pr(\text{blue}) = \frac{2}{8}$

New $\Pr(\text{blue}) = \frac{1}{7}$

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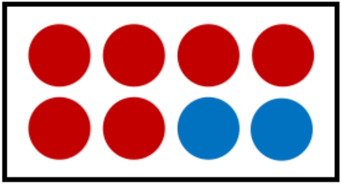

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- This approximates the shape and spread of the sampling distribution.
- We can construct an interval using percentiles of the resampled distribution.

Working through an example

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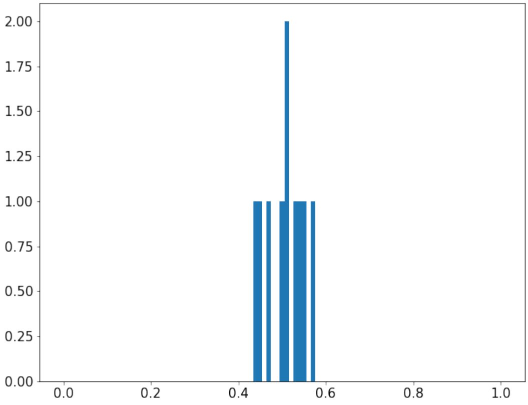
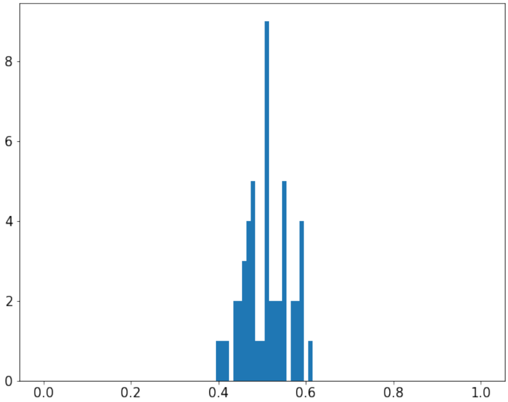
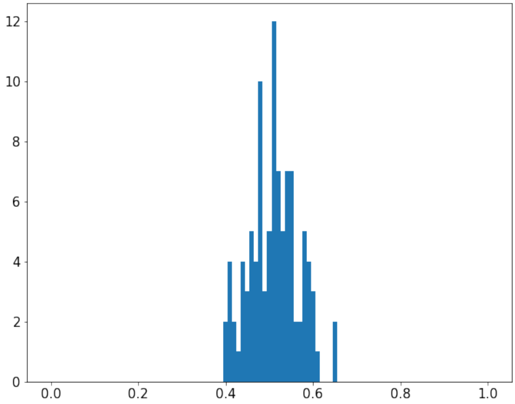
And again. This resample has 57% voting Democrat.



Distribution with 10 resamples.



Distribution with 50 resamples.



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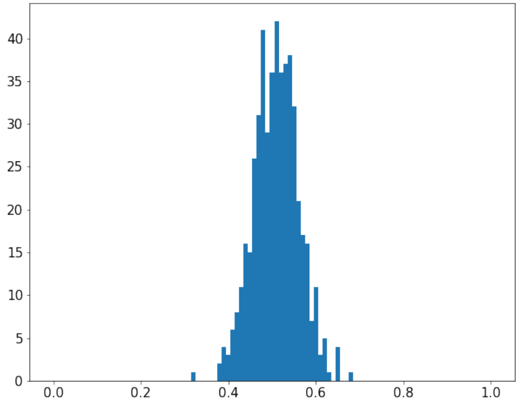
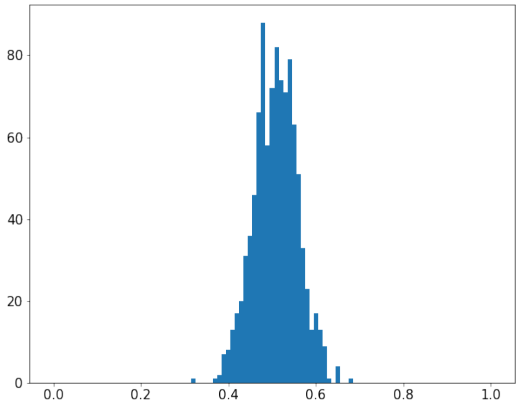
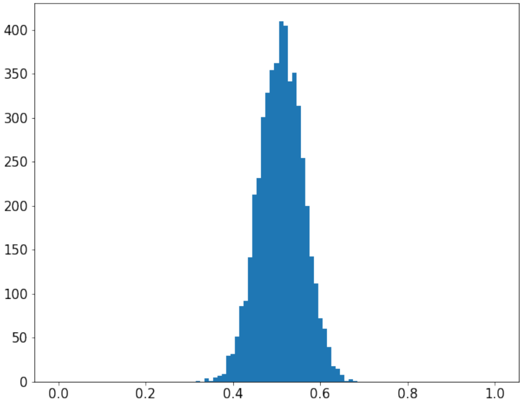


