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Soliton phase shifts in a dissipative lattice

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We measured the amplitude, width, velocity, and phase-shift characteristics of solitons in the *LC* lattice of Hirota and Suzuki [Proc. IEEE **61**, 1483 (1974)], an inexact electrical analog of the Toda lattice. We found that dissipative effects are important in this lattice, and that amplitudes decrease, widths increase, and velocities remain constant as solitons propagate along the lattice. We found that there are distinct families of solitons, distinguished by the shape of the amplitude-width curve, and determined by the reverse bias of the input pulses that generate them. Within a family, the properties of a soliton are determined by the input amplitude and width of its generating pulse. Marked phase shifts occur when solitons of different families collide head-on, and these phase shifts are found to be independent (within experimental error) of the location of the collision on the lattice. Thus, the positional phase of solitons can be used to encode information in a simple way, and the lattice used to perform computation, of which parity checking is a simple example.

INTRODUCTION

Hirota and Suzuki[1] built a network comprising a ladder of inductors (in the series arms) and nonlinear capacitors (in the parallel arms). This nonlinear network supports solitons, and is an inexact electrical analog of the Toda lattice, which is composed of balls of unit mass, connected by nonlinear springs and interacting via an exponential potential.[2] The idea of Hirota and Suzuki's experiment was to use voltage-controlled capacitors to make the equations governing the electrical system equivalent to those governing the mechanical Toda lattice (which, in the continuum limit, reduce to the *K-dV* equation[2]). We should note that the reverse-biased varactor diodes used in our experiment do not exhibit the logarithmic dependence of capacitance on voltage that would make the electrical network an exact analog of the mechanical Toda lattice.

It has been suggested that if the positional phase of a soliton is used to encode information, the phase shifts resulting from collisions can accomplish useful computation.[3,4] This idea arises from work done with cellular automata that support solitonlike structures, and the design of a carry-ripple adder using these pseudosolitons has been described.[4] To carry this idea over to solitons in physical systems, however, it is important that the phase shifts on collision depend only on the types of colliding solitons, and not on the position in the ladder where the collision takes place.

The experiment reported in this paper was motivated by the question of whether the soliton phase shifts in the Hirota–Suzuki lattice are determined only by the identity of the colliding solitons and not the position of the collision. We recreated a ladder similar to that described by Hirota and Suzuki, and observed first the amplitude, width, and velocity characteristics of solitons and then the phase shifts resulting from collisions between two solitons moving in opposite directions along the ladder. A description of the experimental apparatus can be found in the Appendix.

In a dissipative lattice, the soliton amplitude decreases as it travels along the lattice, with a corresponding increase in width. Hirota and Suzuki mention this, but do not describe the effect of this dissipation on velocity or phase shift. Nonetheless, we found that even with dissipation the velocity remains constant along the lattice, so that a particular soliton has an unchanging characteristic velocity. Moreover, we found that the phase change resulting from a collision is indeed dependent only on the particular solitons involved, and not on the amplitudes and widths at the moment of collision. In addition, the velocity of a soliton depends not only on the amplitude but also on the pulse width and reverse bias of the input pulse. It is the reverse bias which seems to be the factor distinguishing families of solitons that exhibit similar behavior.

Finally, we discuss the possible computational uses of solitons, using as an example a parity checker.

VELOCITY, AMPLITUDE, AND WIDTH MEASUREMENTS

There are 80 sections in the lattice; the physical circuit would measure about 2 m if fully extended. For several reasons, among them the following, the amplitude of a soliton diminishes as it travels along the lattice: (1) There is ohmic dissipation in the windings of the coils and the interconnecting wires; (2) The small oscillatory tail that follows the passage of a soliton can briefly forward bias the varactor diodes, causing them to conduct current.

Figure 1 shows the variation of log amplitude with position on the lattice for a particular soliton. Clearly, the amplitude decay is not exponential. Figure 2 compares the amplitude-position curves for solitons generated by initial pulses of the same reverse bias and pulse width, but different amplitude. The pattern in every case is similar; solitons of different amplitudes suffer the same kind of decay. Figure 3 shows a similar comparison for solitons generated from inputs with the same amplitude and pulse width, but different reverse bias. Once again, it is clear that the decay is similar, with solitons of lower reverse bias having higher amplitudes along the lattice.

