Taking Education Online
A Unique Opportunity for the New Millennium

Robert Sedgewick
Princeton University
This talk is dedicated to the memory of Philippe Flajolet

Philippe Flajolet 1948–2011
Starting point

1975: What are the algorithms that everyone should know?

“Everyone” means “everyone using a computer”

- scientists
- engineers
- mathematicians
- software/hardware designers
- cryptanalysts
- COBOL programmers
- ...

Context

- <1 computer per university, on average
- <10 CS departments worldwide

1981: First edition of Algorithms is published
Disseminating knowledge: RS context

Algorithms

Analytic Combinatorics (with P. Flajolet)

Introduction to Computer Science (with K. Wayne)
Taking Education Online

- Frequently asked questions
- Disruptive changes
- A way forward
- Analytic combinatorics
- Questions answered
Frequently asked questions

Q. What does MOOC mean?
A. Massive Open Online Course.
   • Free
   • Designed for large numbers of "students"

Q. What is the business model?

Q. How are our students getting their money's worth if it's free?

Q. Can we afford to participate?

Q. Will this detract from our "brand"?

Q. Can I get credit for this course?

Just one platform (2015):
- 1000+ courses
- 12 million students

the wrong questions

Note: A more disruptive "connectivist" MOOC concept is widely advocated:
• Crowdsourced decisions and assessment
• Aggregation, remixing, and passing forward content
The right questions

Q. How are we going to disseminate knowledge in the future?

Q. Are universities going to take leadership in helping to decide this question?

Q. If we *can* disseminate knowledge for free, isn't it our responsibility to do so?

Q. What is the purpose of a university?
Taking Education Online

- Frequently asked questions
- **Disruptive changes**
- A way forward
- Analytic combinatorics
- Questions answered
Seismic changes are afoot

For a millennium, universities have been considered the main societal hub for knowledge and learning. And for a millennium, the basic structures of how universities produce and disseminate knowledge and evaluate students have survived intact...Today, though, the business of higher education seems to some as susceptible to tech disruption as other information-centric industries.


Business of higher education?? A road to ruin.
Sit in your local coffee shop, and your laptop can tell you a lot. If you want deeper, more local knowledge, you will have to take the narrower path that leads between the lions and up the stairs. There—as in great libraries around the world—you’ll use all the new sources, the library’s and those it buys from others, all the time. You’ll check musicians’ names and dates at Grove Music Online, read Marlowe’s “Doctor Faustus” on Early English Books Online, or decipher Civil War documents on Valley of the Shadow. But these streams of data, rich as they are, will illuminate, rather than eliminate, books and prints and manuscripts that only the library can put in front of you.

**The narrow path still leads, as it must, to crowded public rooms where the sunlight gleams on varnished tables, and knowledge is embodied in millions of dusty, crumbling, smelly, irreplaceable documents and books.**
While Grafton’s reservations about putting knowledge online are well taken, I would also point out that there is quite a bit going on now in the academic world that doesn’t have much to do with old books. Indeed, as the author of many books, I wonder whether perhaps the book is not quite sacred as a means of disseminating knowledge.

**What is the most effective way to produce and disseminate knowledge with today’s technology?** How can we best structure what we know and learn so that students, researchers, and scholars of the future can best understand the work of today’s researchers and scholars?

I think that questions like these are more important and more difficult to address than whether we can put the contents of libraries on the Web.
Disseminating knowledge I: research papers

When is the last time you visited a library to find a paper?
Did you print the papers to read the last time you refereed a conference?

• Color?
• Links to references?
• Links to detailed proofs?
• Simulations?

why?
why not?

“I could read it on my iPad...
...if I had an iPad”
D. E. Knuth (2011)

Question: If it will not be read on paper, why write it as if it will?

Prediction: Someone will soon invent the future (should be easy)
Future of libraries?

