Advanced Topics in Sorting

- complexity
- system sorts
- duplicate keys
- comparators
complexity
- system sorts
- duplicate keys
- comparators
Complexity of sorting

**Computational complexity.** Framework to study efficiency of algorithms for solving a particular problem X.

**Machine model.** Focus on fundamental operations.

**Upper bound.** Cost guarantee provided by some algorithm for X.

**Lower bound.** Proven limit on cost guarantee of any algorithm for X.

**Optimal algorithm.** Algorithm with best cost guarantee for X.

Example: sorting.
- **Machine model = # comparisons**
- **Upper bound = N lg N from mergesort.**
- **Lower bound ?**
Decision Tree

code between comparisons (e.g., sequence of exchanges)
Comparison-based lower bound for sorting

**Theorem.** Any comparison based sorting algorithm must use more than \( N \lg N - 1.44 N \) comparisons in the worst-case.

**Pf.**
- Assume input consists of \( N \) distinct values \( a_1 \) through \( a_N \).
- Worst case dictated by tree height \( h \).
- \( N! \) different orderings.
- (At least) one leaf corresponds to each ordering.
- Binary tree with \( N! \) leaves cannot have height less than \( \lg (N!) \)

\[
\begin{align*}
h & \geq \lg N! \\
 & \geq \lg (N / e)^N \\
 & = N \lg N - N \lg e \\
 & \geq N \lg N - 1.44 N
\end{align*}
\]
Complexity of sorting

**Upper bound.** Cost guarantee provided by some algorithm for X.

**Lower bound.** Proven limit on cost guarantee of any algorithm for X.

**Optimal algorithm.** Algorithm with best cost guarantee for X.

**Example:** sorting.
- Machine model = # comparisons
- Upper bound = \(N \log N\) (mergesort)
- Lower bound = \(N \log N - 1.44 N\)

Mergesort is optimal (to within a small additive factor)

First goal of algorithm design: optimal algorithms
Complexity results in context

Mergesort is optimal (to within a small additive factor)

Other operations?
• statement is only about number of compares
• quicksort is faster than mergesort (lower use of other operations)

Space?
• mergesort is not optimal with respect to space usage
• insertion sort, selection sort, shellsort, quicksort are space-optimal
• is there an algorithm that is both time- and space-optimal?

Nonoptimal algorithms may be better in practice
• statement is only about guaranteed worst-case performance
• quicksort’s probabilistic guarantee is just as good in practice

Lessons
• use theory as a guide
• know your algorithms

Example: Selection

Find the $k^{th}$ largest element.
- Min: $k = 1$.
- Max: $k = N$.
- Median: $k = N/2$.

Applications.
- Order statistics.
- Find the “top $k$”

Use theory as a guide
- easy $O(N \log N)$ upper bound: sort, return $a[k]$
- easy $O(N)$ upper bound for some $k$: min, max
- easy $\Omega(N)$ lower bound: must examine every element

Which is true?
- $\Omega(N \log N)$ lower bound? [is selection as hard as sorting?]
- $O(N)$ upper bound? [linear algorithm for all $k$]
Lower bound may not hold if the algorithm has information about
• the key values
• their initial arrangement

Partially ordered arrays. Depending on the initial order of the input, we may not need \( N \lg N \) compares.

Duplicate keys. Depending on the input distribution of duplicates, we may not need \( N \lg N \) compares.

Digital properties of keys. We can use digit/character comparisons instead of key comparisons for numbers and strings.
Selection: quick-select algorithm

Partition array so that:
• element $a[m]$ is in place
• no larger element to the left of $m$
• no smaller element to the right of $m$
Repeat in one subarray, depending on $m$.

Finished when $m = k$ ← $a[k]$ is in place, no larger element to the left, no smaller element to the right

```java
public static void select(Comparable[] a, int k) {
    StdRandom.shuffle(a);
    int l = 0;
    int r = a.length - 1;
    while (r > l) {
        int i = partition(a, l, r);
        if      (m > k) r = m - 1;
        else if (m < k) l = m + 1;
        else return;
    }
}
```
Quick-select analysis

**Theorem.** Quick-select takes linear time on average.

**Pf.**
- Intuitively, each partitioning step roughly splits array in half.
- $N + N/2 + N/4 + \ldots + 1 \approx 2N$ comparisons.
- Formal analysis similar to quicksort analysis:

\[
C_N = 2N + k \ln \left( \frac{N}{k} \right) + (N - k) \ln \left( \frac{N}{N - k} \right)
\]

Ex: $(2 + 2 \ln 2)N$ comparisons to find the median

**Note.** Might use $\sim N^2/2$ comparisons, but as with quicksort, the random shuffle provides a probabilistic guarantee.

