DRAFT — NOT FOR DISTRIBUTION

The Dynamics of Macroeconomic Policy Instruments: A Monetary Flow Model

Ben Honig *and* Michael Honig
Electrical Engineering & Computer Science Department
Northwestern University

John Morgan
Haas School of Business
University of California at Berkeley

Ken Steiglitz
Computer Science Department
Princeton University

September 3, 2015

1 A Continuous-time, all-cash, no-escrow model

To get started we consider a very simple continuous-time model, an all-cash model with no escrow. Balloon payments are made from revenue. The flow variables are functions of continuous time t, and all the money that flows is cash. Figure 1 shows the corresponding flow graph.

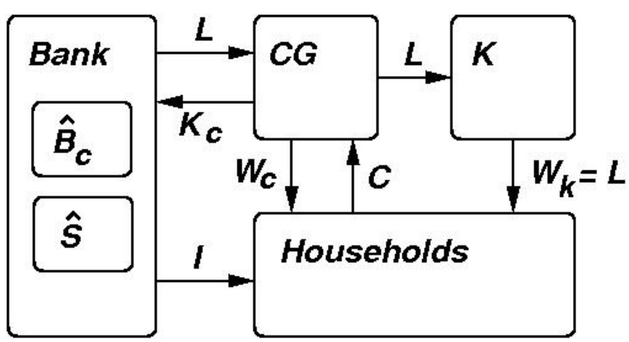

Figure 1: The credit flow transfers in the model. All flow rates are in dollars-per-unit-time.

Plain Latin letters, like S, C, d etc., will now represent flow *rates*, in dollars-per-unit-time. Absolute values in dollars will be denoted with a hat, like the Household account $\hat{S}(t)$, the supply of loanable funds $\hat{S}_{lf}(t)$, and the demand for loanable funds $\hat{D}_{lf}(t)$. The total amount of cash in the system, the monetary base, is denoted by G_0, and is constant throughout any simulation of the flow graph, except when the Fed explicitly injects or extracts cash. The monetary base is also an absolute dollar amount, as is the total cash in the banking sector, $\hat{B}_c$. We loosely speak of "cash at the bank", and similarly for the consumer-good, capital, and household sectors.

The dimensions of every quantity must be correct. A variable like the loan flow, L, for example, has units dollars-per-unit-time, and must be multiplied by something with the units of time to yield an absolute amount in dollars, and so on.

We adjust the loan rate r to the difference between the demand and supply of loanable funds:

$$\dot{r}(t) = k_r \cdot (\hat{D}_{lf}(t) - \hat{S}_{lf}(t)) \ , \tag{1}$$

where k_r is the rate adjustment parameter.

We assume for now that the loans are paid off in full with interest at maturity (a *balloon payment*), and that the rate is determined contractually *at the time the loan is initiated.* That is, we treat the loans as fixed-rate zero-coupon bonds. Total loans $L(t)$ will be distributed according to a discrete distribution where the weight $w_i \geq 0$ is associated with loans of term τ_i, $\sum_{i=1}^{\nu} w_i = 1$. An infinitesimal loan $L_i(t)dt$ initiated at time t when the rate is $r(t)$, and pays $L_i(t) \exp\big(r(t)\tau_i\big)dt$ at time $t + \tau_i$ as a balloon payment. The continuous-time flow corresponding to total capital expenditures is therefore

$$K_c(t) = \sum_{i=1}^{\nu} L_i(t - \tau_i) e^{r(t-\tau_i)\tau_i} \triangleq \ \mathcal{K}_c\{L(t)\} \ , \tag{2}$$

a linear functional of the loan flow $L(t)$. This is the sum of all balloon payments that come due at time t.