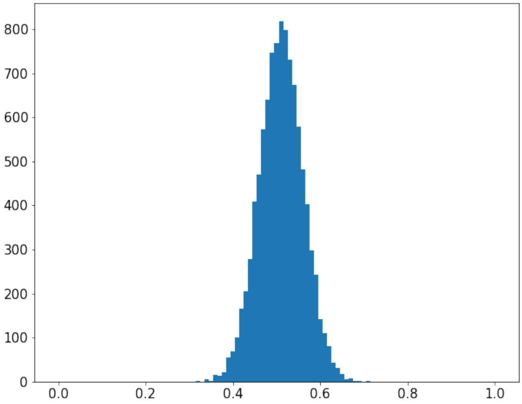
Distribution with 500 resamples.



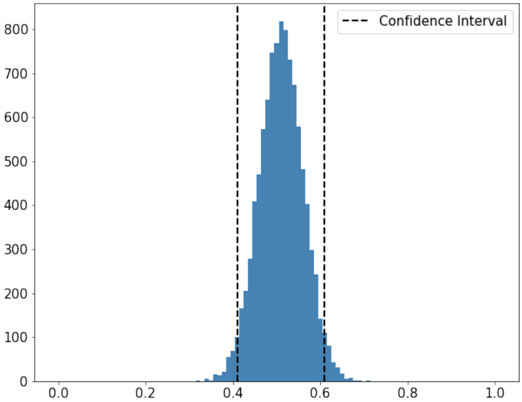
Distribution with 1000 resamples.



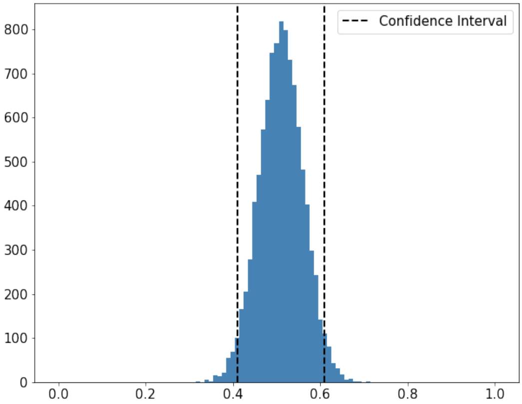
Distribution with 5000 resamples.



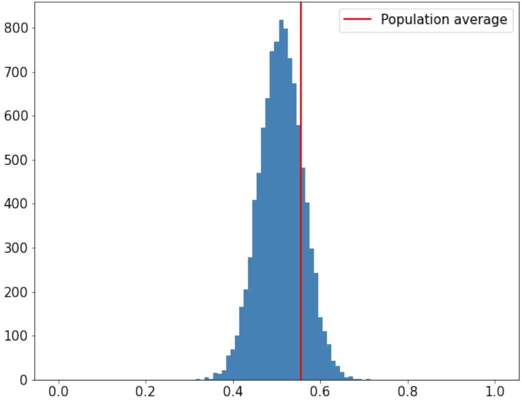
Distribution with 10000 resamples.



We can now compute the 2.5% and 97.5% percentiles.



We can now compute the 2.5% and 97.5% percentiles. This gives us a **confidence interval**: 95% of our values (fraction of Democrats) were between 41% and 61%.



Important: The mean of the resampled distribution does not always approximate the mean of the true population: just the shape and spread approximates that of the true sampling distribution.

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Intuitively,

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- We make the assumption that each of the observations from our sample is drawn randomly from the population.
- Thus, drawing observations from our sample randomly (with replacement) is similar to drawing from the population

Frequentist interpretation of a confidence interval

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- What would this look like?

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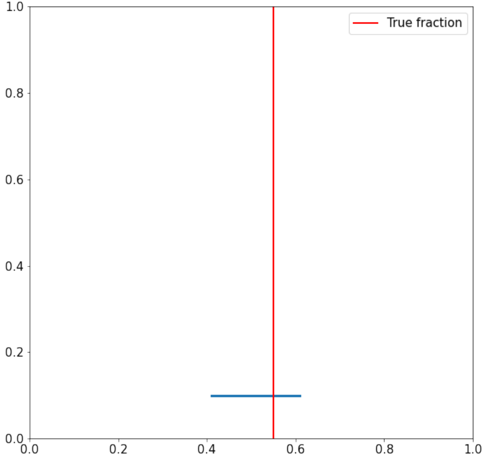