**Theorem.** [Blum, Floyd, Pratt, Rivest, Tarjan, 1973] There exists a selection algorithm that take linear time in the worst case.

**Note.** Algorithm is far too complicated to be useful in practice.

**Use theory as a guide**
- still worthwhile to seek practical linear-time (worst-case) algorithm
- until one is discovered, use quick-select if you don’t need a full sort
complexity
system sorts
duplicate keys
comparators
Sorting Challenge 1

**Problem:** sort a file of huge records with tiny keys.

**Ex:** reorganizing your MP3 files.

**Which sorting method to use?**
1. mergesort
2. insertion sort
3. selection sort
Sorting Challenge 1

**Problem:** sort a file of huge records with tiny keys.

**Ex:** reorganizing your MP3 files.

Which sorting method to use?

1. **mergesort** probably no, selection sort simpler and faster
2. **insertion sort** no, too many exchanges
3. **selection sort** YES, linear time under reasonable assumptions

**Ex:** 5,000 records, each 2 million bytes with 100-byte keys.

- Cost of comparisons: $100 \times 5000^2 / 2 = 1.25$ billion
- Cost of exchanges: $2,000,000 \times 5,000 = 10$ trillion
- Mergesort might be a factor of log (5000) slower.
Sorting Challenge 2

Problem: sort a huge randomly-ordered file of small records.
Ex: process transaction records for a phone company.

Which sorting method to use?
1. quicksort
2. insertion sort
3. selection sort
Sorting Challenge 2

**Problem:** sort a huge randomly-ordered file of small records.
**Ex:** process transaction records for a phone company.

Which sorting method to use?
1. **quicksort**  YES, it’s designed for this problem
2. **insertion sort**  no, quadratic time for randomly-ordered files
3. **selection sort**  no, always takes quadratic time
Sorting Challenge 3

**Problem:** sort a huge number of tiny files (each file is independent)

**Ex:** daily customer transaction records.

Which sorting method to use?
1. quicksort
2. insertion sort
3. selection sort

<table>
<thead>
<tr>
<th>file</th>
<th>record</th>
<th>key</th>
</tr>
</thead>
<tbody>
<tr>
<td>Fox</td>
<td>1</td>
<td>A</td>
</tr>
<tr>
<td>Quilici</td>
<td>1</td>
<td>C</td>
</tr>
<tr>
<td>Chen</td>
<td>2</td>
<td>A</td>
</tr>
<tr>
<td>Furia</td>
<td>3</td>
<td>A</td>
</tr>
<tr>
<td>Kanaga</td>
<td>3</td>
<td>B</td>
</tr>
<tr>
<td>Andrews</td>
<td>3</td>
<td>A</td>
</tr>
<tr>
<td>Rohde</td>
<td>3</td>
<td>A</td>
</tr>
<tr>
<td>Battle</td>
<td>4</td>
<td>C</td>
</tr>
<tr>
<td>Aaron</td>
<td>4</td>
<td>A</td>
</tr>
<tr>
<td>Giasi</td>
<td>4</td>
<td>B</td>
</tr>
</tbody>
</table>
Sorting Challenge 3

Problem: sort a huge number of tiny files (each file is independent)
Ex: daily customer transaction records.

Which sorting method to use?
1. quicksort   no, too much overhead
2. insertion sort   YES, much less overhead than system sort
3. selection sort   YES, much less overhead than system sort

Ex: 4 record file.
- $4 N \log N + 35 = 70$
- $2N^2 = 32$
Sorting Challenge 4

**Problem:** sort a huge file that is already almost in order.

**Ex:** re-sort a huge database after a few changes.

Which sorting method to use?
1. quicksort
2. insertion sort
3. selection sort
Sorting Challenge 4

**Problem:** sort a huge file that is already almost in order.
**Ex:** re-sort a huge database after a few changes.

Which sorting method to use?
1. quicksort  
   probably no, insertion simpler and faster
2. insertion sort  
   YES, linear time for most definitions of "in order"
3. selection sort  
   no, always takes quadratic time

**Ex:**
- ABCDEFGHIJKLMNOPQRSTUVWXYZ
- ZABCDDEFGHIJKLMNOPQRSTUVWXYZ
Sorting Applications

Sorting algorithms are essential in a broad variety of applications

- Sort a list of names.
- Organize an MP3 library.  
  obvious applications
- Display Google PageRank results.
- List RSS news items in reverse chronological order.