Although the amplitude of a soliton decreases along the lattice, its velocity is observed to stay constant within the

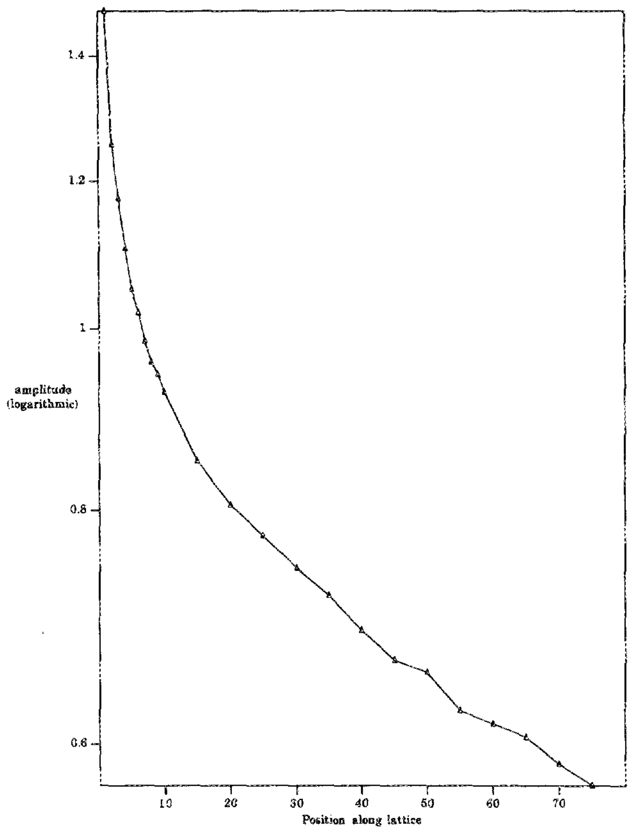

FIG. 1. Amplitude of a typical soliton in volts vs its position along the lattice. Initial amplitude = 4 V; dc bias = − 0.26 V; pulse width = 90 ns; pulse repetition rate = 5 kHz.

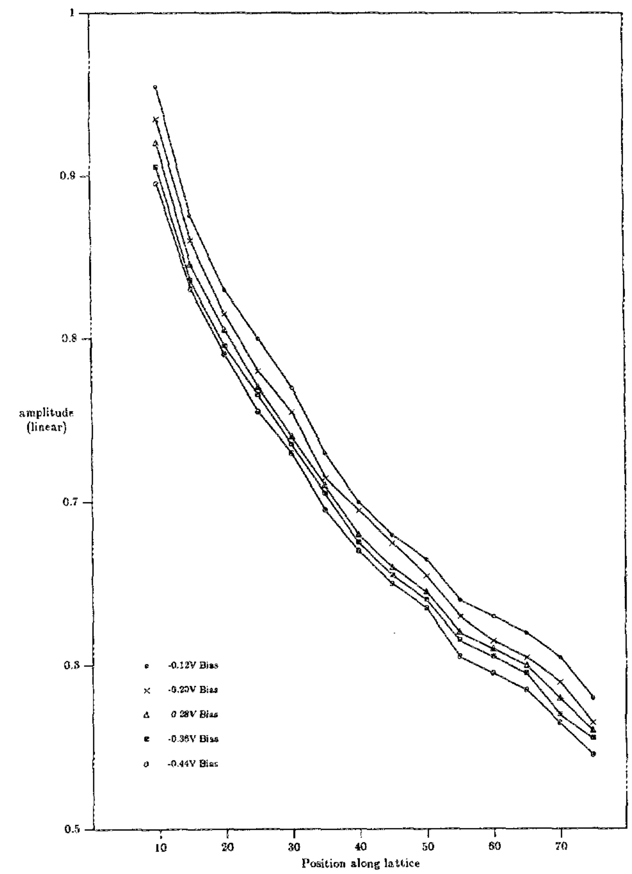

FIG. 3. Amplitude vs position for solitons with various input reverse biases. Input amplitude = 4 V; pulse width = 90 ns; repetition rate = 5 kHz.