1980s

- Students spend significant time in the library
- Faculty members depend on the library for research

2010s

- Students spend significant time online and have *no need* for the library
- Few faculty members in the sciences use the library *at all* for research

2020s?

- A few *book museums* (for Grafton)
- Digital library infrastructure (for everyone else)

How will we disseminate knowledge in the future?
Will universities play a role?
Disseminating knowledge II: Textbooks

We are on a road to ruin

- Prices continue to escalate.
- Students now *rent*, not own books.
- Planned obsolescence? Walled gardens?

Princeton U-store 1950s

Princeton U-store 2010s

Is there room for a good textbook?
Will free web resources prevail?

No books!
Taking Education Online

- Frequently asked questions
- Disruptive changes
- A way forward
- Analytic combinatorics
- Questions answered
A way forward embraces technology to integrate three abstractions that are *here to stay*:

1. A *textbook* for use by students to *learn and study* the details of a subject.
2. A *course* that encourages a community of scholars to learn together.
3. *Web content* for use by students to *explore and interact with* the material.

*Examples.* CS courses at Princeton implemented by RS and Kevin Wayne (*stay tuned*).
The "course" abstraction has been an essential part of education for a millenium and is *here to stay*.

### What is a course?
- *Lectures* to introduce and inspire.
- Assessments (assignments and exams)
- *Precepts* to work in small groups.

### Purpose of a course
- Enable a "community of scholars" to teach and learn a subject.
- Serve as a building block in a *curriculum*.

*Important note:* Certification that a student completes a course is *not* essential.
The "textbook" abstraction has been an essential component in education for centuries and is *here to stay*.

Well-understood since the Greeks.

Enabled for the masses by Gutenberg.

**Relevant for this talk:**

**Purpose of a textbook**

- Articulate what students can reasonably learn about a subject in a semester.
- Provide a reference point for future studies related to the subject.
The "web content" abstraction is emerging as an essential component in education and is *here to stay*

What is web content?
- *Full coverage* integrated with *web search*.
- Always up to date (*dynamic*).
- Content types *not available* in print.

**Issues**
- Basic properties still evolving.
- Free? Who pays?
- Who creates it? Who maintains it?
Example 1: Introduction to CS (RS and Kevin Wayne)

2000s

- Raw material, digests for "story"
- Enrichment materials
- Programming model.

introcs.cs.princeton.edu

Web Content
"Booksite"

10000+ files
2000+ Java programs
50+ animated demos
2.1 million visits in AY2013-14

Textbook

Course

1990s

- Lecture presentations.
- Assignments
- Exercises and Exams
- Precepts.

10000+ files
2000+ Java programs
50+ animated demos
2.1 million visits in AY2013-14
Example 2: Algorithms, 4th edition (RS and Kevin Wayne)

RS: Hey, maybe this could work for "Algorithms"!
KW: No problem, but...it might take some time.

2011
- Code and data repository.
- Enrichment materials
- Course materials.

1977-2013
- Lecture presentations.
- Assignments
- Exercises and Exams
- Precepts.

Textbook

Web Content "Booksite"
Example: Algorithms booksite

http://algs4.cs.princeton.edu/

Web content associated with a book

- Web presence.
- Landing and takeoff for search.
- Code, test data, animations.
- Course materials.
- A living document.
- For use while computing, exploring.
Lecture presentation materials are evolving to new standard of excellence.

"Powerpoints"

This is a horse.

