Email to “Delnoij, J.M.J. (Joyce)” <J.M.J.Delnoij@uu.nl>

RE: Question about overbidding in auctions with spiteful bidders

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1 It ain’t necessarily so

Hello Ms. Delnoij,

I (KS) write on behalf of John Morgan and myself. Thank you for your question—it’s great that you read our paper closely enough to catch a sloppy, and apparently incorrect, claim. Please get back to us with your thoughts on the following, and also keep us informed of your future work.

The good news (for us) is that the point of the remark in the paper is supported by the uniform case, since we say that the effect can be tested by experiment. The bad news is that, I believe, the result is not in general true in the second-price case (but see bleow). I’m guessing that the first-price case goes the same way, but I think you might enjoy looking into that yourself after considering the following argument and John’s remarks.

The equilibrium overbidding in a second-price auction is [1]

$$OB_2(v) = \frac{\int_v^1 (1-F(t))^m dt}{(1-F(v))^m} \ , \tag{1}$$

where $m = (1+\alpha)/\alpha > 0$. Assume that F is increasing, and sufficiently smooth for our purposes.

Let

$$G(t) = (1-F(t))^m \ . \tag{2}$$

Then G is decreasing and smooth iff F is increasing and smooth. We can write

$$OB_2(v) = \frac{\int_v^1 G(t)dt}{G(v)} \ , \tag{3}$$

and the derivative of the overbidding function is

$$OB_2'(v) = -\frac{G'(v)}{G^2(v)} \int_v^1 G(t)\, dt - 1 \ . \tag{4}$$

We want to prove that that $OB_2'(v) \le 0$ for all $0 \le v \le 1$ and appropriate G. Write this

$$\frac{[-G'(v)]}{G^2(v)} \int_v^1 G(t)\, dt \le 1 \ . \tag{5}$$

This checks in the uniform case and all m, when $F(t) = t$, $G(t) = (1-t)^m$, and the left-hand side is $m/(m+1) < 1$.

However, suppose that we choose $G(t)$ so that $G(v)$ goes down very sharply (but smoothly) over a very small interval at $t = v+$. This has a vanishingly small effect on all the factors on the left-hand side of Inequality 5 except $[-G'(v)]$, which can be made arbitrarily large and positive, thus violating the inequality.

The interpretation of this counterexample is that the original density f has a smooth "near-atom" at v. What is the intuition?

Here are further remarks from John:

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...the kind of pathology [envisioned] occurs in a bunch of places in auction
theory and signaling as well. The usual strategy of the economist is to rule
this out through additional assumptions on F. Often, we will assume that the
density of F satisfies a monotone likelihood ratio property.  Sometimes, we'll
assume that F has a strictly increasing hazard rate everywhere or, the strongest
assumption, we'll assume that the density of f is weakly increasing everywhere.
All of these sorts of things rule out G' doing crazy stuff at some point.

I don't think it's worth it to figure out what the right additional assumption
is to make the result fly in more generality, but it may be worth pointing out
[...] that some assumption like this will probably work.
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Best, ken


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References

[1] The spite motive and equilibrium behavior in auctions. Contributions to Economic Analysis & Policy, 2(1), Article 5, 2003.