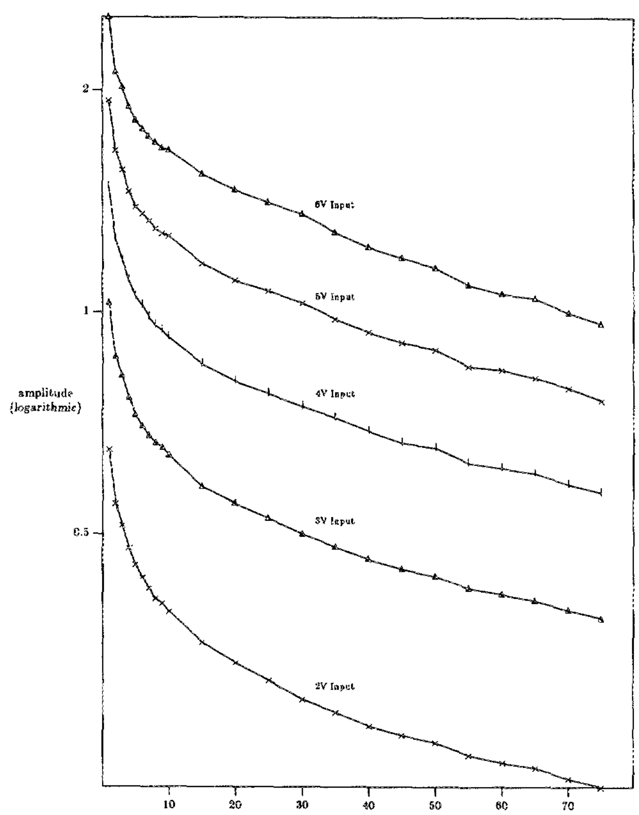

FIG. 2. Amplitude vs position for solitons with various input amplitudes. dc bias = − 0.26 V; pulse width = 90 ns; repetition rate = 5 kHz.

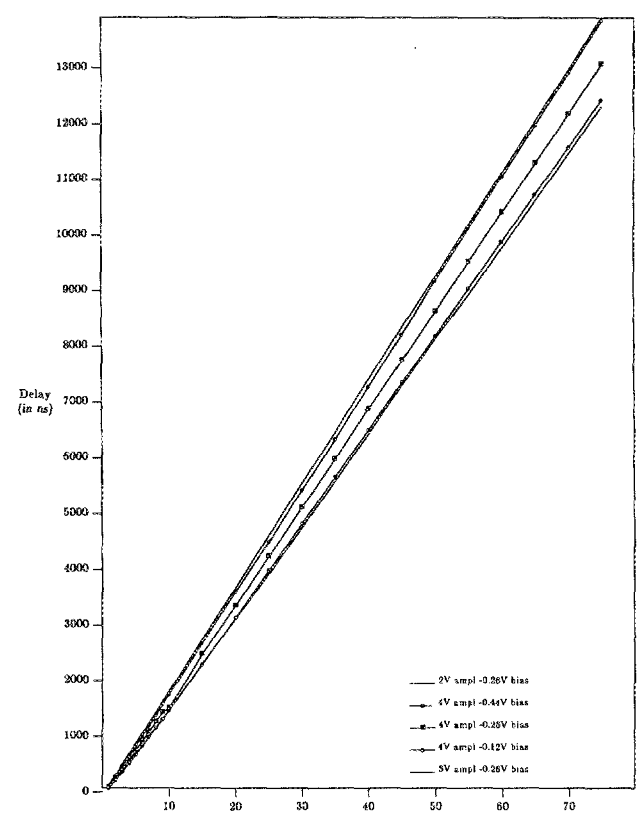

FIG. 4. Delay from input vs position for various input amplitudes and biases. Pulse width = 90 ns; repetition rate = 5 kHz.

limits of experimental error. Figure 4 shows the delay between the input pulse and the observed soliton, at various positions along the lattice and for different solitons. (Note: Although the solitons have a reverse bias, so that they start rising below the x axis, delay measurements are taken at the points where the waveform intersects the x axis for uniformity of measurement. The same applies to width measurements as well, so that the measured widths are smaller than the actual widths of the solitons, but the deviation is uniformly defined.) The linearity of the delay function (and hence the constancy of the velocity) is evident from the graph. The graph also shows the variation of velocity with input amplitude and reverse bias. A higher input amplitude generates a faster soliton (the same was found to be true for pulse width), and one with a smaller input reverse bias is faster than one with a larger bias. It appears as well that although the velocity of a soliton is constant and characterizes that soliton, distinct solitons can be found that have the same velocity, but only if at least two of their defining characteristics (input amplitude, pulse width, and reverse bias) are different. For example, a soliton with a higher input amplitude and higher input reverse bias than another can have the same velocity as that other.