- Find the median.
- Find the closest pair.
- Binary search in a database.  
  problems become easy once items are in sorted order
- Identify statistical outliers.
- Find duplicates in a mailing list.

- Data compression.
- Computer graphics.
- Computational biology.  
  non-obvious applications
- Supply chain management.
- Load balancing on a parallel computer.
  ...

Every system needs (and has) a system sort!
System sort: Which algorithm to use?

Many sorting algorithms to choose from

internal sorts.
• Insertion sort, selection sort, bubblesort, shaker sort.
• Quicksort, mergesort, heapsort, samplesort, shellsort.
• Solitaire sort, red-black sort, splaysort, Dobosiewicz sort, psort, ...

distribution sorts. Poly-phase mergesort, cascade-merge, oscillating sort.

radix sorts.
• Distribution, MSD, LSD.
• 3-way radix quicksort.

parallel sorts.
• Bitonic sort, Batcher even-odd sort.
• Smooth sort, cube sort, column sort.
• GPUsort.
System sort: Which algorithm to use?

Applications have diverse attributes

- Stable?
- Multiple keys?
- Deterministic?
- Keys all distinct?
- Multiple key types?
- Linked list or arrays?
- Large or small records?
- Is your file randomly ordered?
- Need guaranteed performance?

Elementary sort may be method of choice for some combination. Cannot cover all combinations of attributes.

Q. Is the system sort good enough?
A. Maybe (no matter which algorithm it uses).
- complexity
- system sorts
- duplicate keys
- comparators
Duplicate keys

Often, purpose of sort is to bring records with duplicate keys together.
- Sort population by age.
- Finding collinear points.
- Remove duplicates from mailing list.
- Sort job applicants by college attended.

Typical characteristics of such applications.
- Huge file.
- Small number of key values.

Mergesort with duplicate keys: always $\sim N \log N$ compares

Quicksort with duplicate keys
- algorithm goes quadratic unless partitioning stops on equal keys!
- [many textbook and system implementations have this problem]
- 1990s Unix user found this problem in qsort()
Duplicate keys: the problem

Assume all keys are equal.
Recursive code guarantees that case will predominate!

**Mistake:** Put all keys equal to the partitioning element on one side
- easy to code
- guarantees $N^2$ running time when all keys equal

\[
\begin{array}{cccccccc}
C & B & C & B & B & C & B & B
\end{array}
\quad
\begin{array}{cccccccc}
\end{array}
\]

**Recommended:** Stop scans on keys equal to the partitioning element
- easy to code
- guarantees $N \log N$ compares when all keys equal

\[
\begin{array}{cccccccc}
C & B & C & B & B & C & B & B
\end{array}
\quad
\begin{array}{cccccccc}
\end{array}
\]

**Desirable:** Put all keys equal to the partitioning element in place

\[
\begin{array}{cccccccc}
C & C & C & C & B & C & C & C
\end{array}
\quad
\begin{array}{cccccccc}
\end{array}
\]

Common wisdom to 1990s: not worth adding code to inner loop
3-Way Partitioning

3-way partitioning. Partition elements into 3 parts:
- Elements between $i$ and $j$ equal to partition element $v$.
- No larger elements to left of $i$.
- No smaller elements to right of $j$.

![Diagram of 3-way partitioning]

Dutch national flag problem.
- not done in practical sorts before mid-1990s.
- new approach discovered when fixing mistake in Unix qsort()
- now incorporated into Java system sort
Solution to Dutch national flag problem.

3-way partitioning (Bentley-McIlroy).
• Partition elements into 4 parts:
  no larger elements to left of \( i \)
  no smaller elements to right of \( j \)
  equal elements to left of \( p \)
  equal elements to right of \( q \)
• Afterwards, swap equal keys into center.