State of the art presentations (stay tuned)

Connected components in mappings

<table>
<thead>
<tr>
<th>Class</th>
<th>Y, the class of cycles of Cayley trees</th>
</tr>
</thead>
<tbody>
<tr>
<td>EGF</td>
<td>[ Y(z) = \sum_{n \geq 0} \frac{z^n}{n!} N^N \sum_{k \leq N} \frac{N^N}{k!} ]</td>
</tr>
</tbody>
</table>

Example

1 10 2 1 10 2 9 1 2 3 1

Construction

EGF equation

Extract coefficients by Lagrange inversion with \( f(u) = u^a \) and \( f(u) = \ln(1/(1-u)) \)

\[ Y(z) = \ln \frac{1}{1 - C(z)} \]

\[ (2^N) Y(z) = \frac{1}{N} \sum_{0 \leq k < N} \frac{N^N}{k!} = \frac{1}{N} \sum_{0 \leq k < N} \frac{(N-k)!}{N^N} \]

\[ Y_N = N! [z^N] Y(z) = N^{N-1} \sum_{1 \leq k < N} \frac{1}{N^k (N-k)!} = N^{N-1} Q(N) \sim \frac{N^N \sqrt{N}}{2N} \]
Example: Presentation materials for an algorithms lecture (2012)

Elements

- Diagrams of data structures.
- Code.
- Animations.
- Summary Info.
- "Story".

3.3 BALANCED SEARCH TREES

- 2-3 search trees
- red-black BSTs
- B-trees
2011: Time to declare victory?

Introduction to CS enrollments
- *Double* the height of the “bubble”
- 43% of all Princeton students.

“Algorithms” enrollments
- *Three times* the height of the “bubble”
- 17% of all Princeton students.

Q. (2011) When will enrollments start to decline?
2015: Time to declare victory? Not yet!

Introduction to CS enrollments
- *Triple* the height of the “bubble”
- 2/3 of all Princeton students.
- Largest course at Princeton

“Algorithms” enrollments
- *Four times* the height of the “bubble”.
- *Doubled* in the last two years.
- 40% of all Princeton students.
- 4th largest course at Princeton

Q. (2011) When will enrollments start to decline?
A. (2015) Sometime after they stop accelerating!
Primary advantage of booksite model is *scalability* allowing reach to at least an order of magnitude more students handles huge numbers of students worldwide. Instructors use book and booksite as basis for teaching. Individuals are directly accessing book and booksite for self-study.
Next challenge (2011)

RS. *Algorithms for the masses* (ANALCO, San Francisco 2011)

**Summary**

The scientific method is an essential ingredient in programming. Embracing, supporting, and leveraging science in intro CS and algorithms courses can serve large numbers of students.

Proof of concept: First-year courses at Princeton

- 40+% of Princeton students in a single intro course
- 25+% of Princeton students in a single *algorithms* course

Next goals:

- 80+% of *all college students* in an intro CS course
- 50+% of *all college students* in an algorithms course

*Algorithms for the masses*

Confession: No idea how we would get there...
2012: MOOCs go mainstream

Q. (Jan. 2012) Are you interested in teaching online?
RS+KW. No. (Too much work to do it properly.)

Q. (Apr. 2012) Trustees want it: we're doing it anyway. Are you in?
RS+KW. An offer we cannot refuse...

With apologies to our actual administrators

An online platform for the "course" abstraction

"Algorithms, Part I" (Summer 2012)

Immediate realization:
Our model is perfectly suited to go online.
Putting "Algorithms" online

Lectures (RS and Kevin Wayne)
- Developed "4th edition" of presentations.
- Recorded in studio.

Quizzes for self-assessments (Kevin Wayne and Josh Hug)
- Developed an populated "random" question generation process.
- Innovative work, more research needed.

Scripts for programming assignments (Kevin Wayne and Josh Hug)
- Extended "grading scripts" to evaluate student programs on a massive scale.
- Innovative work, more research needed.
Integrated model

extends our reach to at least *another* order of magnitude more students

Handles huge numbers of students worldwide. Instructors use book, *course*, and booksite as basis for teaching. Individuals are directly accessing all three for self-study.
Extending our reach

Booksite handled 1.5 million hits in AY 2013-2014.
Online courses are enrolling 300,000+ students per year.
Book sales are increasing sharply.
Taking Education Online

• Frequently asked questions
• Disruptive changes
• A way forward
• **Analytic combinatorics**
• Questions answered
RS: Hey, maybe this could work for "Analytic Combinatorics"!