The existence of distinct families of solitons is revealed by plotting amplitude versus width curves (amplitude of a soliton at a particular position along the lattice versus its width—measured as described previously—at that position) for various solitons. Figure 5 shows the amplitude-width curves for three solitons having different input amplitudes but the same input reverse bias. Naturally, the highest curve is for the soliton with the greatest input amplitude and the greatest width is attained by the one with the lowest amplitude, but the general shape of the curve is the same in all three cases. Figure 6 shows the same three curves, along with the curves for another set of three solitons having a different reverse bias. This graph suggests that there are different families of solitons that exhibit qualitatively similar behavior, and that these families are distinguished by the reverse bias of their generating pulses (this suggestion is borne out in the following discussion of collisions). Within a family, the variations are as described above, with variations in input pulse width having effects similar to variations in amplitude; across families, it is found that solitons with lower reverse biases have greater amplitudes as well as greater widths along the lattice.

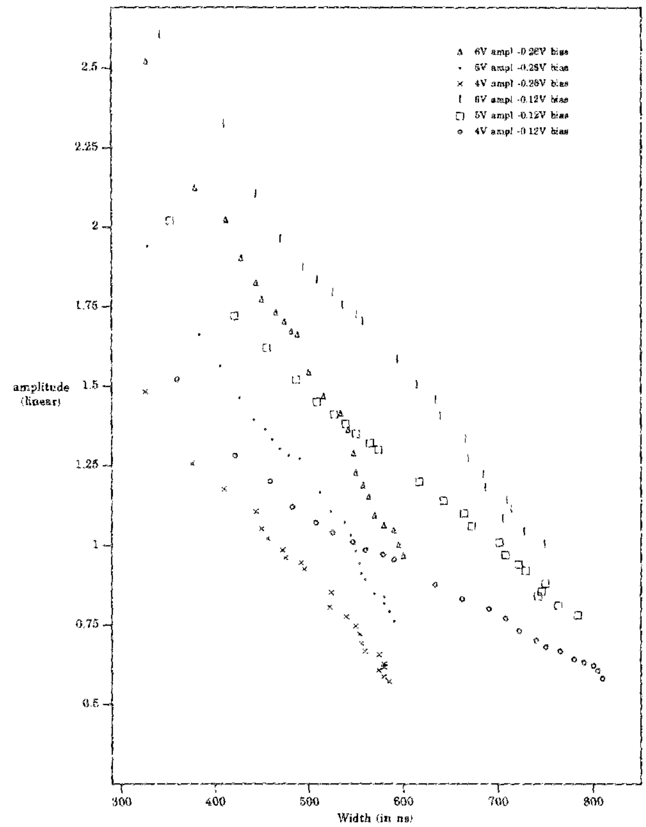

FIG. 6. Amplitude vs width at particular positions for solitons with two different input reverse biases and three different input amplitudes. Pulse width = 90 ns; repetition rate = 5 kHz.

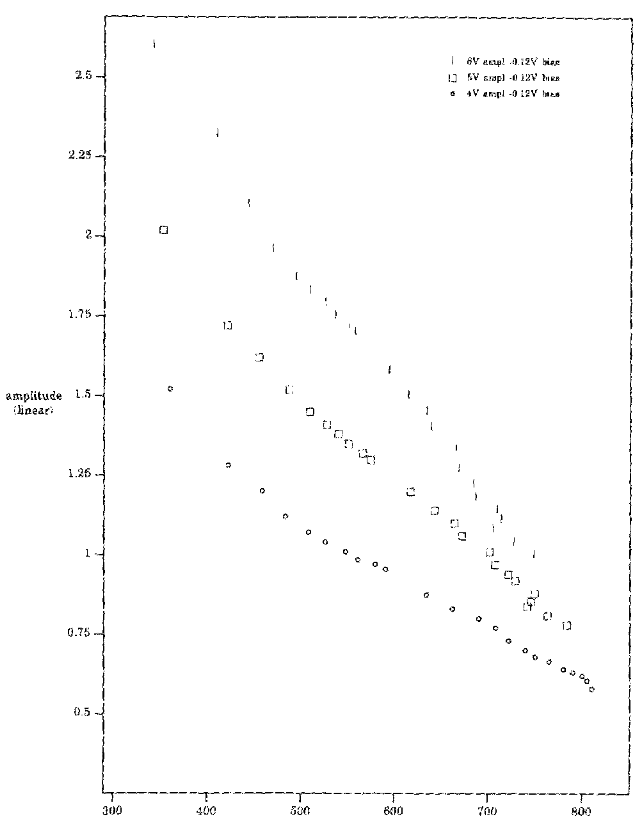

FIG. 5. Amplitude vs width at particular positions for solitons with different amplitudes and the same input reverse bias. dc bias = −0.12 V; pulse width = 90 ns; repetition rate = 5 kHz.