All the right properties.
• in-place.
• not much code.
• linear if keys are all equal.
• small overhead if no equal keys.
private static void sort(Comparable[] a, int l, int r) {
    if (r <= l) return;
    int i = l-1, j = r;
    int p = l-1, q = r;

    while(true) {
        while (less(a[++i], a[r])) ;
        while (less(a[r], a[--j])) if (j == l) break;
        if (i >= j) break;
        if (eq(a[i], a[r])) exch(a, ++p, i);
        if (eq(a[j], a[r])) exch(a, --q, j);
    }
    exch(a, i, r);

    j = i - 1;
    i = i + 1;
    for (int k = l ; k <= p; k++) exch(a, k, j--);
    for (int k = r-1; k >= q; k--) exch(a, k, i++);

    sort(a, l, j);
    sort(a, i, r);
}
Theorem. [Sedgewick-Bentley] Quicksort with 3-way partitioning is optimal for random keys with duplicates.

Proof (beyond scope of 226).
• generalize decision tree
• tie cost to entropy
• note: cost is linear when number of key values is $O(1)$

**Bottom line:** Randomized Quicksort with 3-way partitioning reduces cost from linearithmic to linear (!) in broad class of applications
3-way partitioning animation
complexity
system sorts
duplicate keys
comparators
Generalized compare

**Comparable** interface: sort uses type’s `compareTo()` function:

```java
class Date implements Comparable<Date> {
    private int month, day, year;

    public Date(int m, int d, int y) {
        month = m;
        day   = d;
        year  = y;
    }

    public int compareTo(Date b) {
        Date a = this;
        if (a.year  < b.year  ) return -1;
        if (a.year  > b.year  ) return +1;
        if (a.month < b.month) return -1;
        if (a.month > b.month) return +1;
        if (a.day   < b.day  ) return -1;
        if (a.day   > b.day  ) return +1;
        return 0;
    }
}
```
Generalized compare

**Comparable** interface: sort uses type's `compareTo()` function:

- **Problem 1:** Not type-safe
- **Problem 2:** May want to use a different order.
- **Problem 3:** Some types may have no “natural” order.

**Ex.** Sort strings by:
- Natural order.  
- Case insensitive.  
- French.  
- Spanish.

```
String[] a;
...
Arrays.sort(a);
Arrays.sort(a, String.CASE_INSENSITIVE_ORDER);
Arrays.sort(a, Collator.getInstance(Locale.FRENCH));
Arrays.sort(a, Collator.getInstance(Locale.SPANISH));
```

Import java.text.Collator;

ch and rr are single letters
Generalized compare

**Comparable** interface: sort uses type’s `compareTo()` function:

**Problem 1:** Not type-safe

**Problem 2:** May want to use a different order.

**Problem 3:** Some types may have no “natural” order.

**A bad client**

```java
public class BadClient {
    public static void main(String[] args) {
        int N = Integer.parseInt(args[0]);
        Comparable[] a = new Comparable[N];
        ...
        a[i] = 1;
        ...
        a[j] = 2.0;
        ...
        Insertion.sort(a);
    }
}
```

Exception ... java.lang.ClassCastException: java.lang.Double
at java.lang.Integer.compareTo(Integer.java:35)
**Generalized compare**

**Comparable** interface: sort uses type’s `compareTo()` function:

- **Problem 1:** Not type-safe
- **Problem 2:** May want to use a different order.
- **Problem 3:** Some types may have no “natural” order.

**Fix:** generics

```java
public class Insertion {
    public static <Key extends Comparable<Key>>
        void sort(Key[] a) {
        int N = a.length;
        for (int i = 0; i < N; i++)
            for (int j = i; j > 0; j--)
                if (less(a[j], a[j-1])) exch(a, j, j-1);
                else break;
    }
}
```

Client can sort array of any `Comparable` type: `Double[]`, `File[]`, `Date[]`, ...

Necessary in system library code; not in this course (for brevity)
Generalized compare

**Comparable** interface: sort uses type’s `compareTo()` function:

Problem 1: Not type-safe
Problem 2: May want to use a different order.
Problem 3: Some types may have no “natural” order.

**Solution:** Use **Comparator** interface

**Comparator** interface. Require a method `compare()` so that `compare(v, w)` is a total order that behaves like `compareTo()`.