Prevailing wisdom: No way—interactive chalktalk is time-honored and needed for math.

RS: Seriously?
Presentation elements: Analytic Combinatorics

Mathematical derivations

Cayley trees

<table>
<thead>
<tr>
<th>Class</th>
<th>( \mathcal{C} ), the class of labelled rooted unordered trees</th>
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</table>

EGF

\[
\mathcal{C}(z) = \sum_{C \in \mathcal{C}} \frac{z^{|C|}}{|C|!} = \sum_{N \geq 0} \frac{C^N}{N!} z^N
\]

Example

6 2 1 1 2 2 5 1

1 2 3 4 5 6 7 8

"Builds" control pace.
Details to support reasoning are included.

Drawings of combinatorial objects

Classic example of the symbolic method

Q. How many trees with \( N \) nodes?

\( G_1 = 1 \)
\( G_2 = 1 \)
\( G_3 = 2 \)
\( G_4 = 5 \)
\( G_5 = 14 \)

"Story"

Analytic combinatorics overview

A. SYMBOIC METHOD
1. OGFs
2. EGFs
3. MCFs

B. COMPLEX ASYMPTOTICS
4. Rational & Meromorphic
5. Applications of R&M
6. Singularity Analysis
7. Applications of SA
8. Saddle point
1. Combinatorial structures and OGFs

http://ac.cs.princeton.edu
"An Introduction to Analytic Combinatorics" Online Lectures

### Part I: Analysis of Algorithms

1. Introduction
2. Recurrences
3. Generating Functions
4. Asymptotic Analysis
5. Analytic combinatorics
6. Trees
7. Permutations
8. Strings and Tries
9. Words and Mappings

### Part II: Analytic Combinatorics

1. Ordinary GFs
2. Exponential GFs
3. Bivariate GFs
4. Meromorphic Asymptotics
5. MA applications
6. Singularity Analysis
7. SA Applications
8. Saddle Point
9. Advanced Topics

~500 slides
2013: Mission accomplished

AN INTRODUCTION TO THE ANALYSIS OF ALGORITHMS

People who analyze algorithms have double happiness. First of all they experience the sheer beauty of elegant mathematical patterns that surround elegant computational procedures. Then they receive a practical payoff when their theories make it possible to get other jobs done more quickly and more economically. — D. E. Knuth

This book is under development (Spring 2012). No promises.

Textbook. The textbook An Introduction to the Analysis of Algorithms by Robert Sedgewick and Philippe Flajolet [Amazon · Inform IT] overviews the primary techniques used in the mathematical analysis of algorithms. The material covered draws from classical mathematical topics, including discrete mathematics, elementary real analysis, and combinatorics, as well as from classical computer science topics, including algorithms and data structures.

- Chapter 1: Analysis of Algorithms considers the general motivations for algorithmic analysis and relationships among various approaches to studying performance characteristics of algorithms.
- Chapter 2: Recurrence Relations concentrates on fundamental mathematical properties of various types of recurrence relations which arise frequently when analyzing an algorithm through a direct mapping from a recursive representation of a program to a recursive representation of a function describing its properties.
- Chapter 3: Generating Functions introduces a central concept in the average-case analysis of algorithms: generating functions — a necessary and natural link between the algorithms that are our objects of study and analytic methods that are necessary to discover their properties.
- Chapter 4: Asymptotic Approximations examines methods of deriving approximate solutions to problems or of approximating exact solutions, which allow us to develop concise and precise estimates of quantities of interest when analyzing algorithms.
- Chapter 5: Trees investigates properties of many different types of trees, fundamental structures that arise implicitly and explicitly in many practical algorithms. Our goal is to provide access to results from an extensive literature on the combinatorial analysis of trees, while at the same time providing the groundwork for a host of algorithmic applications.
- Chapter 6: Permutations surveys combinatorial properties of permutations (orderings of the natural numbers) and their applications to fundamental and widely-used sorting algorithms.
- Chapter 7: String and Trie Trees studies basic combinatorial properties of strings, sequences of characters, and introduces algorithms that process strings ranging from fundamental methods at the heart of the field to highly sophisticated methods with a host of important applications.
- Chapter 8: Words and Maps covers global properties of words (σ-letter strings from an n-letter alphabet) and in classical applied algorithmics. The chapter also covers random maps (σ-letter words from an n-letter alphabet) and their applications.