To relate absolute cash values to flows and vice-versa, we need to establish a time scale. To do this, let the total credit $\hat{X}(t)$ at time t be defined as the total of all outstanding loans:

$$\hat{X}(t) = \sum_{i=1}^{\nu} \int_{t-\tau_i}^{t} L_i(t) \, d\tau \triangleq \ \hat{\mathcal{X}}\{L(t)\} \ . \tag{3}$$

In equilibrium,

$$\hat{\mathcal{X}} = \overline{\tau} \cdot L \ , \tag{4}$$

where we define

$$\overline{\tau} \triangleq \sum_{i=1}^{\nu} w_i \tau_i \ , \tag{5}$$

the weighted average term of a loan. We denote the equilibrium value of a state variable by omitting explicit dependence on time t. When we write $\hat{X} = \overline{\tau} L$ we are converting from the equilibrium flow rate L to the absolute equilibrium cash value $\hat{X}$. The time $\overline{\tau}$ thus represents the effective time scale for the "average lifetime of credit", and we can also write, for example, $\hat{W}_c = \overline{\tau} \cdot W_c$, and similarly for other state variables.

We will also need the equilibrium value of K_c, which is

$$K_c = \overline{\omega} \cdot L \ , \tag{6}$$

where we define

$$\overline{\omega} \triangleq \sum_{i=1}^{\nu} w_i e^{r\tau_i} \ . \tag{7}$$

Whereas $\overline{\tau}$ has the dimensions of time, $\overline{\omega}$ is dimensionless; it represents the average appreciation of a loan.

The flow of interest that is paid by the CG sector, $I(t)$, is

$$I(t) = K_c(t) - L(t) - B_c(t) \triangleq \mathcal{I}\{L(t)\} \ , \tag{8}$$

where $B_c(t)$ is the net flow of cash into the Bank reservoir of cash, $\hat{B}_c(t)$. We can move between flow and absolute Bank cash simply by integrating or differentiating:

$$\hat{B}_c(t) = \int_{-\infty}^{t} B_c(t) \, dt \tag{9}$$

$$B_c(t) = \dot{\hat{B}}_c(t) \ , \tag{10}$$

and similarly for other variables. In equilibrium, $\hat{B}_c(t)$ is a constant, which implies that $B_c = 0$.

The supply of loanable funds is the difference between the Bank cash and the required reserve:

$$\hat{S}_{lf}(t) = \hat{B}_c(t) - f\hat{S}(t) \ , \tag{11}$$

where $\hat{S}(t)$ is the Household Savings, a demand-deposit account that accrues interest at the rate $s(t)$, meaning that

$$I(t) = s(t)\hat{S}(t) \ . \tag{12}$$

2 All-cash equilibrium

We will Solve for the state variables in terms of known functions of r and s, looking for two equations in those two unknowns.

$$\hat{S}_{lf} = \hat{B}_c - f\hat{S} = \hat{D}_{lf} = \overline{\tau}L \ , \tag{13}$$

or, normalizing by L,

$$\hat{S}_{lf}/L = \overline{\tau}/\lambda - f\hat{S}/L = \hat{D}_{lf}/L = \overline{\tau} \ , \tag{14}$$

where we choose the dimensionless parameter $\lambda \triangleq \overline{\tau}L/\hat{B}_c < 1$. From this we can solve for $\hat{S}/L$:

$$\frac{\hat{S}}{L} = \frac{\overline{\tau}/f}{1/\lambda - 1} \ . \tag{15}$$

From $I = K_c - L$ we get

$$I/L = \overline{\varpi} - 1 \ , \tag{16}$$

and, since $I/L = s\hat{S}/L$,

$$\boxed{s = f(1/\lambda - 1)(\overline{\varpi} - 1)/\overline{\tau}} \quad \text{...Condition 1} \ , \tag{17}$$

a relation between r and s.