**Advantage.** Separates the definition of the data type from definition of what it means to compare two objects of that type.
- add any number of new orders to a data type.
- add an order to a library data type with no natural order.
Generalized compare

**Comparable interface:** sort uses type’s `compareTo()` function:

**Problem 2:** May want to use a different order.
**Problem 3:** Some types may have no “natural” order.

**Solution:** Use `Comparator` interface

**Example:**

```java
class ReverseOrder implements Comparator<String> {
    public int compare(String a, String b) {
        return -a.compareTo(b);
    }
}
```

reverse sense of comparison

```java
Arrays.sort(a, new ReverseOrder());
...```

**Generalized compare**

Easy modification to support comparators in our sort implementations

- pass comparator to `sort()`, `less()`
- use it in `less()`

Example: (insertion sort)

```java
public static void sort(Object[] a, Comparator comparator) {
    int N = a.length;
    for (int i = 0; i < N; i++)
        for (int j = i; j > 0; j--)
            if (less(comparator, a[j], a[j-1]))
                exch(a, j, j-1);
            else break;
}

private static boolean less(Comparator c, Object v, Object w) {
    return c.compare(v, w) < 0;
}

private static void exch(Object[] a, int i, int j) {
    Object t = a[i]; a[i] = a[j]; a[j] = t;
}
```
Generalized compare

Comparators enable multiple sorts of single file (different keys)

Example. Enable sorting students by name or by section.

```java
Arrays.sort(students, Student.BY_NAME);
Arrays.sort(students, Student.BY_SECT);
```

<table>
<thead>
<tr>
<th>Name</th>
<th>Section</th>
<th>Phone</th>
<th>Room</th>
</tr>
</thead>
<tbody>
<tr>
<td>Andrews</td>
<td>3</td>
<td>664-480-0023</td>
<td>097 Little</td>
</tr>
<tr>
<td>Battle</td>
<td>4</td>
<td>874-088-1212</td>
<td>121 Whitman</td>
</tr>
<tr>
<td>Chen</td>
<td>2</td>
<td>991-878-4944</td>
<td>308 Blair</td>
</tr>
<tr>
<td>Fox</td>
<td>1</td>
<td>884-232-5341</td>
<td>11 Dickinson</td>
</tr>
<tr>
<td>Furia</td>
<td>3</td>
<td>766-093-9873</td>
<td>101 Brown</td>
</tr>
<tr>
<td>Gazsi</td>
<td>4</td>
<td>665-303-0266</td>
<td>22 Brown</td>
</tr>
<tr>
<td>Kanaga</td>
<td>3</td>
<td>898-122-9643</td>
<td>22 Brown</td>
</tr>
<tr>
<td>Rohde</td>
<td>3</td>
<td>232-343-5555</td>
<td>343 Forbes</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Name</th>
<th>Section</th>
<th>Phone</th>
<th>Room</th>
</tr>
</thead>
<tbody>
<tr>
<td>Fox</td>
<td>1</td>
<td>884-232-5341</td>
<td>11 Dickinson</td>
</tr>
<tr>
<td>Chen</td>
<td>2</td>
<td>991-878-4944</td>
<td>308 Blair</td>
</tr>
<tr>
<td>Andrews</td>
<td>3</td>
<td>664-480-0023</td>
<td>097 Little</td>
</tr>
<tr>
<td>Furia</td>
<td>3</td>
<td>766-093-9873</td>
<td>101 Brown</td>
</tr>
<tr>
<td>Kanaga</td>
<td>3</td>
<td>898-122-9643</td>
<td>22 Brown</td>
</tr>
<tr>
<td>Rohde</td>
<td>3</td>
<td>232-343-5555</td>
<td>343 Forbes</td>
</tr>
<tr>
<td>Battle</td>
<td>4</td>
<td>874-088-1212</td>
<td>121 Whitman</td>
</tr>
<tr>
<td>Gazsi</td>
<td>4</td>
<td>665-303-0266</td>
<td>22 Brown</td>
</tr>
</tbody>
</table>
Generalized compare

Comparators enable multiple sorts of single file (different keys)

Example. Enable sorting students by name or by section.