Booksite. Reading a book and surfing the web are two different activities: This booksite is intended to provide supplementary material (Excerpts, Exercise solutions, Lecture slides), and while browsing the web; the textbook is for your use when initially learning new material and when reviewing for an exam. The booksite consists of the following elements:

- Excerpts. A condensed version of the text narrative, for reference while online.
- Exercise solutions. Solutions to selected exercises.
- Java, Sage, and Python code. Validation of analytic results.
2013: Mission accomplished

Analysis of Algorithms
Six offerings since 2013
Ten 90-minute lectures.
25,000+ registrants each time

Analytic Combinatorics
Six offerings since 2013
Ten 90-minute lectures.
10,000+ registrants each time
Q. Can an advanced subject such as *Analytic Combinatorics* be taught effectively online?
A. Absolutely! Indeed, advanced subjects may be a *sweet spot* for MOOCs.
Could this work for "Analytic Combinatorics"?

Enrollments worldwide in Analytic Combinatorics courses

YES! 70,000+ registrants (!)
Taking Education Online

• Frequently asked questions
• Disruptive changes
• A way forward
• Analytic combinatorics

• Questions answered
Questions answered

Q. What is a MOOC?
A. An **irresistible opportunity** for individuals and institutions who want to disseminate knowledge to **vastly expand** their reach.

Q. Can an introductory course such as *Algorithms* be taught effectively online?
A. Absolutely! Indeed, such subjects are a **sweet spot** for MOOCs.

Q. Can an advanced course such as *Analytic Combinatorics* be taught effectively online?
A. Absolutely! Indeed, such subjects may **also** be a **sweet spot** for MOOCs.

Q. That would seem to cover most courses, right?
A. *Anyone* interested in disseminating knowledge can put a good course online nowadays.
A. No university can afford to teach **all** advanced subjects.
A. *Every* faculty member teaches at least one good course with original material.
Questions answered (faculty)

Q. Isn't developing an online course time-consuming and difficult?
A. Yes! 50-100 hours of preparation per lecture.

A. Yes! My workflow requires skill in Unix, emacs, Mac OS X, DropBox, Illustrator, Indesign, Java, C, TeX, video capture, KeyNote, Acrobat, HTML, MathJax, Mathematica, PostScript, and a dozen other tools I can't even name.

A. It's less time-consuming than writing a book.

Q. I think my administration will fire me and use your lectures.
A. I'm wondering about that myself!
A. Cut costs by firing teachers? I think not.
A. You can use a blended model, where students watch my lectures and you make sure they understand the material and you evaluate your students.
A. Does it make sense for all of us to be preparing and delivering lectures on Quicksort, hashing, and a dozen other topics every semester? Just use mine.
A. Your students are already watching my lectures.
Questions answered (students)

Q. How much does it cost for me to take a MOOC?
A. It *must be* free. If not, it's not a MOOC.
A. Well, for the best learning outcome you should buy the book if it's offered at a reasonable price (and maybe the lectures, too).

Q. Who grades my assignments?
A. Most students self-assess (we provide answers).

Q. Can I get credit for taking a MOOC?
A. For *free*? Not likely.
A. You can pay to take a test.
A. You can take a MOOC as part (or all) of a course at a university.

*Observation*: The vast majority of MOOC students don't care about credit!