PHASE SHIFTS ON COLLISION

We now come to the main point of our experiment: the measurement of phase shifts resulting from head-on collisions between two solitons. The results, tabulated in Table I, show that the phase shift is dependent on the identities of the colliding solitons. If the colliding solitons belong to the same family (i.e., have the same reverse bias), the phase shifts are very small. Identical solitons suffer almost no phase shift on collision, and the phase shift of a soliton is found to increase (i.e., the soliton is delayed) with increasing input amplitude

TABLE I. Phase shifts on collision with different solitons.

Input values of measured soliton				
Amplitude:		4 V		
dc bias:		−0.26 V		
Width:		90 ns		
Repetition rate:		5 kHz		

Observations

Input values of other soliton: dc bias (V)	Amplitude (V)	Width (ns)	Pos. of collison	Phase shift (ns)
−0.26	4	90	40	0
−0.26	4	120	41	−14
−0.26	4	160	42	−30
−0.26	6	90	41	+34
−0.26	2	90	39	+18
−0.12	4	90	40	−262
−0.44	4	90	40	+366
−0.58	4	90	40	+704
−0.66	4	90	39	+926
−0.66	5	90	39	+935
−0.66	6	90	40	+958
−0.66	8	90	41	+968
−0.66	4	160	40	+896
−0.66	4	50	36	+942
−0.66	8	120	43	+918
−0.66	8	160	44	+886
−0.66	8	50	38	+1006

of the other soliton, and to decrease (i.e., the soliton gains time) with increasing input pulse width of the other. This shows that even within a family the phase shift is not dependent only on the velocity of the soliton collided with—for both increases in amplitude and increases in pulse width cause increases in velocity—but separately on its input amplitude and pulse width. It is, therefore, possible for two nonidentical solitons of the same family to undergo no phase change on collision (just like two identical solitons) if their input amplitudes and pulse widths are fine-tuned for this purpose.

Colliding solitons of different families suffer large phase shifts: the larger the difference in reverse bias, the greater the shift. Given the two reverse biases, however, the same results hold for variations in input amplitude and pulse width, although the effect of a variation in reverse bias (a change of family) on the phase shift is far greater than that of a variation in amplitude or width. A final result regarding the dependence of phase shifts on the particular identities of the colliding solitons is that when two solitons of different families collide, the one with the smaller bias is delayed and that with the larger bias gains time.

Despite the intricate dependence of phase shift on input amplitudes, pulse widths and reverse biases, our experiment validates the encoding of computation in the phase shifts suffered by colliding solitons. For it is found that the phase shift on collision is indeed independent (within the limits of experimental error) of the location of the collision on the lattice (and therefore of the dissipation of amplitude along the lattice). For a description of how the same solitons were made to collide at different points on the lattice, please see the Appendix. The results of these measurements for two particular solitons are shown in Table II.

TABLE II. Collisions with same soliton at different locations.

Parameter	Input values: Measured soliton	Other soliton
Amplitude:	4 V	4 V
dc bias:	−0.12 V	−0.26 V
Pulse width:	90 ns	90 ns

Observations

Position of collision	Phase shift (ns)
45	188
50	186
55	188
60	188
65	190

DESIGN FOR A PARITY CHECKER

The independence of phase shifts of the location of collisions can be exploited to design a parity checker. The idea is simple. We send a parity-checker soliton down one end of the lattice and input the data bits encoded in solitons at the other end. The phase of the checker soliton when it arrives at the other end of the lattice will indicate whether the parity of the data bits is even or odd, as long as a suitable encoding method is used. One example of an encoding scheme is as follows. A "0" bit is represented by one soliton (i.e., one input pulse) and a "1" bit by two, so that if a collision with a 0 causes a phase shift of x in the checker soliton, collision with a 1 will cause a shift of $2x$. After passing through all the data solitons, the phase shift of the checker soliton (mod x) will be 0 or 1 if the data stream had an even or odd number of 0's, respectively.

The above scheme can be tested by the following arrangement: The checker soliton can be input at one end of the lattice with a pulse generator, and the data stream by a burst mode pulse generator, triggered by the first generator, at the other end. The result can be decoded by amplifying the output checker soliton (so that it can drive a logic gate) and ANDing it with a clock of appropriate frequency so that the result of the AND operation will be 1 if and only if the checker soliton undergoes an even number of collisions. However, reflections become troublesome when a soliton undergoes many collisions during one run down the lattice, and an interesting problem is to find appropriate termination impedances when both ends of the lattice are used as inputs.