```java
public class Student {
    private String name;
    private int section;
    ...

    private static class ByName implements Comparator<Student> {
        public int compare(Student a, Student b) {
            return a.name.compareTo(b.name);
        }
    }

    private static class BySect implements Comparator<Student> {
        public int compare(Student a, Student b) {
            return a.section - b.section;
        }
    }
}
```

only use this trick if no danger of overflow
Generalized compare problem

A typical application
- first, sort by name
- then, sort by section

Arrays.sort(students, Student.BY_NAME);

Arrays.sort(students, Student.BY_SECT);

<table>
<thead>
<tr>
<th>Name</th>
<th>Section</th>
<th>Phone</th>
<th>Room</th>
</tr>
</thead>
<tbody>
<tr>
<td>Andrews</td>
<td>3</td>
<td>664-480-0023</td>
<td>097 Little</td>
</tr>
<tr>
<td>Battle</td>
<td>4</td>
<td>874-088-1212</td>
<td>121 Whitman</td>
</tr>
<tr>
<td>Chen</td>
<td>2</td>
<td>991-878-4944</td>
<td>308 Blair</td>
</tr>
<tr>
<td>Fox</td>
<td>1</td>
<td>884-232-5341</td>
<td>11 Dickinson</td>
</tr>
<tr>
<td>Furia</td>
<td>3</td>
<td>766-093-9873</td>
<td>101 Brown</td>
</tr>
<tr>
<td>Gazsi</td>
<td>4</td>
<td>665-303-0266</td>
<td>22 Brown</td>
</tr>
<tr>
<td>Kanaga</td>
<td>3</td>
<td>898-122-9643</td>
<td>22 Brown</td>
</tr>
<tr>
<td>Rohde</td>
<td>3</td>
<td>232-343-5555</td>
<td>343 Forbes</td>
</tr>
</tbody>
</table>

@#%&@@!! Students in section 3 no longer in order by name.

A stable sort preserves the relative order of records with equal keys. Is the system sort stable?
Stability

Q. Which sorts are stable?
- Selection sort?
- Insertion sort?
- Shellsort?
- Quicksort?
- Mergesort?

A. Careful look at code required.

Annoying fact. Many useful sorting algorithms are unstable.

Easy solutions.
- add an integer rank to the key
- careful implementation of mergesort

Open: Stable, inplace, optimal, practical sort??
Java system sorts

Use theory as a guide: Java uses both mergesort and quicksort.
• Can sort array of type Comparable or any primitive type.
• Uses quicksort for primitive types.
• Uses mergesort for objects.

```java
import java.util.Arrays;
public class IntegerSort {
    public static void main(String[] args) {
        int N = Integer.parseInt(args[0]);
        int[] a = new int[N];
        for (int i = 0; i < N; i++)
            a[i] = StdIn.readInt();
        Arrays.sort(a);
        for (int i = 0; i < N; i++)
            System.out.println(a[i]);
    }
}
```

Q. Why use two different sorts?
A. Use of primitive types indicates time and space are critical
A. Use of objects indicates time and space not so critical
Arrays.sort() for primitive types

**Bentley-McIlroy.** [Engineeering a Sort Function]
- Original motivation: improve qsort() function in C.
- Basic algorithm = 3-way quicksort with cutoff to insertion sort.
- Partition on Tukey's ninther: median-of-3 elements, each of which is a median-of-3 elements.

Why use ninther?
- better partitioning than sampling
- quick and easy to implement with macros
- less costly than random

*Good idea? Stay tuned.*
Achilles heel in Bentley-McIlroy implementation (Java system sort)

Based on all this research, Java’s system sort is solid, right?

McIlroy’s devious idea. [A Killer Adversary for Quicksort]

- Construct malicious input while running system quicksort, in response to elements compared.
- If $p$ is pivot, commit to $(x < p)$ and $(y < p)$, but don’t commit to $(x < y)$ or $(x > y)$ until $x$ and $y$ are compared.

Consequences.

- Confirms theoretical possibility.
- Algorithmic complexity attack: you enter linear amount of data; server performs quadratic amount of work.
Achilles heel in Bentley-McIlroy implementation (Java system sort)

A killer input:
- blows function call stack in Java and crashes program
- would take quadratic time if it didn’t crash first

250,000 integers between 0 and 250,000

Java’s sorting library crashes, even if you give it as much stack space as Windows allows.

Attack is not effective if file is randomly ordered before sort
System sort: Which algorithm to use?

Applications have diverse attributes

- Stable?
- **Multiple keys?**
- Deterministic?
- Keys all distinct?
- Multiple key types?
- Linked list or arrays?
- Large or small records?
- Is your file randomly ordered?
- Need guaranteed performance?

Elementary sort may be method of choice for some combination. Cannot cover all combinations of attributes.

Q. Is the system sort good enough?
A. Maybe (no matter which algorithm it uses).