1 sample

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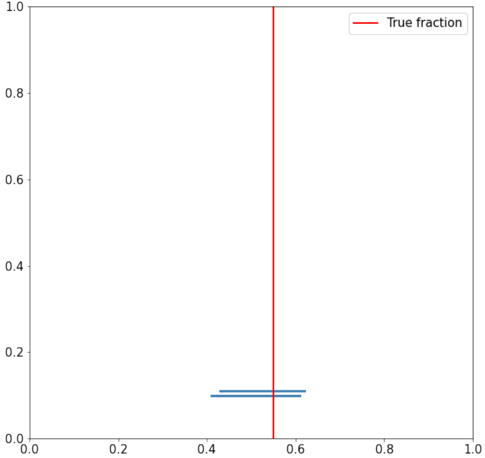

2 samples

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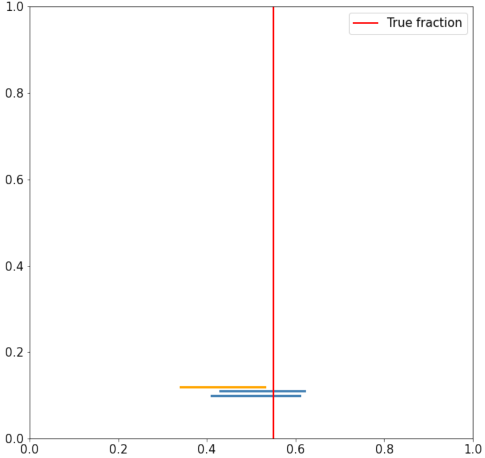

3 samples

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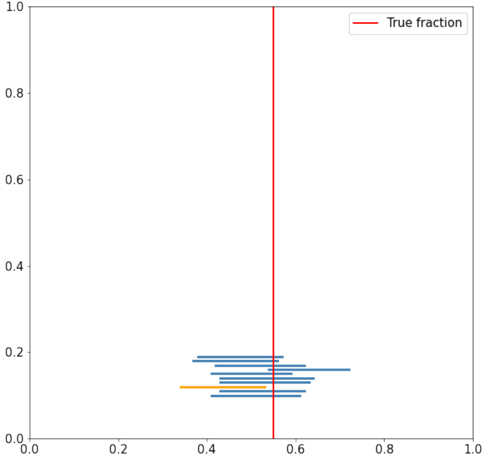

10 samples

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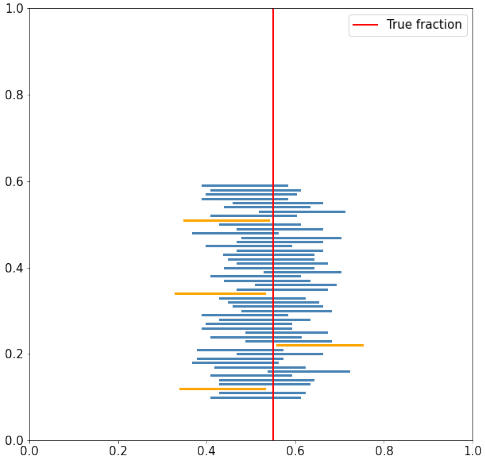

50 samples

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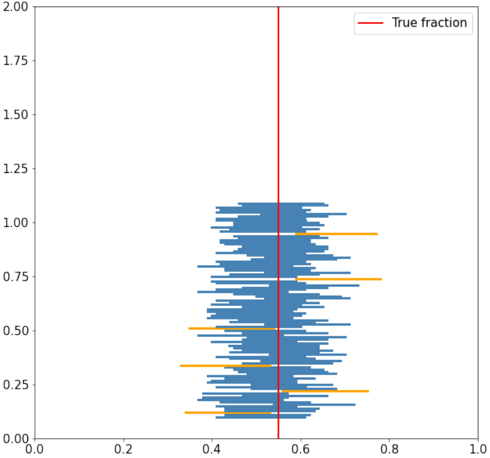
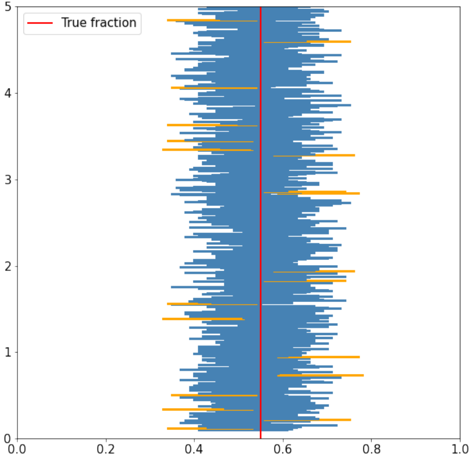


100 samples

Frequentist interpretation of a confidence interval



500 samples

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In general, if we compute percentiles at $\frac{\alpha}{2}$ and $1 - \frac{\alpha}{2}$, the true value lies within the confidence interval with a probability of $1 - \alpha$.

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- More traditional methods of constructing confidence intervals have better properties/guarantees but these require stronger assumptions.
- We'll talk more about these on Wednesday.

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Wednesday:

- Getting guarantees about confidence intervals (with more assumptions)