SUMMARY

Our results, then, can be summarized as follows:

(1) The velocity (as well as the amplitude at a particular point on the lattice) of a soliton in the Hirota–Suzuki lattice depends on the amplitude, pulse width, and reverse bias of the input pulse that generates it, being greater for larger amplitude, for larger pulse width, and for smaller reverse bias. Also, the velocity of a particular soliton remains constant along the lattice, being unaffected by the decrease in amplitude.

(2) The amplitude-width curves are similar for solitons with the same input reverse bias, but differ in shape for solitons with different input biases, so that families of solitons are characterized chiefly by their input reverse biases.

(3) Phase shifts on head-on collision between solitons depend primarily on the families of the solitons, and, given the families, on the particular input amplitudes and pulse widths.

(4) Given particular colliding solitons, the phase shift on collision is independent of the position on the lattice where the collision occurs, so that the phase shifts can be used to perform computation, a simple example being a parity checker.

The independence of velocity of soliton amplitude along the lattice [result (2)] is similar to the behavior of solitons in the nonlinear Schrödinger equation with dissipation.[5] Theoretical or computational verification of our results requires further work on modeling the Hirota–Suzuki lattice with dissipation. It would also be interesting to see if more sophisticated computation can be implemented using similar phase-shift encoding. Fiber optic transmission lines, which support envelope solitons, are fast and small enough to offer an attractive medium for such an application.

ACKNOWLEDGMENTS

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APPENDIX: APPARATUS

The Hirota–Suzuki nonlinear LC lattice used in the experiment is low pass in the small-signal limit; that is, the inductors are in the series arms and the capacitors in the parallel arms (see Fig. 7). Altogether there were 80 four-terminal sections, each consisting of an inductor and a capacitor. We used reverse-biased varactor diodes (Phillips ECG 618) for the voltage-dependent capacitors, and iron-core coils (J. W. Miller Part No. 4628) for the linear inductors. The diodes had a measured capacitance of 440 pF at 1.2 V and a minimum Q of 200 at 1 V; the inductance was 39 μH with a minimum Q of 70 at the testing frequency (2.5 MHz). In this lattice, solitons are observed for a certain range of input pulse amplitudes, widths, and reverse biases. The influence of the oscillatory tail of a soliton can be diminished by increasing the input pulse width—which raises the amplitude of the soliton—but the width must be kept small enough (less than 300 ns for most of our observations) so that multiple solitons are not generated.

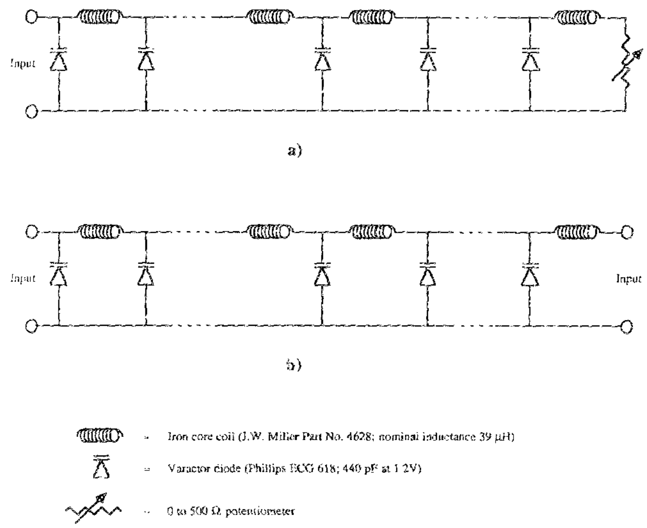

FIG. 7. Experimental setup: (a) for soliton properties and (b) for collisions.

Owing to the nonlinearity of the lattice, finding a suitable termination impedance is a problem. It is essential to minimize reflection from the ends of the lattice, especially when observing collisions, for collisions with the reflections will affect the observed phase shifts. Our experiment was divided into two parts: observing the properties of solitons and measuring phase shifts on collision. For the first part, we used a single input pulse generator (Hewlett Packard 8116A) at one end and a fine-tuned potentiometer as a termination impedance at the other [see Fig. 7(a)]. For most cases, a termination resistance of about 210 Ω was found to be satisfactory. For the second part, we used a pulse generator at each end, with one triggered by the other, to observe collisions [see Fig. 7(b)]. In this case, using a potentiometer at each end would greatly affect the input properties of the lattice, so the output impedances of the pulse generators were used as terminations. This was not very effective in suppressing reflection, and a better method will have to be found in order to implement computational schemes such as the parity checker.

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