We next stipulate that the CG Firms allocate $h(\beta_K, \beta_L)C$ of their revenue to capital; in other words, $K_c = hC$. That determines $C = K_c/h$ and, from credit flow-balance, $W_c = C - K_c = (1/h - 1)K_c$. (When $h = 1/2$, revenue is split equally between wages W_c and capital expenditures K_c.) We aim for $\hat{S} = g(s)\mathcal{W}$, where the Wealth is defined by

$$\hat{\mathcal{W}} = \overline{\tau}W_c + \overline{\tau}W_k + \hat{S} \ , \tag{18}$$

scaling the income flows to totals using $\overline{\tau}$. With the Savings/Consumption split determined by $g(s)$, this implies

$$\hat{S} = \frac{\overline{\tau}}{1-g}(W_c + L) \ , \tag{19}$$

using L for W_k and using g to denote the function $g(s)$ evaluated at equilibrium s. Finally, using $\hat{S} = (\overline{\varpi} - 1)L/s$ and $W_c = (1/h - 1)\overline{\varpi}L$ gives a second condition relating r and s:

$$\boxed{s = \frac{(1-g)(\overline{\varpi} - 1)/\overline{\tau}}{[\overline{\varpi}(1/h - 1) + 1]}} \quad \text{...Condition 2} \ . \tag{20}$$

2.1 Numerical example

Figure 2.1 shows the predicted equilibrium values r and s vs. f in the continuous-time all-cash case. The parameters are $e_1 = 200$, $\tau_{max} = 200$, $\lambda = 0.35$.

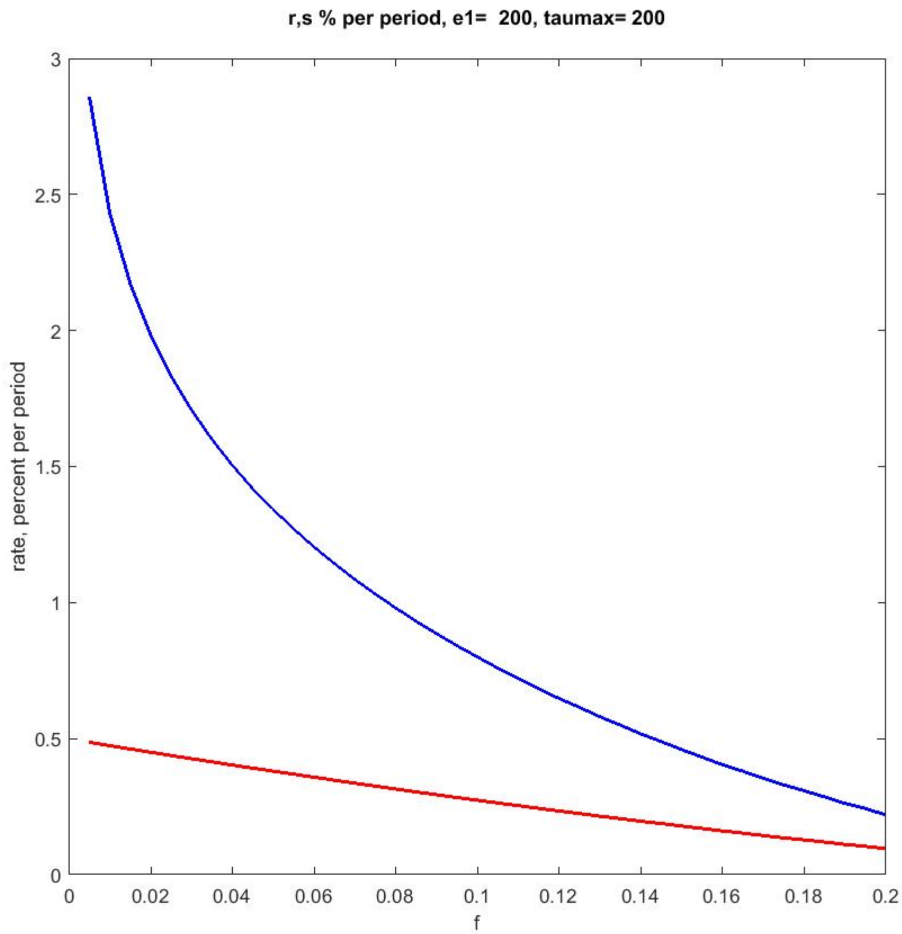

Figure 2: Predicted equilibrium values r and s vs. f in the continuous-time all-cash case; $e_1 = 200$, $\tau_{max} = 200$, $\lambda = 0.35$.

Dummy reference [1].

References

[1] Dummy reference to make LaTex happy.