Q. If you're giving away content, what am I getting for the money at University X?
A. Universities make sure that you learn as much as you are able.
A. Universities certify that you did so.
Questions answered (administrators)

Q. Will MOOCs save us money?
A. NO. Quite the contrary.
A. They will improve the quality of your teaching, whether or not that is your goal.

Q. How?
A. Many online courses are now better than the ones you now offer. Use them!
A. Everyone's education now has a significant online component.
A. Developing a course into a MOOC is certain to improve it.
A. Vast online discussion groups are better than disinterested TAs.
A. Many (most?) students prefer online videos to traditional lectures.
A. Scheduling constraints will become virtually nonexistent.
A. Offloading the preparation removes a disincentive for researchers who teach.
A. You can vastly extend the reach of your institution at comparatively little cost.
**Questions answered (administrators)**

**Q.** Why aren't more faculty developing online courses?

**A.** You are providing only a tiny fraction of the resources needed.

**Q.** What can we do to improve things?

**A.** *Embrace technology.*

- Develop a *teaching class* of professors who can teach blended classes.
- Provide support and real incentives for teaching and content creation.
- Invest in *research* at the interface of education and technology.
- Attract and provide resources to the best and brightest professors in the field.
- Develop *academic leadership* for the effort.

**Q.** How much will it cost our institution to embrace online education?

**A.** Less than you are spending on many things that are less central to your mission.

**A.** *You need to plan to invest in this at the scale you are investing in the library.*

**A.** Can you afford to *not* embrace online education?
Online lectures on *Computer Science, An Interdisciplinary Approach*

- Precept
- Textbook
- Booksite
- Online course

Book, course, and booksite are 21st century resources. Intended audience: Teachers, students, and anyone wanting to learn CS.
# Computer Science Online Lectures

## Part I: Programming

<table>
<thead>
<tr>
<th>1. Basics</th>
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<tbody>
<tr>
<td>2. Loops/Conditionals</td>
</tr>
<tr>
<td>3. Arrays</td>
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<tr>
<td>4. Input/Output</td>
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<tr>
<td>5. Functions</td>
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<tr>
<td>6. Recursion</td>
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<td>7. Performance</td>
</tr>
<tr>
<td>8. Using Data Types</td>
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<tr>
<td>9. Creating Data Types</td>
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<tr>
<td>10. Languages</td>
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~500 slides

## Part II: Programming

<table>
<thead>
<tr>
<th>1. Searching/Sorting</th>
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<td>2. Stacks/Queues</td>
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<tr>
<td>3. Symbol Tables</td>
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<tr>
<td>4. REs/DFAs</td>
</tr>
<tr>
<td>5. Computability</td>
</tr>
<tr>
<td>6. Intractability</td>
</tr>
<tr>
<td>7. A Computing Machine</td>
</tr>
<tr>
<td>8. von Neumann Machines</td>
</tr>
<tr>
<td>9. Combinational Circuits</td>
</tr>
<tr>
<td>10. CPU Circuit</td>
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~500 slides
“But ... there’s a tsunami coming.”

“[Universities,] like newspapers and music companies and much of traditional media a little more than a decade ago are sailing in seemingly placid waters.”

(from John Hennessy in an interview for an article by Ken Auletta the New Yorker, 2012)
What happened to the tsunami?

I think the bloom is now off the rose, and now is going to be the time when some really hard-nosed thinking has to be done about the true value of these online courses.

Shirley Tilghman, 2013

Stumbling blocks

- Institutions are trying to take control (and failing).
- Misplaced focus on credentials and degrees.
- Content creation is the province of individuals.
- Bad business models, created prematurely.

Result: Plenty of money sitting on the table.

RS: Looks like a tsunami to me!

- Tens of thousands of pages of online content
- 100+ hours of lecture videos.
- Reaching millions of individuals.
- 1990s: Lucky to be able to teach my own children.
- 2030s: Will be teaching my own grandchildren.
Taking Education Online
A Unique Opportunity for the New Millenium

Robert Sedgewick
Princeton University